



Exercise 3.1

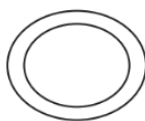
1. Given here are some figures



1



2



3



4



5



6



7



8

Classify each of them on the basis of the following.

- (a) Simple curve
- (b) Simple closed curve
- (c) Polygon
- (d) Convex polygon
- (e) Concave polygon

Answer:

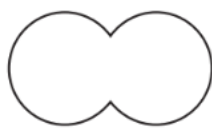
(a) Simple curve - A simple curve is a curve that does not cross itself and can be both open and closed.



1



2



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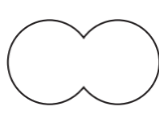
(b) Simple closed curve - In simple closed curves the shapes are closed by line segments or by a curved line.



1



2



5



6



7

(c) Polygon - A simple closed curve made up of only line segments is called a polygon.



1



2



(d) Convex polygon - A convex polygon is a closed figure where all its interior angles are less than 180° and the vertices are pointing outwards.



(e) Concave polygon - A concave polygon is defined as a polygon with one or more interior angles greater than 180°



2. How many diagonals does each of the following have?

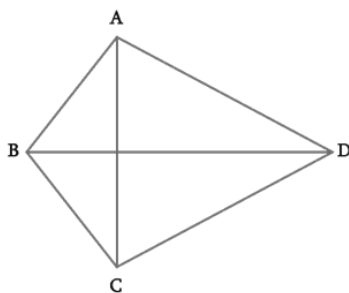
(a) A convex quadrilateral

(b) A regular hexagon

(c) A triangle

Answer: A diagonal is a line segment connecting two non-consecutive vertices of a polygon.

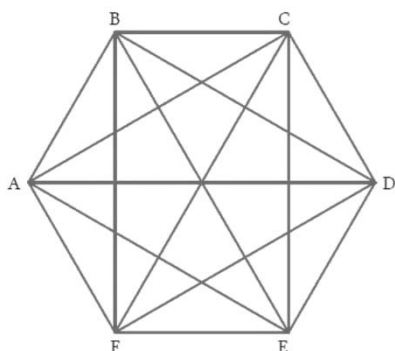
Draw the given polygon and mark the vertices and then, draw lines joining the two non-consecutive vertices. From this, we can calculate the number of diagonals.



(a) Convex quadrilateral - A convex quadrilateral has two diagonals

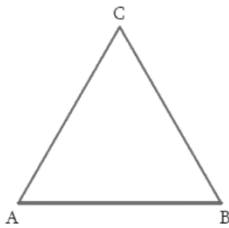
Here, AC and BD are two diagonals.

(b) A regular hexagon



(a) Regular Hexagon

Here, the diagonals are AD, AE, BD, BE, FC, FB, AC, EC, and FD. Totally there are 9 diagonals.

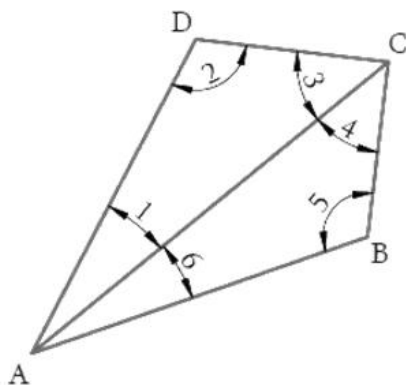


(c) A triangle

A triangle has no diagonal because there are no two non-consecutive vertices.

3. What is the sum of the measures of the angles of a convex quadrilateral? Will this property hold if the quadrilateral is not convex? (Make a non-convex quadrilateral and try!)

Answer: Let ABCD be a convex quadrilateral. We draw a diagonal AC which divides the quadrilateral into two triangles.



ABCD is a convex quadrilateral made of two triangles $\triangle ABC$ and $\triangle ADC$.

We know that the sum of the angles of a triangle is 180 degrees. So:

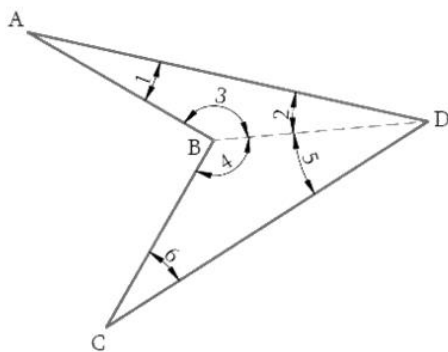
$$\angle 6 + \angle 5 + \angle 4 = 180^\circ \text{ [sum of the angles of } \triangle ABC = 180^\circ]$$

$$\angle 1 + \angle 2 + \angle 3 = 180^\circ \text{ [sum of the angles of } \triangle ADC = 180^\circ]$$

Adding we get,

$$\angle 6 + \angle 5 + \angle 4 + \angle 1 + \angle 2 + \angle 3 = 180^\circ + 180^\circ = 360^\circ$$

Hence, the sum of measures of the angles of a convex quadrilateral is 360° . Yes, even if a quadrilateral is not convex then, this property applies.



Let ABCD be a non-convex or a concave quadrilateral. Join BD, which also divides the quadrilateral into two triangles.

Using the angle sum property of a triangle, on triangles $\triangle ABD$ and $\triangle BCD$

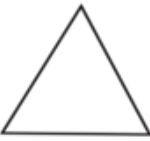
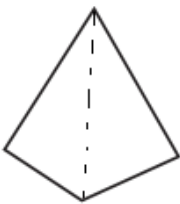
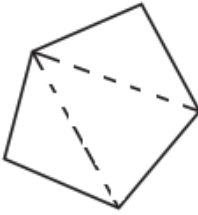
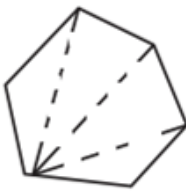
$$= 180^\circ + 180^\circ \text{ [sum of angles of triangles } \triangle ABD \text{ and } \triangle BCD]$$

$$= 360^\circ$$

Therefore, the sum of all the interior angles of this quadrilateral will also be 360° .



4. Examine the table. (Each figure is divided into triangles and the sum of the angles deduced from that)

Figure				
Side	3	4	5	6
Angle Sum	$1 \times 180^\circ$ $= (3 - 2) \times 180^\circ$	$2 \times 180^\circ$ $= (4 - 2) \times 180^\circ$	$3 \times 180^\circ$ $= (5 - 2) \times 180^\circ$	$4 \times 180^\circ$ $= (6 - 2) \times 180^\circ$

What can you say about the angle sum of a convex polygon with number of sides?

- (a) 7 (b) 8 (c) 10 (d) n

Answer: From the table, it can be observed that the angle sum of a convex polygon of n sides is $(n - 2) \times 180$

- (a) When $n = 7$

Then angle sum of a polygon $= (n - 2) \times 180^\circ = (7 - 2) \times 180^\circ = 5 \times 180^\circ = 900^\circ$

- (b) When $n = 8$

Then angle sum of a polygon $= (n - 2) \times 180^\circ = (8 - 2) \times 180^\circ = 6 \times 180^\circ = 1080^\circ$

- (c) When $n = 10$

Then angle sum of a polygon $= (n - 2) \times 180^\circ = (10 - 2) \times 180^\circ = 8 \times 180^\circ = 1440^\circ$

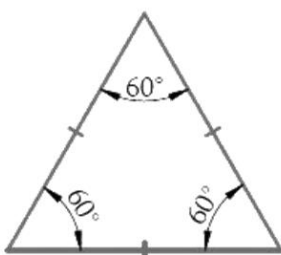
- (d) When $n = n$

Then angle sum of a polygon $= (n - 2) \times 180^\circ$

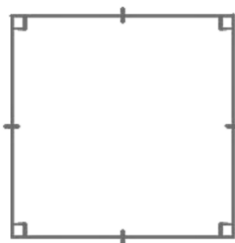
5. What is a regular polygon?

