Exercise 8.1

Write the correct answer in each of the following:

1. Three angles of a quadrilateral are 75º, 90º and 75º. The fourth angle is

(A) 90^0 (B) 95^0 (C) 105^0 (D) 120^0

Answer: (D) 120^0

Explanation: According to the question,

Three angles of quadrilateral are 75°, 90° and 75°

Consider the fourth angle to be x.

We know that,

Sum of all angles of a quadrilateral = 360°

⇒ 75° + 90° + 75° + x = 360°

⇒ 240° + x = 360°

⇒ x = 360° – 240°

⇒ x = 120°

Hence, the fourth angle is 120°.

2. A diagonal of a rectangle is inclined to one side of the rectangle at 25^0. The acute angle between the diagonals is

(A) 55^0 (B) 50^0 (C) 40^0 (D) 25^0

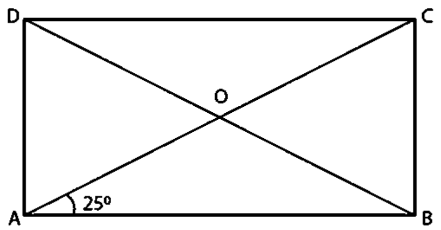
Answer: (B) 50^0

Explanation: According to the question,

A diagonal of a rectangle is inclined to one side of the rectangle at 25^0

i.e., Angle between a side of rectangle and its diagonal = 25°

Consider the acute angle between diagonals to be = x



We know that diagonals of a rectangle are equal in length i.e., AC = BD

Dividing RHS and LHS by 2,

⇒ ½ AC = ½ BD

Since, O is mid-point of AC and BD

⇒ OD = OC

Since, angles opposite to equal sides are equal

⇒ ∠y = 25°

We also know that,

Exterior angle is equal to the sum of two opposite interior angles.

So, ∠BOC = ∠ODC + ∠OCD

⇒ ∠x = ∠y + 25°

⇒ ∠x = 25° + 25°

⇒ ∠x = 50°

Hence, the acute angle between diagonals is 50°.

3. ABCD is a rhombus such that ∠ACB = 40^0. Then ∠ADB is

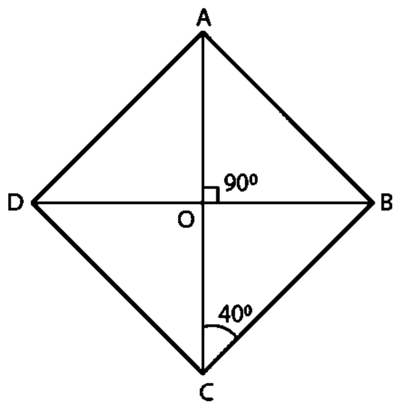
(A) 40^0 (B) 45^0 (C) 50^0 (D) 60^0

Answer: (C) 50^0

Explanation: According to the question,

ABCD is a rhombus

∠ACB = 40°



∵ ∠ACB = 40°

⇒ ∠OCB = 40°

∵ AD ∥ BC

⇒ ∠DAC = ∠BCA = 40° [Alternate interior angles]

⇒ ∠DAO = 40°

Since, diagonals of a rhombus are perpendicular to each other

We have,

∠AOD = 90°

We know that,

Sum of all angles of a triangle = 180°

⇒ ∠AOD + ∠ADO + ∠DAO = 180°

⇒ 90° + ∠ADO + 40° = 180°

⇒ 130° + ∠ADO = 180°

⇒ ∠ADO = 180° – 130°

⇒ ∠ADO = 50°

⇒ ∠ADB = 50°

Hence, ∠ADB = 50°

4. The quadrilateral formed by joining the mid-points of the sides of a quadrilateral PQRS, taken in order, is a rectangle, if

(A) PQRS is a rectangle

(B) PQRS is a parallelogram

(C) diagonals of PQRS are perpendicular

(D) diagonals of PQRS are equal.

Answer: (C) diagonals of PQRS are perpendicular

Explanation: Let the rectangle be ABCD,

We know that,

Diagonals of rectangle are equal

∴ AC = BD

⇒ PQ = QR

∴ PQRS is a rhombus

Diagonals of a rhombus are perpendicular.

Hence, diagonals of PQRS are perpendicular

5. The quadrilateral formed by joining the mid-points of the sides of a quadrilateral

PQRS, taken in order, is a rhombus, if

(A) PQRS is a rhombus

(B) PQRS is a parallelogram

(C) diagonals of PQRS are perpendicular

(D) diagonals of PQRS are equal.

Answer: (D) diagonals of PQRS are equal.

Explanation: Since, ABCD is a rhombus

We have,

AB = BC = CD = DA

Now,

Since, D and C are midpoints of PQ and PS

By midpoint theorem,

We have,

DC = ½ QS

Also,

Since, B and C are midpoints of SR and PS

By midpoint theorem

We have,

BC = ½ PR

Now, again, ABCD is a rhombus

∴ BC = CD

⇒ ½ QS = ½ PR

⇒ QS = PR

Hence, diagonals of PQRS are equal

6. If angles A, B, C and D of the quadrilateral ABCD, taken in order, are in the ratio

3:7:6:4, then ABCD is a

(A) rhombus **(B) parallelogram** **(C) trapezium** **(D) kite**

Answer: (C) trapezium

Explanation: As angles A, B, C and D of the quadrilateral ABCD, taken in order, are in the ratio 3: 7: 6: 4, We have the angles A, B, C and D = 3x, 7x, 6x and 4x.

Now, sum of the angle of a quadrilateral = 360^o.

$3x + 7x + 6x + 4x = 360^o$

$\Rightarrow 20x = 360^o$

$\Rightarrow x = 360 \div 20 = 18^o$

So, the angles A, B, C and D of quadrilateral ABCD are,

$\angle A = 3\times18^o = 54^o$,

$\angle B = 7\times18^o = 126^o$

$\angle C = 6\times18^o = 108^o$

$\angle D = 4\times18^o = 72^o$

AD and BC are two lines cut by a transversal CD

Now, sum of angles ∠C and ∠D on the same side of transversal,

$\angle C + \angle D = 108^o + 72^o = 180$

Hence, AD|| BC

So, ABCD is a quadrilateral in which one pair of opposite sides are parallel.

Hence, ABCD is a trapezium.

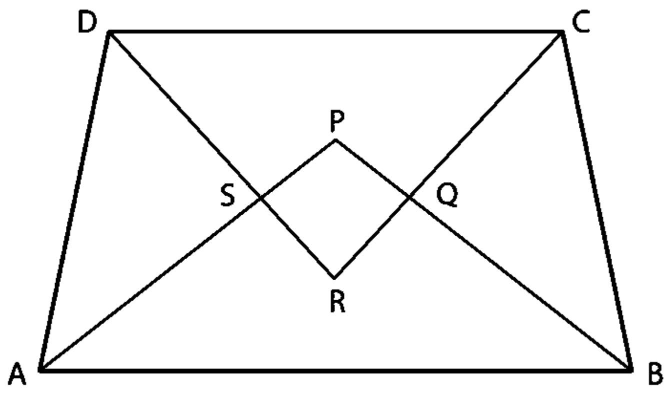
7. If bisectors of ∠A and ∠B of a quadrilateral ABCD intersect each other at P, of ∠B and ∠C at Q, of ∠C and ∠D at R and of ∠D and ∠A at S, then PQRS is a

(A) rectangle (B) rhombus

(C) parallelogram (D) quadrilateral whose opposite angles are supplementary

Answer: (D) quadrilateral whose opposite angles are supplementary

Explanation:



We know that,

Sum of all angles of a quadrilateral = 360°

$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360°$

Dividing LHS and RHS by 2,

$\Rightarrow \frac{1}{2} (\angle A + \angle B + \angle C + \angle D) = \frac{1}{2} \times 360° = 180°$

Since, AP, PB, RC and RD are bisectors of ∠A, ∠B, ∠C and ∠D

⇒ ∠PAB + ∠ABP + ∠RCD + ∠RDC = 180° … (1)

We also know that,

Sum of all angles of a triangle = 180°

∠PAB + ∠APB + ∠ABP = 180°

⇒ ∠PAB + ∠ABP = 180° – ∠APB …(2)

Similarly,

∴ ∠RDC + ∠RCD + ∠CRD = 180°

⇒ ∠RDC + ∠RCD = 180° – ∠CRD …(3)

Substituting the value of equations (2) and (3) in equation (1),

180° – ∠APB + 180° – ∠CRD = 180°

⇒ 360° – ∠APB – ∠CRD = 180°

⇒ ∠APB + ∠CRD = 360° – 180°

⇒ ∠APB + ∠CRD = 180° …(4)

Now,

∠SPQ = ∠APB [vertically opposite angles]

∠SRQ = ∠DRC [vertically opposite angles]

Substituting in equation (4),

⇒ ∠SPQ + ∠SRQ = 180°

Hence, PQRS is a quadrilateral whose opposite angles are supplementary.

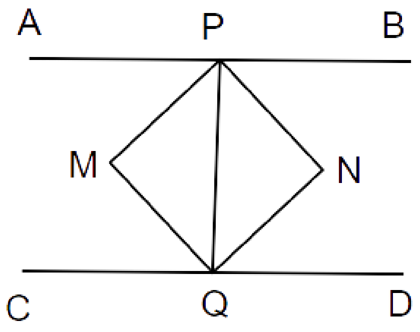
8. If APB and CQD are two parallel lines, then the bisectors of the angles APQ, BPQ, CQP and PQD form

(A) a square (B) a rhombus

(C) a rectangle (D) any other parallelogram

Answer: (C) a rectangle

The required figure is as follows:



Lines APB and CQD are parallel.