State the name of a regular polygon of (i) 3 sides (ii) 4 sides (iii) 6 sides

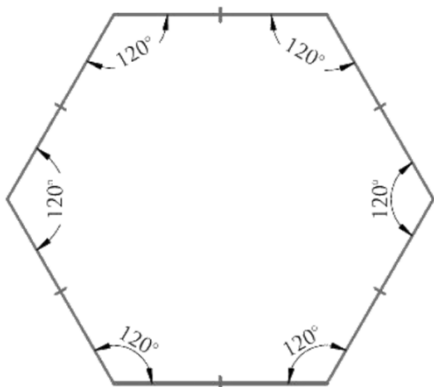
Answer: Regular polygon - A polygon having all sides of equal length and the interior angles of equal measure is known as a regular polygon i.e. a regular polygon is both 'equiangular' and 'equilateral'.



- (a) **3 sides** = polygons having three sides is called a triangle. A regular 3-sided polygon is an equilateral triangle.

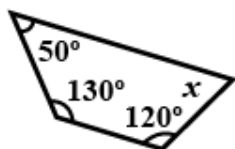


(b) 4 sides = polygons having four sides is called a quadrilateral. A regular 4 sided polygon is a square.

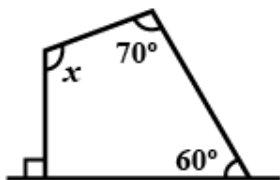


(c) 6 sides = a regular polygon having six sides is called a Hexagon.

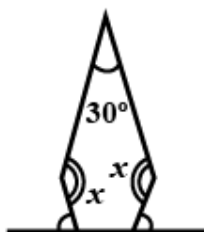
6. Find the angle measure x in the following figures:



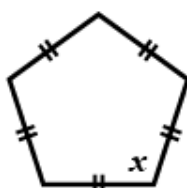
(a)



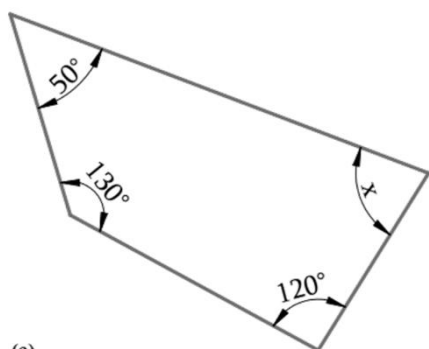
(b)



(c)



(d)



(a)

Answer: (a) The given figure has 4 sides and hence it is a quadrilateral

Using the angle sum property of a quadrilateral,

$$50^\circ + 130^\circ + 120^\circ + x = 360^\circ$$

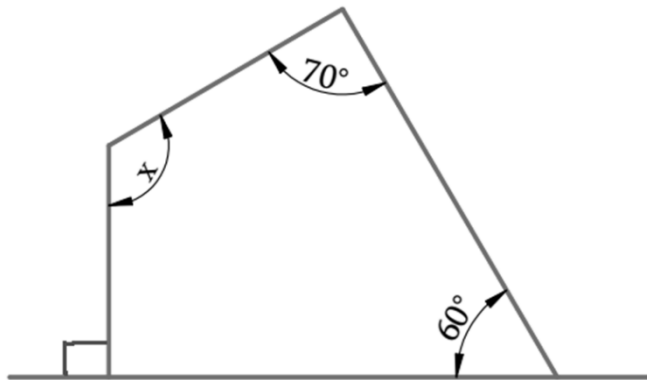
$$300^\circ + x = 360^\circ$$

$$x = 360^\circ - 300^\circ$$



$$x = 60^\circ$$

(b) Using the angle sum property of a quadrilateral.



The line is perpendicular to the base line
hence, the angle formed is 90° .

$$90^\circ + 60^\circ + 70^\circ + x = 360^\circ$$

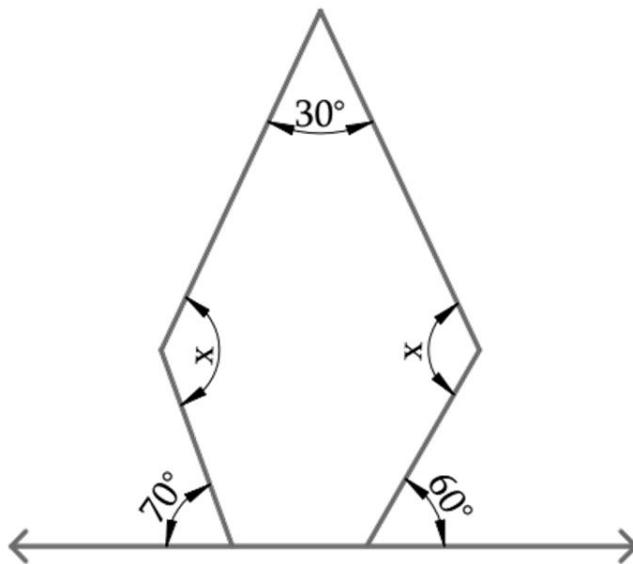
$$220^\circ + x = 360^\circ$$

$$220^\circ + x = 360^\circ$$

$$x = 360^\circ - 220^\circ$$

$$x = 140^\circ$$

(c) The given figure is a pentagon (number of sides =5)



(c)

$$\text{Angle sum of a polygon} = (n - 2) \times 180^\circ$$

$$= (5 - 2) \times 180^\circ$$

$$= 3 \times 180^\circ$$

$$= 540^\circ$$

Sum of the interior angle of pentagon is 540° .

Angles at the bottom are linear pairs.

Thus, First base interior angle,

$$= 180^\circ - 70^\circ \text{ (angle of straight line is } 180^\circ \text{)}$$

$$= 110^\circ$$

Also, Second base interior angle,

$$= 180^\circ - 60^\circ$$

$$= 120^\circ$$

$$\text{Angle sum property of a polygon} = (n - 2) \times 180^\circ$$

$$\rightarrow 30^\circ + x + 110^\circ + 120^\circ + x = 540^\circ$$

$$\rightarrow 2x + 260^\circ = 540^\circ$$

$$\rightarrow 2x = 540^\circ - 260^\circ$$

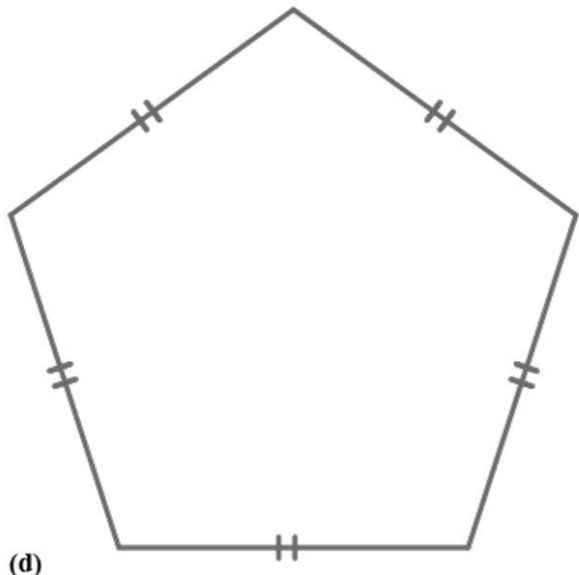
$$\rightarrow 2x = 280^\circ$$



$$\rightarrow x = 280^\circ/2$$

Therefore, $x = 140^\circ$

(d) The given figure is pentagon (number of sides =5)



Sum of the interior angle of pentagon is 540°

Angle sum of a polygon

$$= (n - 2) \times 180^\circ$$

$$= (5 - 2) \times 180^\circ$$

$$= 3 \times 180^\circ$$

$$= 540^\circ$$

Let each angle of the polygon be x since it is a regular pentagon.