∴∠APQ = ∠PQD

∴∠APQ=∠PQD ..(alternate interior angles)

Here, MP and NQ are the angle bisectors of ∠APQ and ∠PQD

∠APQ = 2∠NQP

Dividing both the sides by 2 we get,

∠MPQ = ∠NQP

But these are alternate angles. So, MP || QN.

Similarly,

∠BPQ = ∠CQP

So, PN || QM.

Thus, quadrilateral PNQM is a parallelogram.

As, CQD is a line.

∴∠CQD = 180°

∠CQP + ∠PQD = 180°

MQ and NQ are the bisectors of ∠CQP and ∠PQD

2∠MPQ + 2∠NQP = 180°

2 (∠MPQ + ∠NQP) = 180°

∠MQN = 90°

Therefore, quadrilateral PNQM is a rectangle.

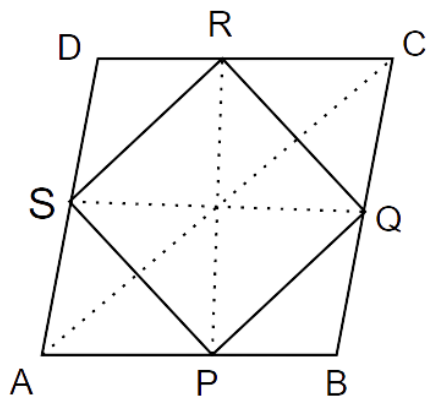
9. The figure obtained by joining the mid-points of the sides of a rhombus, taken in order, is

(A) a rhombus (B) a rectangle

(C) a square (D) any parallelogram

Answer: (B) a rectangle

The required figure is as follows:



In ΔABC,

P and Q are the midpoints of AB and BC respectively.

By Midpoint theorem,

PQ || AC and PQ = $\frac{1}{2}AC$ ..(i)

Similarly, In ΔADC,

S and R are the midpoints of AD and DC respectively.

By Midpoint theorem,

AC || SR and SR = $\frac{1}{2}$ AC ..(ii)

From (i) and (ii) we get,

PQ || SR and PQ = SR

Thus, quadrilateral PQRS is a parallelogram.

Now, ASQB is a parallelogram. So, AB = SQ.

Also, PRCB is a parallelogram. So, BC = PR.

But, BC = AB (Sides of a rhombus are equal)

AB = PR

So, SQ = PR.

A parallelogram with equal diagonals is a rectangle.

Therefore, the figure obtained by joining the mid-points of the sides of a rhombus will be a rectangle.

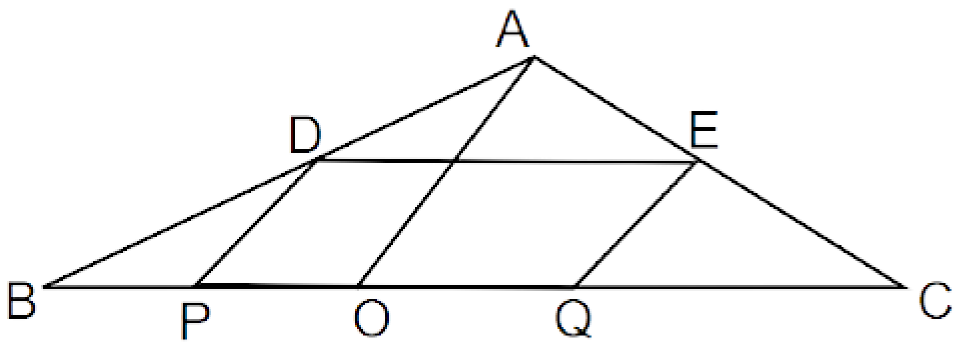
10. D and E are the mid-points of the sides AB and AC of ABC and O is any point on side BC. O is joined to A. If P and Q are the mid-points of OB and OC respectively, then DEQP is

(A) a square (B) a rectangle

(C) a rhombus (D) a parallelogram

Answer: (D) a parallelogram

The required figure is as follows:



Since the line segment joining the mid-points of any two sides of a triangle is parallel to the third side and is half of it.

So, DE || BC i.e. DE || PQ ..(i)

Now, In ΔAOB

Points D and P are the midpoints of AB and BO respectively.

Thus, DP || AO ..(ii)

Similarly,

In ΔAOC ,

Points E and Q are the midpoints of AC and CO respectively.

Thus, EQ || AO ..(iii)

From (ii) and (iii)

DP || EQ ..(iv)

From (i) and (iv)

DE || PQ and DP || EQ (opposite sides of quadrilateral DEQP)

This quadrilateral DEQP is a parallelogram.