$$\text{Angle sum of a polygon} = x + x + x + x + x = 540^\circ$$

$$5x = 540^\circ$$

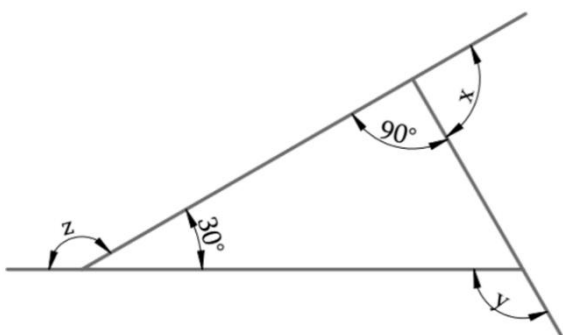
$$x = 540^\circ/5$$

$$x = 108^\circ$$

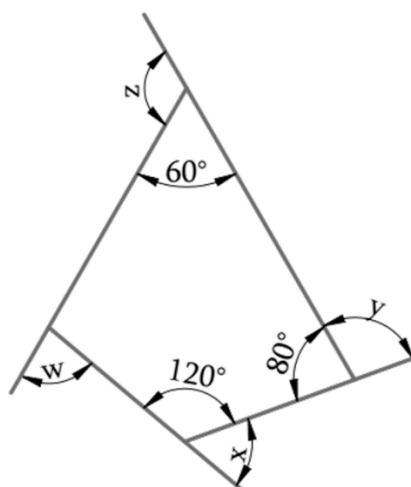
The given pentagon is a regular polygon i.e. it is both 'equilateral' and 'equiangular'. Thus, the measure of each interior angle of the pentagon is equal.

Hence each interior angle is 108°

7. a) Find $x + y + z$

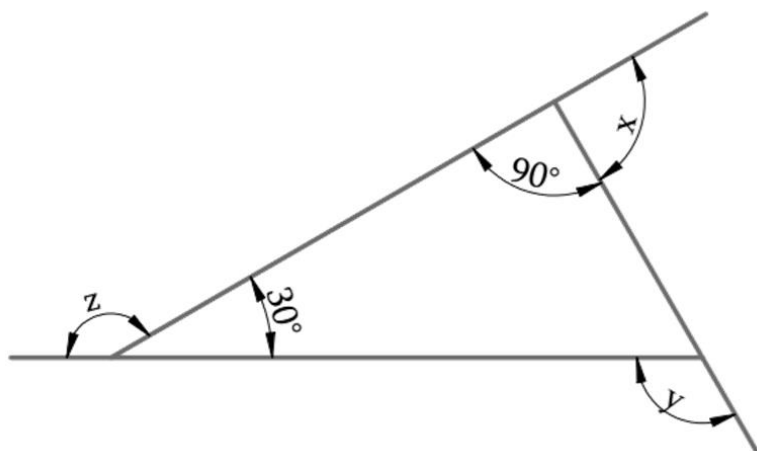


b) Find $x + y + z + w$





Answer: The unknown angles can be estimated by using the angle sum property of a quadrilateral and triangle accordingly.



(a)

Sum of linear pair of angles is $= 180^\circ$

$$x + 90^\circ = 180^\circ \text{ (Linear pair)}$$

$$x = 180^\circ - 90^\circ$$

$$x = 90^\circ$$

Solving for z

$$z + 30^\circ = 180^\circ \text{ (Linear pair)}$$

$$z = 180^\circ - 30^\circ$$

$$z = 150^\circ$$

Solving for y

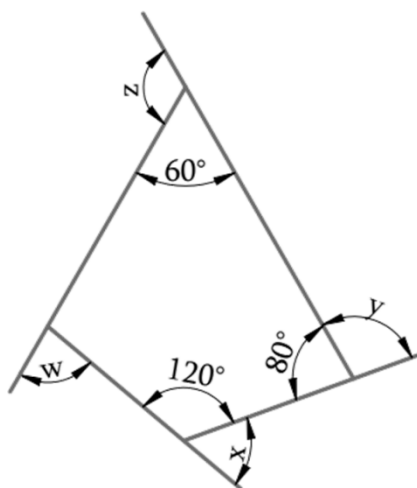
$$y = 90^\circ + 30^\circ \text{ (Exterior angle theorem)}$$

$$y = 120^\circ$$

$$x + y + z = 90^\circ + 120^\circ + 150^\circ$$

$$= 360^\circ$$

(b)



The sum of the measures of all the interior angles of a quadrilateral is 360° . Using the angle sum property of a quadrilateral,

Let n be the fourth interior angle of the quadrilateral.

$$60^\circ + 80^\circ + 120^\circ + n = 360^\circ$$

$$260^\circ + n = 360^\circ$$

$$n = 360^\circ - 260^\circ$$

$$n = 100^\circ$$

Sum of linear pair of angles is 180°

$$w + 100^\circ = 180^\circ \text{ ----- 1}$$

$$x + 120^\circ = 180^\circ \text{ ----- 2}$$

$$y + 80^\circ = 180^\circ \text{ ----- 3}$$



$$z + 60^\circ = 180^\circ \text{ ----- 4}$$

Adding equation (1), (2), (3) and (4),

$$w + 100^\circ + x + 120^\circ + y + 80^\circ + z + 60^\circ = 180^\circ + 180^\circ + 180^\circ + 180^\circ$$

$$w + x + y + z + 360^\circ = 720^\circ$$

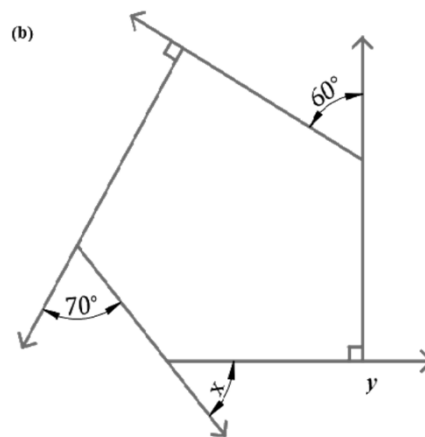
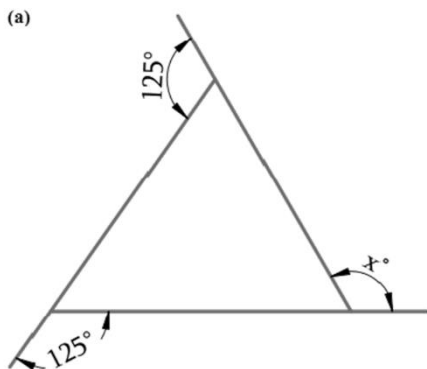
$$w + x + y + z = 720^\circ - 360^\circ$$

$$w + x + y + z = 360^\circ$$

Thus, the sum of the measures of the external angles of any polygon is 360° .

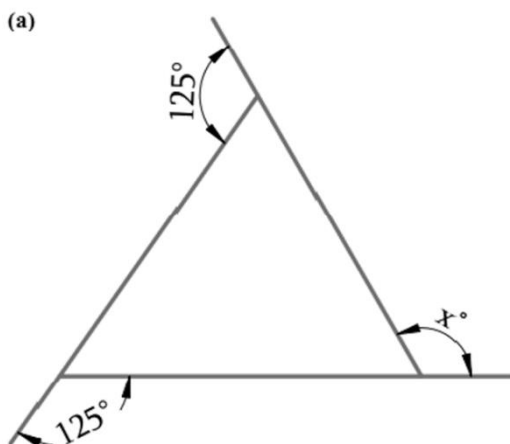
Exercise 3.2

1. Find x in the following figures



Answer: We know that the sum of the measures of the exterior angles of any polygon is 360° . So we will equate all the angle sum to 360° and find out the unknown angle.