11. The figure formed by joining the mid-points of the sides of a quadrilateral ABCD, taken in order, is a square only if,

(A) ABCD is a rhombus

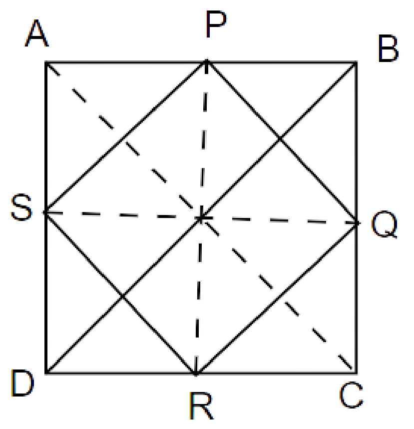
(B) diagonals of ABCD are equal

(C) diagonals of ABCD are equal and perpendicular

(D) diagonals of ABCD are perpendicular

Answer: (C) diagonals of ABCD are equal and perpendicular

The required figure is as follows:



Given: In quadrilateral ABCD,

P, Q, R and S are the mid-points of sides AB, BC, CD and AD respectively.

Thus, PQRS is a square.

So, PQ = RQ = RS = SP ..(i)

And PR = SQ

But, SQ = AB and PR = BC

Thus, AB = BC

Hence, all the sides of quadrilateral ABCD are equal.

So, the quadrilateral ABCD can either be a rhombus or a square.

In ΔABC,

By mid-point theorem,

PQ || AC so, PQ= $\frac{1}{2}$ AC .. (ii)

Similarly, In ΔADBΔADB

By mid-point theorem,

SP || DB so, SP= $\frac{1}{2}\ DB$..(iii)

But PQ = SP ..(from i)

From (ii) and (iii),

$\frac{1}{2}$ DB = $\frac{1}{2}$ AC

∴ DB = AC

Here the diagonals of quadrilateral ABCD are equal. So, quadrilateral ABCD is a square.

In a square the diagonals are equal and perpendicular.

12. The diagonals AC and BD of a parallelogram ABCD intersect each other at the point O. If ∠DAC = 32^0 and ∠AOB = 70^0, then ∠DBC is equal to

(A) 24^0 (B) 86^0 (C) 38^0 (D) 32^0

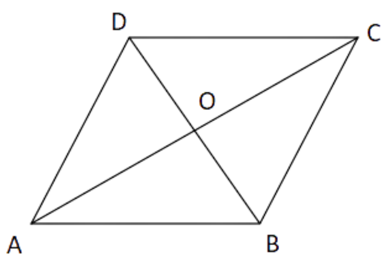
Answer: (C) 38^0

Given:

∠DAC = 32^o

∠AOB = 70^o

AD is parallel to BC and AC is the transversal.



$\angle DAC = \angle ACB$

(alternate interior angles)

$\therefore \angle ACB = 32^o$

i.e.

$\therefore \angle OCB = 32^o$

In ΔBOC

$\angle BOA$ is an exterior angle.

So, by exterior angle property,

$\Rightarrow \angle BOA = \angle OCB + \angle OBC$

$\Rightarrow 70^o = \angle OBC + 32^o$

$\Rightarrow \angle OBC = 38^o$

Thus, $\angle DBC = 38^o$

13. Which of the following is not true for a parallelogram?

(A) opposite sides are equal

(B) opposite angles are equal

(C) opposite angles are bisected by the diagonals

(D) diagonals bisect each other.

Answer: (C) opposite angles are bisected by the diagonals

In a parallelogram opposite sides and angles are equal. The diagonals bisect each other. Thus, opposite angles bisected by the diagonals is not true.

14. D and E are the mid-points of the sides AB and AC respectively of ABC. DE is produced to F. To prove that CF is equal and parallel to DA, we need an additional information which is

(A) $\angle DAE = \angle EFC$

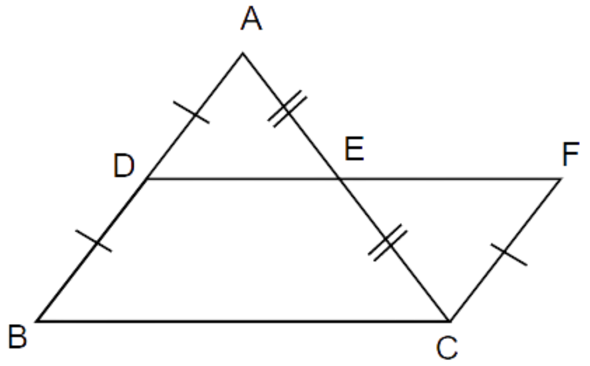
(B) AE = EF

(C) DE = EF

(D) $\angle ADE = \angle ECF$

Ans: (C) DE = EF

The required figure is as follows:



Given: AD = DB and AE = EC

CF is parallel to AD and D is the midpoint of AB

Thus, AD = DB = FC

Now, In Δ AED and Δ EFC ,

(i) AD = FC ..(given)

(ii) ∠AED = ∠FEC (vertically opposite angles)

(iii) AE = CE ..(E is the midpoint of AC)

Thus, by SAS test

ΔAED ≅ ΔEFC

DE = EF (C.P.C.T)

∠DAE = ∠ECF (these are alternate angles formed by lines AD and CF and AC is the transversal)

Thus, AD || CF

The additional information we need is DE = EF.

Short Answer Questions with Reasoning

Sample Question 1: ABCD is a parallelogram. If its diagonals are equal, then find the value of

∠ABC.

Answer: A parallelogram with equal diagonals is a rectangle. So, ABCD is a rectangle. Hence, ∠ABC=90^o

Sample Question 2: Diagonals of a rhombus are equal and perpendicular to each other. Is this statement true? Give reason for your answer.

Answer: The given statement is false. It is because in a rhombus the diagonals are perpendicular bisectors of each other but they are not equal.

Sample Question 3: Three angles of a quadrilateral ABCD are equal. Is it a parallelogram? Why or why not?

Answer: The quadrilateral ABCD will not be a parallelogram if the three equal angles are acute angles. Because then the fourth angle will be greater than the three angles.

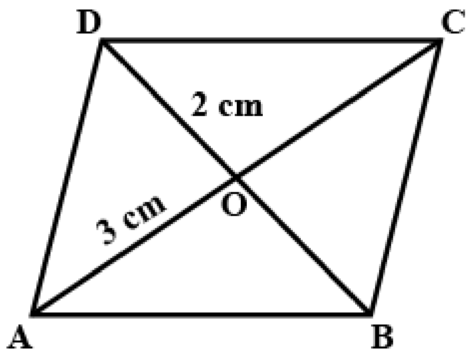
Sample Question 4: Diagonals AC and BD of a quadrilateral ABCD intersect each other at O such that OA: OC = 3: 2. Is ABCD a parallelogram? Why or why not?

Answer: Diagonals of a parallelogram are perpendicular bisectors of each other. But, here the ratio of OA to OC is not equal. Hence, quadrilateral ABCD is not a parallelogram.

Exercise 8.2:

Q1. Diagonals AC and BD of a parallelogram ABCD intersect each other at O. If OA = 3 cm and OD = 2 cm, determine the lengths of AC and BD.

Answer:



Given

ABCD is a parallelogram with AC and BD as the diagonals intersecting at O

OA = 3 cm

OD = 2 cm

As the diagonals of a parallelogram bisect each other

AO = OC

BO = OD

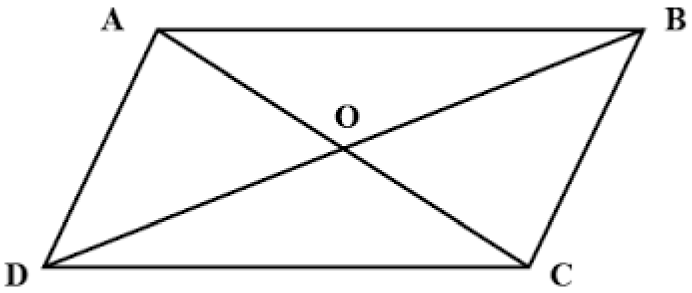
AC = 2 × OA = 2 × 3 = 6 cm

BD = 2 × OD = 2 × 2 = 4 cm

Therefore, the lengths of AC and BD are 6 cm and 4 cm.

Q2. Diagonals of a parallelogram are perpendicular to each other. Is this statement true? Give reason for your answer.

Answer: Consider a parallelogram



AC and BD are the diagonals

It intersects each other at the point O

The properties of a parallelogram are

a. The opposite sides of a parallelogram are parallel.

b. The opposite sides of a parallelogram are equal.

c. The opposite angles of a parallelogram are equal.

d. The diagonals of a parallelogram bisect each other.

e. Same-side interior angles supplement each other.

f. The diagonals divide the parallelogram into two congruent triangles.

We know that

Diagonals of a parallelogram bisect each other but not at 90°.

So the diagonals are not perpendicular

Therefore, the statement is false.

Q3. Can the angles 110^0, 80^0, 70^0 and 95^0 be the angles of a quadrilateral? Why or why not?

Answer: The angles given are 110º, 80º, 70º and 95º

We know that the sum of all the angles of a quadrilateral is 360º

So the sum of the given angles is

110º + 80º + 70º + 95º = 355º ≠ 360º

Therefore, 110º, 80º, 70º and 95º cannot be the angles of a quadrilateral.

4. In quadrilateral ABCD, ∠A + ∠D = 180º. What special name can be given to this quadrilateral?

Answer: In quadrilateral ABCD,

∠A + ∠D = 180º

We know that

The sum of co-interior angles in a trapezium is 180^0

Therefore, the given quadrilateral is a trapezium.

5. All the angles of a quadrilateral are equal. What special name is given to this quadrilateral?

Answer:

From the question we know that

All the angles of a quadrilateral are equal

Consider the angle of the quadrilateral as x

As the sum of all angles of a quadrilateral is 360º

We can write it as

x + x + x + x = 360º

4x = 360º

Dividing both sides by 4

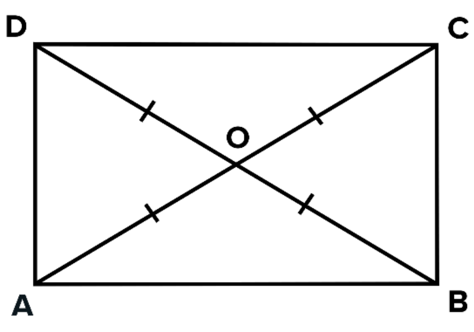
x = 360º/4

x = 90º

Therefore, the given quadrilateral is a rectangle.