(a)

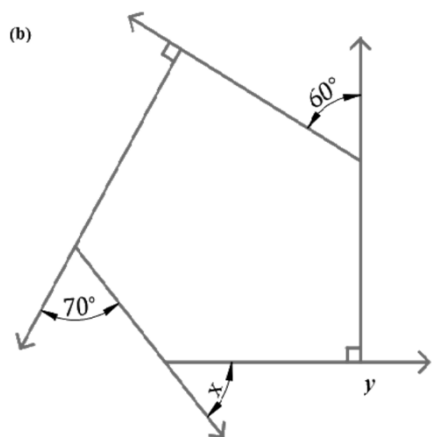


Sum of the measures of the external angles,

$$125^\circ + 125^\circ + x^\circ = 360^\circ$$

$$250^\circ + x^\circ = 360^\circ$$

$$x^\circ = 110^\circ$$



(b) $y = 180^\circ - 90^\circ$ [linear pair angles]

$y = 90^\circ$

Sum of the measures of the external angles is 360° ,

$60^\circ + 90^\circ + 70^\circ + x + y = 360^\circ$

$60^\circ + 90^\circ + 70^\circ + x + 90^\circ = 360^\circ$

$310^\circ + x = 360^\circ$

$x = 50^\circ$

2. Find the measure of each exterior angle of a regular polygon of

(i) 9 sides (ii) 15 sides

Answer: Irrespective of the number of sides of a regular polygon, the measure of each exterior angle is equal and the sum of the measure of all the exterior angles of the regular polygon is equal to 360° .

(i) 9 sides

The total sum of all exterior angles = 360°

Each exterior angle = Sum of exterior angles / Number of sides

$$= \frac{360^\circ}{9}$$

$= 40^\circ$

Each exterior angle = 40°

(ii) 15 sides

The total sum of all exterior angles = 360°

Each exterior angle = Sum of exterior angles / Number of sides

$$= \frac{360^\circ}{15} = 24^\circ$$

Each exterior angle = 24°

3. How many sides does a regular polygon have if the measure of an exterior angle is 24° ?

Answer: Total sum of all the exterior angles of the regular polygon = 360°

Let number of sides be = n .

Measure of each exterior angle = 24°



Number of sides = Sum of exterior angles / each exterior angle

$$= \frac{360^0}{24} = 15$$

Thus, the regular polygon has 15 sides.

4. How many sides does a regular polygon have if each of its interior angles is 165° ?

Answer: Total sum of all the exterior angles of a regular polygon = 360°

Let number of sides be = n.

Measure of each interior angle = 165°

Measure of each exterior angle = $180^\circ - 165^\circ = 15^\circ$ [Since, an interior and an exterior angle forms a linear pair]

$$\text{Number of sides} = \frac{\text{Sum of exterior angles}}{\text{Each exterior angle}}$$

$$= \frac{360^0}{15} = 24$$

Hence, the regular polygon has 24 sides

5. (a) Is it possible to have a regular polygon with measure of each exterior angle as 22° ?

(b) Can it be an interior angle of a regular polygon? Why?

Answer: We know that, total sum of all the exterior angles of a regular polygon = 360°

Let the number of sides be = n.

(a) Measure of each exterior angle = 22°

$$\text{Number of sides} = \frac{\text{Sum of exterior angles}}{\text{Each exterior angle}}$$

$$= \frac{360^0}{22^0}$$

$$= 16.36$$

Thus, we cannot have a regular polygon with an exterior angle of 22° as the number of sides is not a whole number.

(b) Measure of each interior angle = 22°

Measure of each exterior angle = $(180 - 22)^\circ = 158^\circ$



$$\text{Number of sides} = \frac{\text{Sum of exterior angles}}{\text{Each exterior angle}}$$

$$= \frac{360^\circ}{1582^\circ} = 2.27$$

Thus, we cannot have a regular polygon with an interior angle of 22° as the number of sides is not a whole number

6. (a) What is the minimum interior angle possible for a regular polygon? Why?

(b) What is the maximum exterior angle possible for a regular polygon?

Answer:

(a) Consider a regular polygon having the least number of sides (i.e., an equilateral triangle)

We know that the sum of all the angles of a triangle = 180°

$$x + x + x = 180^\circ$$

$$3x = 180^\circ$$

$$x = \frac{360^\circ}{3}$$

$$x = 60^\circ$$

Thus, the minimum interior angle possible for a regular polygon = 60°

(b) We know that the exterior angle and an interior angle will always form a linear pair. Thus, the exterior angle will be maximum when the interior angle is minimum.

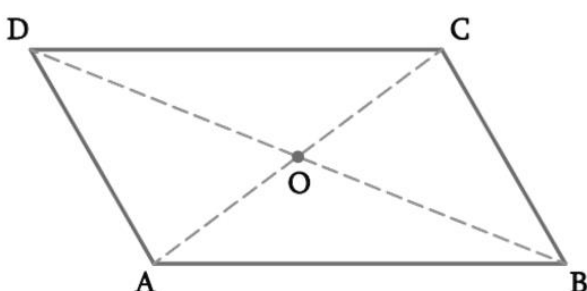
Consider the interior angle to be 60° since an equilateral triangle is a regular polygon having maximum exterior angle because it consists of the least number of sides

$$\text{Exterior angle} = 180^\circ - 60^\circ = 120^\circ$$

Therefore, the maximum exterior angle possible for a regular polygon is 120° .

Exercise 3.3

1. Given a parallelogram ABCD. Complete each statement along with the definition or property



used

(i) AD =

(ii) $\angle DCB = \dots\dots$

(iii) OC =

(iv) $m \angle DAB + m \angle CDA = \dots\dots$



Answer:

(i) The opposite sides of a parallelogram are of equal length.

Thus, $AD = BC$

(ii) In a parallelogram, opposite angles are equal in measure

Thus, $\angle DCB = \angle DAB$

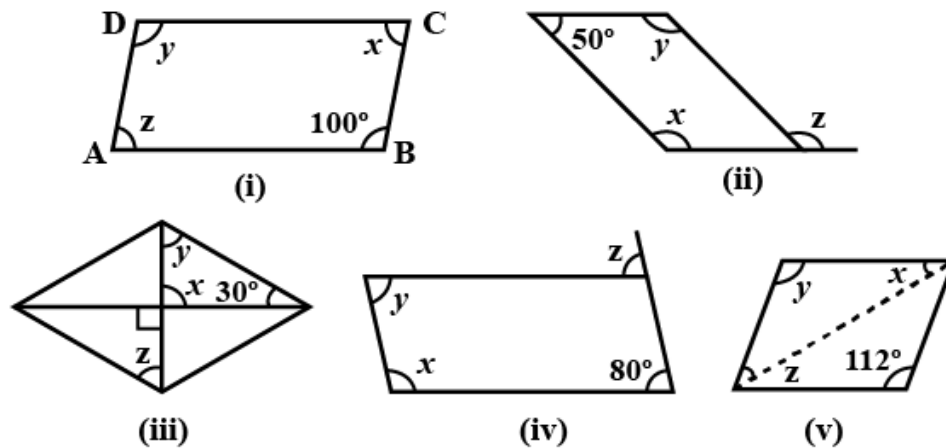
(iii) In a parallelogram, diagonals bisect each

Hence, $OC = OA$

(iv) In a parallelogram, adjacent angles are supplementary to each other.

Hence, $m\angle DAB + m\angle CDA = 180^\circ$

2. Consider the following parallelograms. Find the values of the unknowns x , y , z .



Answer:



(i) Since $\angle D$ is opposite to $\angle B$, so, $y = 100^\circ$
(Opposite angles of a parallelogram are equal)

$\angle C + \angle B = 180^\circ$ (The adjacent angles in a parallelogram are supplementary)

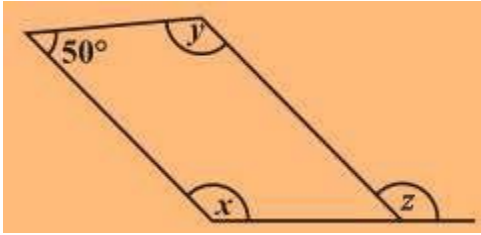
$x + 100^\circ = 180^\circ$ (The adjacent angles in a parallelogram are supplementary)

Therefore,

$$x = 180^\circ - 100^\circ = 80^\circ$$

$x = z = 80^\circ$ (Since opposite angles of a parallelogram are equal)

Thus, $x = 80^\circ$, $y = 100^\circ$, $z = 80^\circ$



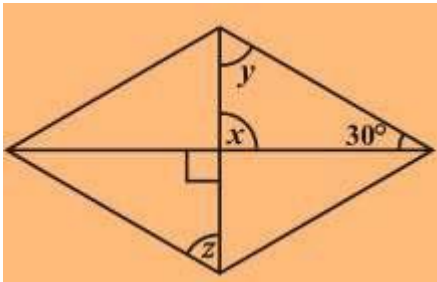
(ii) $x + 50^\circ = 180^\circ$ (The adjacent angles in a parallelogram are supplementary)

$$\begin{aligned}x &= 180^\circ - 50^\circ \\&= 130^\circ\end{aligned}$$

$x = y = 130^\circ$ (Since opposite angles of a parallelogram are equal)

$x = z = 130^\circ$ (Corresponding angles)

Thus, $x = y = z = 130^\circ$



(iii) $x + y + 30^\circ = 180^\circ$ (Angle sum property of a triangle)

$x = 90^\circ$ (Vertically opposite angle)

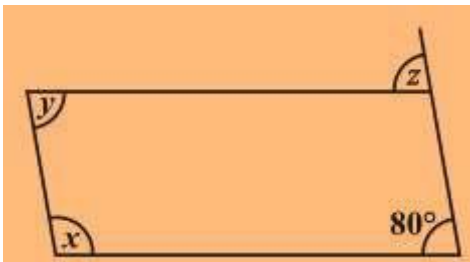
$$90^\circ + y + 30^\circ = 180^\circ$$

$$y + 120^\circ = 180^\circ$$

$$y = 60^\circ$$

$z = y = 60^\circ$ (Alternate interior angles are equal)

$x = 90^\circ, y = z = 60^\circ$



(iv) $z = 80^\circ$ (Corresponding angles)

$y = 80^\circ$ (Since opposite angles of a parallelogram are equal)

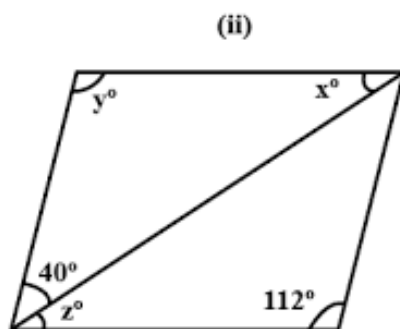
$x + y = 180^\circ$ (Adjacent angles of a parallelogram are supplementary)

$$x + 80^\circ = 180^\circ$$

$$x = 180^\circ - 80^\circ$$

$$x = 100^\circ$$

Thus, $x = 100^\circ, y = 80^\circ, z = 80^\circ$



(v) $y = 112^\circ$ (Since opposite angles of a parallelogram are equal)

$x + y + 40^\circ = 180^\circ$ (Angle sum property of a triangle)

$$x + 112^\circ + 40^\circ = 180^\circ$$

$$x + 152^\circ = 180^\circ$$

$$x = 180^\circ - 152^\circ$$

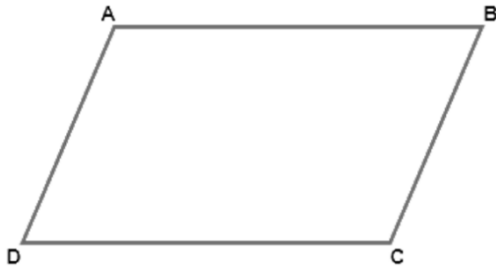


$$x = 28^\circ$$

$$z = x = 28^\circ \text{ (Alternate interior angles)}$$

$$\text{Thus, } x = 28^\circ, y = 112^\circ, z = 28^\circ$$

3. Can a quadrilateral ABCD be a parallelogram if



(i) $\angle D + \angle B = 180^\circ$?

(ii) $AB = DC = 8 \text{ cm}$, $AD = 4 \text{ cm}$ and $BC = 4.4 \text{ cm}$?

(iii) $\angle A = 70^\circ$ and $\angle C = 65^\circ$?

Answer:

(i) Using the angle sum property of a quadrilateral,

$$\angle A + \angle B + \angle D + \angle C = 360^\circ$$

$$\angle A + \angle C + 180^\circ = 360^\circ \text{ (Since it's given that } \angle D + \angle B = 180^\circ \text{)}$$

$$\angle A + \angle C = 360^\circ - 180^\circ$$

$$\angle A + \angle C = 180^\circ \text{ (Opposite angles should also be of same measures.)}$$

For $\angle D + \angle B = 180^\circ$, is a parallelogram.

If the following conditions are fulfilled, then ABCD is a parallelogram. The sum of the measures of the adjacent angles should be 180° and opposite angles should also be of the same measure.

Hence, using the given condition $\angle D + \angle B = 180^\circ$ we can say that yes, it may or may not be a parallelogram.

(ii) Property of parallelogram: The opposite sides of a parallelogram are of equal length.

Here, $AD = 4 \text{ cm}$ and $BC = 4.4 \text{ cm}$

Opposite sides AD and BC are of different lengths. So, ABCD is not a parallelogram

(iii) Property of a parallelogram: In a parallelogram opposite angles are equal.

So, $\angle A = 70^\circ$ and $\angle C = 65^\circ$

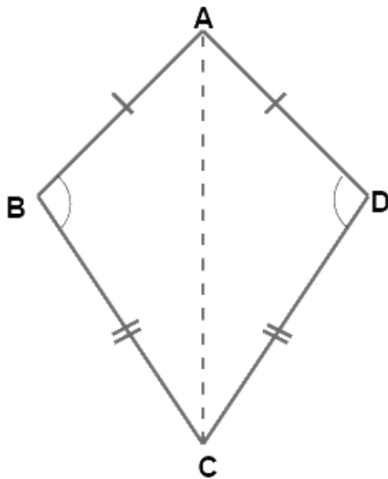
Opposite angles are not equal. So, ABCD is not a parallelogram.



4. Draw a rough figure of a quadrilateral that is not a parallelogram but has exactly two opposite angles of equal measure.

Answer:

ABCD is quadrilateral whose opposite angles are equal as shown below.



ABCD is a kite where $\angle B = \angle D$

In a kite, the angle between unequal sides is equal.

Let's prove the same.

Draw a line from A to C and we will get two triangles with common base AC.

In $\triangle ABC$ and $\triangle ADC$ we have,

$AB = AD$,

$BC = CD$

AC is common to both

$\triangle ABC \cong \triangle ADC$ (Using SSS congruence)

Hence corresponding parts of congruent triangles are equal.

Therefore $\angle B = \angle D$ (By CPCT)

However, the quadrilateral ABCD is not a parallelogram as the measures of the remaining pair of opposite angles, $\angle A$ and $\angle C$ are not equal since they form angles between equal sides.

5. The measures of two adjacent angles of a parallelogram are in the ratio 3:2. Find the measure of each of the angles of the parallelogram.

Answer: Given that the adjacent angles of a parallelogram are in the ratio 3:2.

Thus, the angles are $3x$ and $2x$ respectively.