6. Diagonals of a rectangle are equal and perpendicular. Is this statement true? Give reason for your answer.

Answer: Consider a rectangle ABCD



The diagonals of the rectangle are AD and BC

We know that

The diagonals are equal but need not be perpendicular

Therefore, the statement is false.

7. Can all the four angles of a quadrilateral be obtuse angles? Give reason for your answer.

Answer:

An obtuse angle is greater than 90º

From the angle sum property, the sum of all four angles of a quadrilateral is 360º

If we consider all the four angles more than 90º, the sum will be more than 360º

So all the four angles cannot be obtuse in a quadrilateral.

Therefore, all the four angles cannot be obtuse.

Exercise 8.3 :

1. One angle of a quadrilateral is of 108^0 and the remaining three angles are equal. Find each of the three equal angles.

Answer:

Given, one angle of a quadrilateral is 108^0

The remaining three angles are equal.

We have to find each of the three equal angles.

We know that the sum of all the angles of a quadrilateral is always equal to 360 degrees.

Let the other angle be x

So, $108 + x + x + x = 360^0$

$108^0 + 3x = 360^0$

$3x = 360^0 - 108^0$

$3x = 252^0$

$x = 252^0/3$

$x = 84^0$

Therefore, each of the three equal angles are equal to 84^0

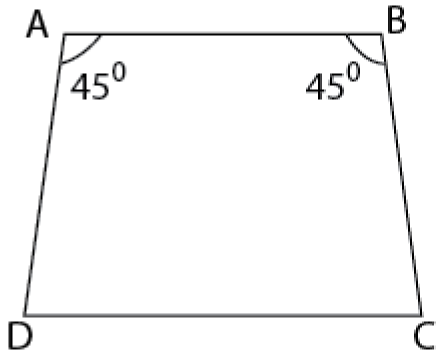
2. ABCD is a trapezium in which AB || DC and ∠A = ∠B = 45^0. Find angles C and D of the trapezium.

Answer: Given, ABCD is a trapezium

AB || DC

$\angle A = \angle B = 45^0$

We have to find the angles C and D of the trapezium



We know that opposite angles in a quadrilateral are always supplementary.

So, $\angle B + \angle C = 180^0$

Given, $\angle B = 45^0$

$45 + \angle C = 180^0$

$\angle C = 180^0 - 45^0$

$\angle C = 135^0$

Similarly, $\angle A + \angle D = 180^0$

Given, $\angle A = 45^0$

$45^0 + \angle D = 180^0$

$\angle D = 180^0 - 45^0$

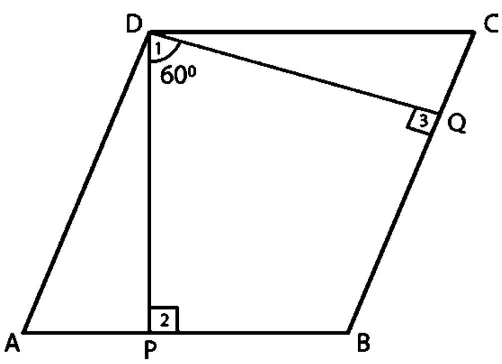
$\angle D = 135^0$

Therefore, the angles C and D of trapezium is equal to 135^0

3. The angle between two altitudes of a parallelogram through the vertex of an obtuse angle of the parallelogram is 60º. Find the angles of the parallelogram.

Answer: Given, the angle between two altitudes of a parallelogram through the vertex of an obtuse angle of the parallelogram is 60º

We have to find the angles of the parallelogram.



Consider a parallelogram ABCD

∠ADC and ∠ABC are the two obtuse angles of the parallelogram

DQ and DP are the two altitudes of the parallelogram

DP ⊥ AB

DQ ⊥ BC.

∠PDQ = 60

In quadrilateral DPBQ,

We know that according to the quadrilateral angle sum property, the sum of all the four interior angles is 360 degrees.

Sum of all interior angles of a quadrilateral is = 360^0

We have,

∠PDQ + ∠Q + ∠P + ∠B = 360^0

60 + 90 + 90 + ∠B = 360^0

240 + ∠B = 360^0

∠B = 360º - 240^0

∠B = 120^0

Since, opposite angles in parallelogram are equal,

∠B = ∠D = 120^0

Since, opposite sides are parallel in parallelogram,

AB||CD

Also, since sum of adjacent interior angles is 180 degrees

∠B + ∠C = 180^0

120 + ∠C = 180^0

∠C = 180^0 - 120^0

∠C = 60^0

Since, opposite angles in parallelogram are equal,

∠C = ∠A = 60^0

Therefore, areas of the parallelogram are 60^0, $120^{0,}$ 60^0 and 120^0

4. ABCD is a rhombus in which altitude from D to side AB bisects AB. Find the angles of the rhombus.

Answer: Given, ABCD is a rhombus

The altitude from D to side AB bisects AB

We have to find the angles of the rhombus.