We know that the sum of the measures of adjacent angles is 180° for a parallelogram.

$$\angle A + \angle B = 180^\circ$$

$$3x + 2x = 180^\circ$$

$$5x = 180^\circ$$

$$x = \frac{180^\circ}{5}$$

$$x = 36^\circ$$



Thus, one of the angles = $3x$

$$3(36^\circ) = 108^\circ$$

The other angle is $2x$

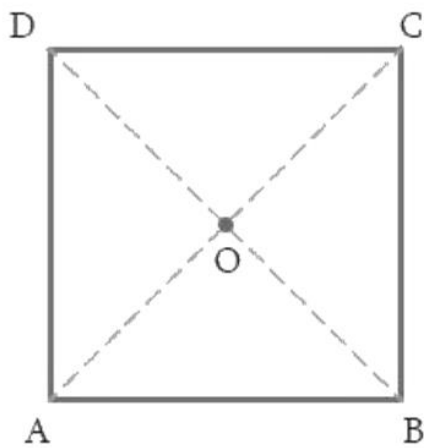
$$2(36^\circ) = 72^\circ$$

The other two angles are 72° and 108° since opposite angles of a parallelogram are equal.

Thus, the measures of the angles of the parallelogram are 108° , 72° , 108° , and 72°

6. Two adjacent angles of a parallelogram have equal measure. Find the measure of each of the angles of the parallelogram

Answer: Let ABCD be a parallelogram as shown below.



In parallelogram ABCD,

- $\angle A$ and $\angle D$ are supplementary since DC is parallel to AB and with transversal DA. (Adjacent angles of a parallelogram are supplementary)
- $\angle A$ and $\angle B$ are supplementary since AD is parallel to BC and with transversal BA. (Adjacent angles of a parallelogram are supplementary)

$$\text{Sum of adjacent angles} = 180^\circ$$

Let each adjacent angle be x

Since the adjacent angles in a parallelogram are supplementary,

$$x + x = 180^\circ$$

$$2x = 180^\circ$$

$$x = \frac{180^\circ}{2} = 90^\circ$$

Hence, each adjacent angle is 90 degrees

$$\angle A = \angle B = 90^\circ$$

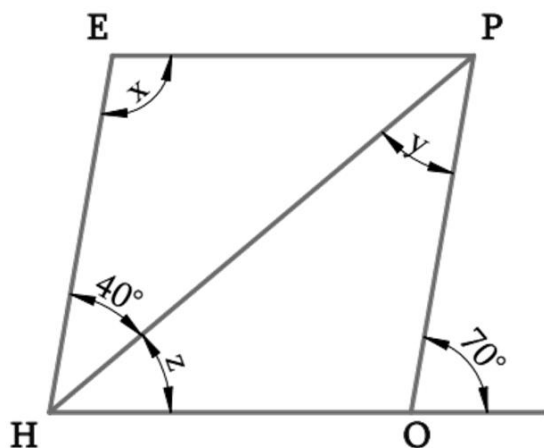
$$\angle C = \angle A = 90^\circ$$

$$\angle D = \angle B = 90^\circ$$

Thus, each angle of the parallelogram measures 90° .



7. The adjacent figure HOPE is a parallelogram. Find the angle measures x , y , and z . State the properties you use to find them



Answer:

According to the given figure,

$\angle HOP + 70^\circ = 180^\circ$ since they form a linear pair

$$\angle HOP = 180^\circ - 70^\circ$$

$$\angle HOP = 110^\circ$$

$\angle O = \angle E$ since opposite angles in a parallelogram are equal

$$\text{Thus, } x = 110^\circ$$

$\angle EHP = \angle HPO$ since they are alternate interior angles

$$\text{Thus, } y = 40^\circ$$

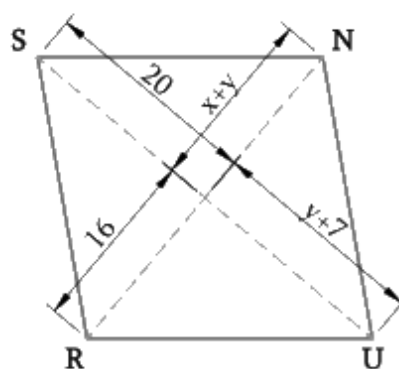
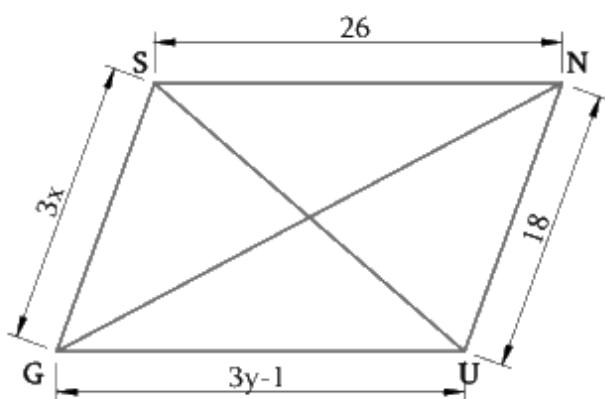
$z + 40^\circ = 70^\circ$ since they form corresponding angles

$$z = 70^\circ - 40^\circ$$

$$z = 30^\circ$$

$$\text{Thus, } x = 110^\circ, y = 40^\circ, z = 30^\circ$$

8. The following figures GUNS and RUNS are parallelograms. Find x and y . (Lengths are in cm)



Answer:

(i) GUNS

In a parallelogram, the opposite sides have equal lengths



In GUNS, $SG = NU$

$$3x = 18$$

$$x = \frac{18}{3}$$

$$x = 6$$

Also $SN = GU$

$$26 = 3y - 1$$

$$3y = 26 + 1$$

$$y = \frac{27}{3}$$

$$y = 9$$

Hence, the measures of x and y are 6 cm and 9 cm respectively in GUNS.

(ii) RUNS

The diagonals of a parallelogram bisect each other.

Thus, in parallelogram RUNS,

Considering diagonal SU, $y + 7 = 20$

$$y = 20 - 7$$

$$y = 13$$

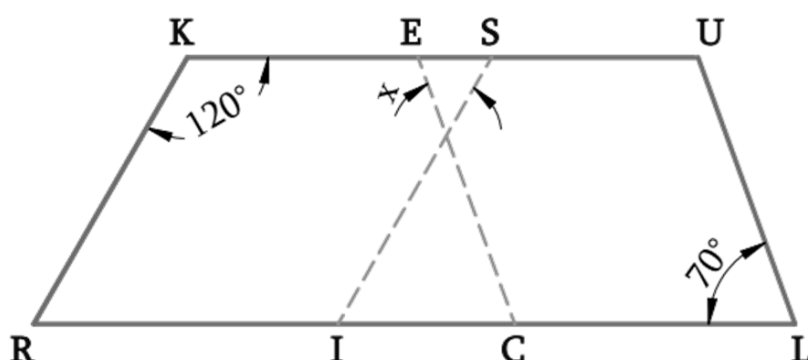
Considering diagonal RN, $x + y = 16$

$$x + 13 = 16$$

$$x = 3$$

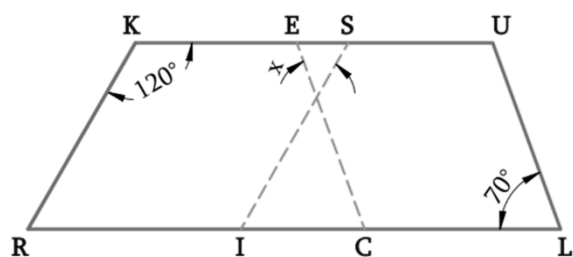
Hence, the measures of x and y are 3 cm and 13 cm respectively in RUNS.