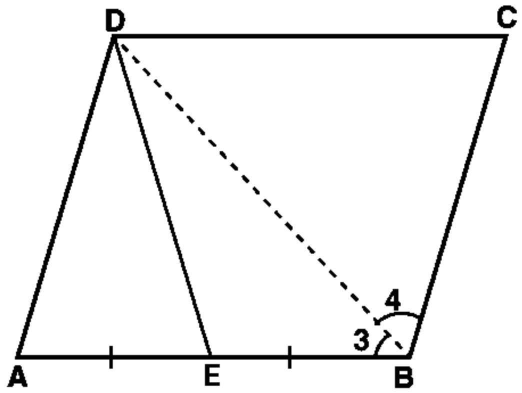
We know that all the sides of the rhombus are equal in length.

Let the sides AB = BC = CD = AD = x

The altitude from D bisects AB at L.

So, AL = x/2 and LB = x/2

Join DB



Considering triangles ALD and BLD,

As DL is the perpendicular bisector of AB

∠DLA = ∠DLB = 90

Also, AL = BL = x/2

Common side = DL

SAS criterion states that if two sides of one triangle are proportional to two sides of another triangle and their included angles are congruent, then the triangles are congruent.

By SAS criteria, the triangles ALD and BLD are congruent.

The Corresponding Parts of Congruent Triangles are Congruent (CPCTC) theorem states that when two triangles are congruent, then their corresponding sides and angles are also congruent or equal in measurements.

By CPCTC,

AD = BD

Now, in triangle ADB AD = AB = DB = x

Therefore, ADB is an equilateral triangle

We know that the angles of an equilateral triangle is always equal to 60 degrees

So, ∠A = ∠ADB = ∠ABD = 60º -------------- (1)

Considering triangle DBC,

DB = BC = CD = x

Therefore, DBC is an equilateral triangle

So, ∠C = ∠CBD = ∠DBC = 60^0 ------------------- (2)

From (1) and (2), ∠A = ∠C

By angle sum property,

∠C + ∠B + ∠D = 180^0

60 + ∠B + ∠D = 180^0

We know, ∠B = ∠D

∠B = ∠D = 180 - 60^0

∠B = ∠D = 120^0

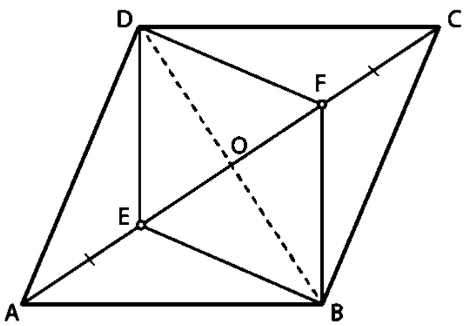
Therefore, the angles of the rhombus are ∠A = ∠C = 60^0 and ∠B = ∠D = 120^0

5. E and F are points on diagonal AC of a parallelogram ABCD such that AE = CF. Show that BFDE is a parallelogram.

Answer: Given, ABCD is a parallelogram

E and F are points on diagonal AC of parallelogram ABCD such that AE = CF

We have to show that BFDE is a parallelogram.



Join the other diagonal BD of the parallelogram.

The diagonal BD meets AC at O

We know that diagonals of a parallelogram bisect each other.

From the figure,

OA = OC

OD = OB

Given, AE = CF

From the figure,

OA - AE = OE

OC - CF = OF

So, OE = OF

Therefore, BDEF is a parallelogram as the diagonals EF and BD bisect each other at O.

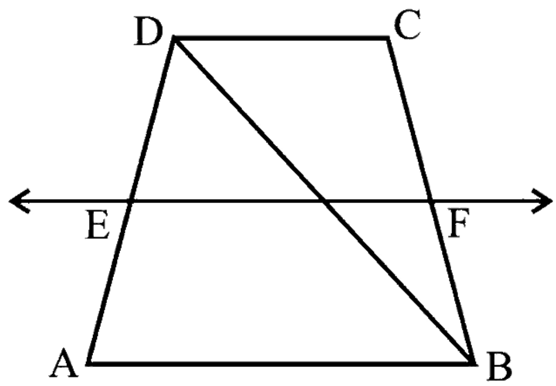
6. E is the mid-point of the side AD of the trapezium ABCD with AB || DC. A line through E drawn parallel to AB intersect BC at F. Show that F is the mid-point of BC. [Hint: Join AC]

Answer: Given, ABCD is a trapezium

E is the midpoint of the side AD with AB parallel to DC.

A line through E drawn parallel to AB intersect BC at F.

We have to show that F is the midpoint of BC



The midpoint theorem states that "The line segment in a triangle joining the midpoint of two sides of the triangle is said to be parallel to its third side and is also half of the length of the third side."