9. In the above figure both RISK and CLUE are parallelograms. Find the value of x





Answer:



In parallelogram RISK

$\angle RKS + \angle ISK = 180^\circ$ (Adjacent angles are supplementary)

$$120^\circ + \angle ISK = 180^\circ$$

$$\angle ISK = 180^\circ - 120^\circ$$

$$\angle ISK = 60^\circ$$

$\angle I = \angle K$ (Opposite angles of parallelogram are equal)

$$= 120^\circ$$

In parallelogram CLUE,

$\angle L = \angle E$ (Opposite angles of parallelogram are equal)

$$= 70^\circ$$

The sum of the measures of all the interior angles of a triangle is 180° .

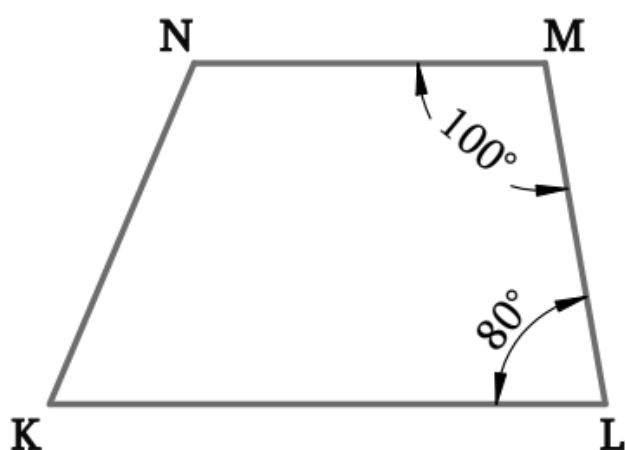
$$x + 60^\circ + 70^\circ = 180^\circ$$

$$x + 130^\circ = 180^\circ$$

$$x = 180^\circ - 130^\circ$$

$$x = 50^\circ$$

10. Explain how this figure is a trapezium. Which of its two sides are parallel? (Fig 3.32)



Answer: In the given figure KLMN

Since two pairs of adjacent angles that form pairs of consecutive interior angles are supplementary,

$$\text{i.e., } \angle L + \angle M = 180^\circ$$

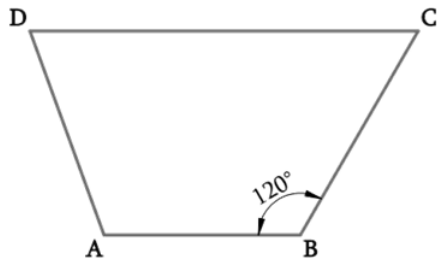


Thus, $80^\circ + 100^\circ = 180^\circ$

Therefore, NM is parallel to KL

Hence, KLMN is a trapezium with a pair of parallel sides KL and NM.

11. Find $m\angle C$ in Fig 3.33 if $\overline{AB} \parallel \overline{DC}$



Answer:

Figure ABCD is a Trapezium, in which AB is parallel to DC.

Here,

$\angle B + \angle C = 180^\circ$ (pair of adjacent angles are supplementary)

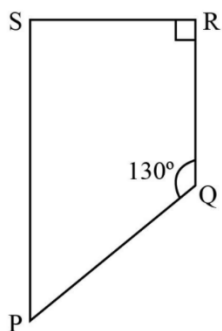
$$120^\circ + \angle C = 180^\circ$$

$$\angle C = 180^\circ - 120^\circ$$

$$\angle C = 60^\circ$$

Therefore, $m\angle C = 60^\circ$

12. Find the measure of $\angle P$ and $\angle S$ if $SP \parallel RQ$ in Fig 3.34. (If you find $m\angle R$, is there more than one method to find $m\angle P$?)



Answer: Given, SP is parallel to RQ and SR is the transversal drawn to these lines. Hence, SPQR is a trapezium.

$$\angle S + \angle R = 180^\circ$$

$$\angle S + 90^\circ = 180^\circ \text{ [Since, } \angle R = 90^\circ \text{ in the given figure]}$$



$$\angle S = 180^\circ - 90^\circ$$

$$\angle S = 90^\circ$$

Using the angle sum property of a quadrilateral,

$$\angle S + \angle P + \angle Q + \angle R = 360^\circ$$

$$90^\circ + \angle P + 130^\circ + 90^\circ = 360^\circ$$

$$\angle P + 310^\circ = 360^\circ$$

$$\angle P = 360^\circ - 310^\circ$$

$$\angle P = 50^\circ$$

Alternate Method:

$$\angle P + \angle Q = 180^\circ \text{ (adjacent angles with } SP \parallel RQ)$$

$$\angle P + 130^\circ = 180^\circ$$

$$\angle P = 180^\circ - 130^\circ$$

$$\angle P = 50^\circ$$

Also,

$$\angle S + \angle R = 180^\circ \text{ (adjacent angles)}$$

$$\angle S + 90^\circ = 180^\circ$$

$$\angle S = 180^\circ - 90^\circ$$

$$\angle S = 90^\circ$$

Exercise 3.4

1. State whether True or False.

- (a) All rectangles are squares**
- (b) All rhombuses are parallelograms**
- (c) All squares are rhombuses and also rectangles**
- (d) All squares are not parallelograms**
- (e) All kites are rhombuses**
- (f) All rhombuses are kites**
- (g) All parallelograms are trapeziums**
- (h) All squares are trapeziums**



Answer:

	Shapes	True / False	Reason
A	All rectangles are squares.	False	A rectangle need not have all sides equal hence it is not square.
B	All rhombuses are parallelograms	True	Since the opposite sides of a rhombus are equal and parallel to each other, it is also a parallelogram
C	All squares are rhombuses and are also rectangles.	True	All squares are rhombuses as all sides of a square are of equal lengths. All squares are also rectangles as each internal angle is 90 degrees.
D	All squares are not parallelograms.	False	The opposite sides of a parallelogram are of equal length hence squares with all sides equal are parallelograms.
E	All kites are Rhombuses.	False	A rhombus has all sides of equal length whereas a kite does not have all sides of equal length.
F	All rhombuses are kites.	True	Since all rhombuses have equal sides and diagonals bisect each other.
G	All parallelograms are trapeziums.	True	Since all trapeziums have a pair of parallel sides which is true for parallelograms as well.
H	All squares are Trapeziums.	True	All trapeziums have a pair of parallel sides, hence all squares can be trapezium.

2. Identify all the quadrilaterals that have:

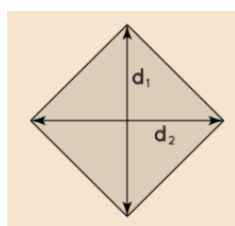
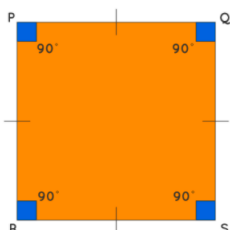
(a) four sides of equal length (b) four right angles

Answer:

(a) Four sides of equal length - Rhombus and Square are the quadrilaterals with four sides of equal length.

(b) Four right angles - Square and Rectangle are the quadrilaterals with four right angles.

Below are the diagrams shown for a square, rhombus, and a rectangle.





3. Explain how a square is (i) a quadrilateral

(ii) a parallelogram (iii) a rhombus (iv) a rectangle

Answer:

(i)	Quadrilateral	A square is a quadrilateral since it has four sides.
(ii)	Parallelogram properties- (i) Opposite sides are equal. (ii) Opposite angles are equal. (iii) Diagonals bisect one another.	A square is a parallelogram since it contains both pairs of opposite sides which are equal, opposite angles are equal and its diagonals bisect each other.
(iii)	Rhombus properties- i) A rhombus is a parallelogram with all sides of equal length. ii) The diagonals of a rhombus are perpendicular bisectors of one another.	A square is a rhombus since i) All four sides are of the same length. ii) The diagonals of a square are perpendicular bisectors of each other.
(iv)	Rectangle properties- Being a parallelogram, the rectangle has opposite sides of equal length and its diagonals bisect each other and opposite angles are 90 degrees each.	A square is a rectangle since each interior angle measures 90 degrees, opposite sides are of equal length and its diagonals bisect each other.

4. Name the quadrilaterals whose diagonals

(i) bisect each other (ii) are perpendicular bisectors of each other

(iii) are equal

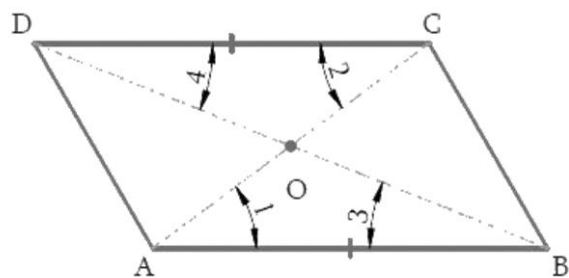
Answer:

(i) Bisect each other: The diagonals of a parallelogram, rhombus, rectangle and square bisect each other.

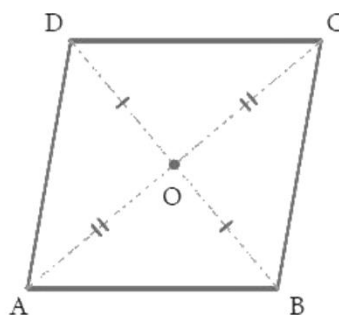
(Diagram below)



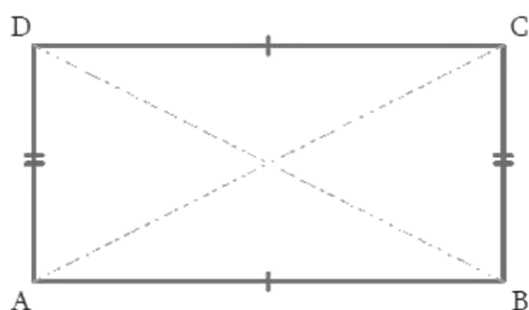
Parallelogram:



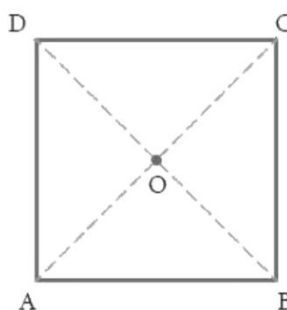
Rhombus:



Rectangle:

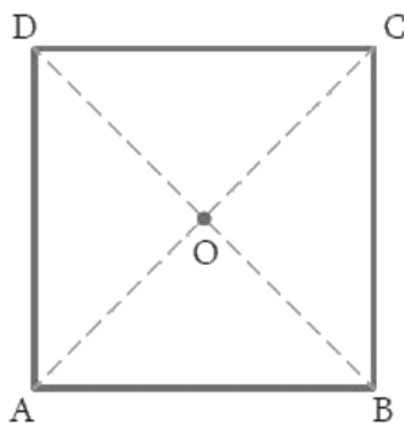
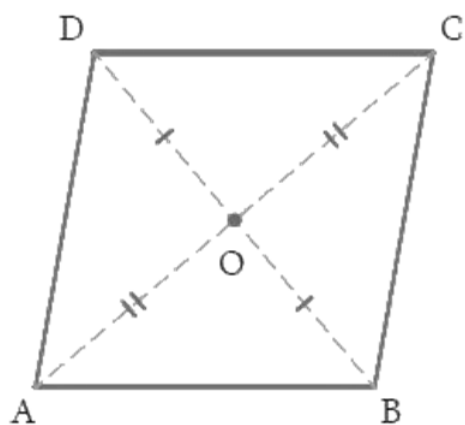


Square



(ii) are perpendicular bisectors of each other: The diagonals of a square and rhombus are perpendicular bisectors of each other.

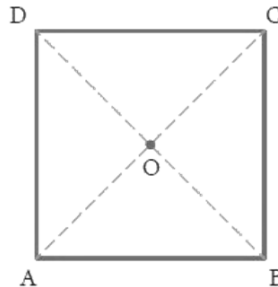
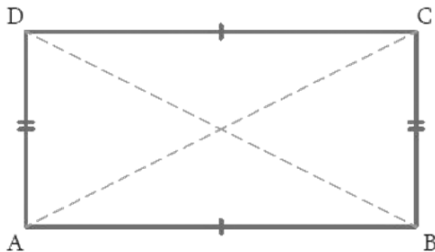
Rhombus and Square





(iii) **are equal:** The diagonals of a rectangle and square are equal.

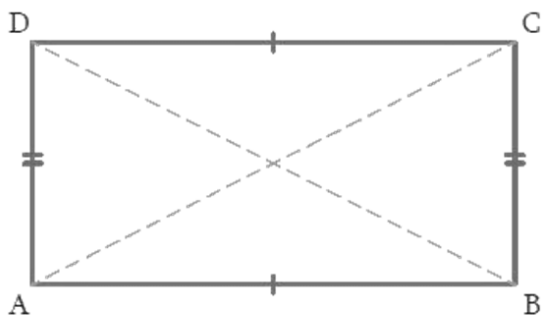
Rectangle and Square



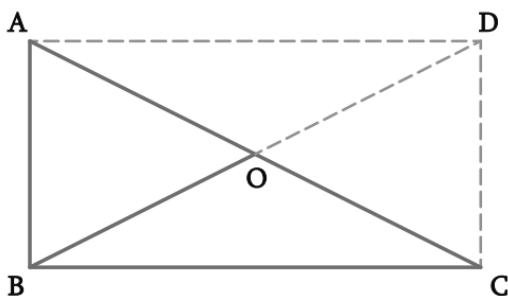
5. Explain why a rectangle is a convex quadrilateral.

Answer: Polygons that are convex have no portions of their diagonals in their exteriors. Also, all the interior angles of a convex quadrilateral are lesser than 180 degrees.

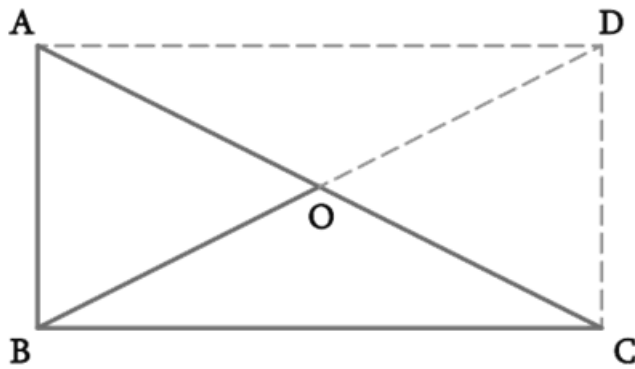
A rectangle is a convex quadrilateral since its vertex are raised and both of its diagonals lie in its interior. Also, each interior angle of a rectangle measures 90 degrees. Hence, none of the angles is a reflex angle. So, a rectangle is considered a convex quadrilateral.



6. ABC is a right-angled triangle and O is the mid point of the side opposite to the right angle. Explain why O is equidistant from A, B and C. (The dotted lines are drawn additionally to help you).



Answer: ABCD is a rectangle as opposite sides are equal and parallel to each other and all the interior angles measure 90° .



Thus, $AD \parallel BC$, $AB \parallel DC$ and $AD = BC$, $AB = DC$

In a rectangle, diagonals are of equal length and they bisect each other.

Hence, $AO = OC = BO = OD$

Thus, two right triangles make a rectangle where O is an equidistant point from A, B, C, and D because O is the mid-point of the two diagonals of the rectangle ABCD.

Therefore, O is equidistant from A, B, C, and D.