Considering triangle ADC,

E is the midpoint of AD

So, OE is parallel to DC

i.e.,OE || DC

By midpoint theorem,

O is the midpoint of AC

Considering triangle CBA,

O is the midpoint of AC

So, OF is parallel to AB

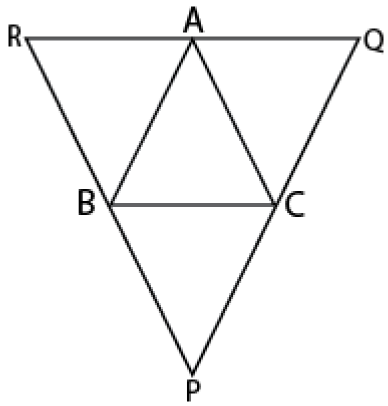
i.e.,OF || AB

By midpoint theorem,

F is the midpoint of BC

Therefore, it is shown that F is the midpoint of BC.

7. Through A, B and C, lines RQ, PR and QP have been drawn, respectively parallel to sides BC, CA and AB of a Δ ABC as shown in Fig.8.5. Show that BC = 1/2 QR.



Answer: Given, ABC is a triangle

Lines RQ, PR and QP are drawn through A, B and C parallel to sides BC, CA and AB of the triangle ABC.

We have to show that BC = 1/2 QR

Given, RQ || BC

PR || AC

QP || AB

Considering quadrilateral BCAR,

BR || CA

RA || BC

We know the opposite sides of a parallelogram are parallel and congruent.

So, BCAR is a parallelogram.

BC = AR ------------------ (1)

Considering quadrilateral BCQA,

BC || AQ

AB || QC

So, BCQA is a parallelogram

BC = AQ ------------------ (2)

Adding (1) and (2),

BC + BC = AR + AQ

2BC = AR + AQ

From the figure,

AR + AQ = RQ

So, 2BC = RQ

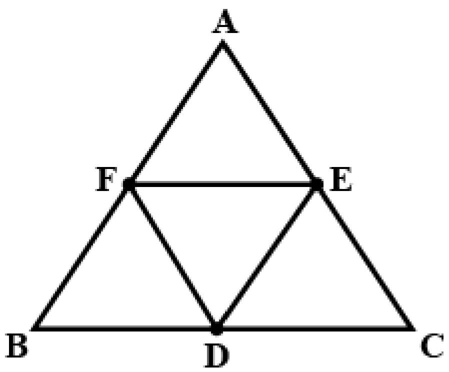
Therefore, BC = 1/2 QR

8. D, E and F are the mid-points of the sides BC, CA and AB, respectively of an equilateral triangle ABC. Show that Δ DEF is also an equilateral triangle.

Answer: Given, ABC is an equilateral triangle

D, E and F are the midpoints of the sides BC, CA and AB

We have to show that DEF is also an equilateral triangle



Considering triangle ABC,

E and F are the midpoints of AC and AB

The midpoint theorem states that “The line segment in a triangle joining the midpoint of two sides of the triangle is said to be parallel to its third side and is also half of the length of the third side.”

By Midpoint theorem,

EF || BC

EF = 1/2 BC ------- (1)

DF || AC

DF = 1/2 AC ----------- (2)

DE || AB

DE = 1/2 AB ----------- (3)

Since ABC is an equilateral triangle

The sides AB = BC = AC

Dividing by 2,

1/2 AB = 1/2 BC = 1/2 AC

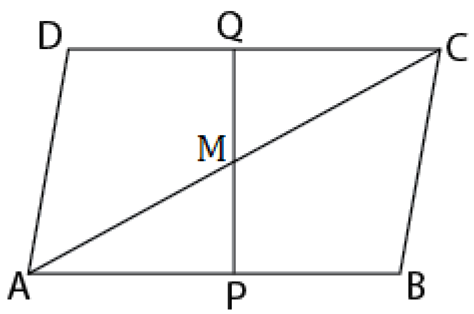
From (1), (2) and (3)

DE = EF = DF

This implies the sides of triangle DEF are equal.

Therefore, DEF is an equilateral triangle.

9. Points P and Q have been taken on opposite sides AB and CD, respectively of a parallelogram ABCD such that AP = CQ (Fig. 8.6). Show that AC and PQ bisect each other.



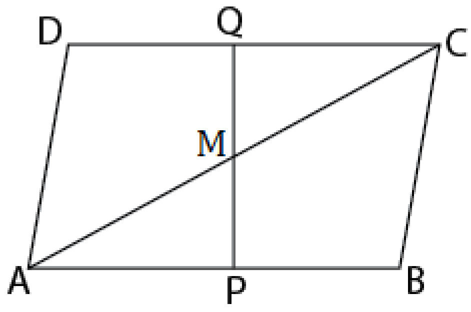
Answer:

Given, ABCD is a parallelogram

The points P and Q lie on the opposite sides AB and CD of the parallelogram

Given, AP = CQ

We have to show that AC and PQ bisect each other.



Considering triangles AMP and CMQ,

We know that alternate interior angles are equal.

So, ∠PAM = ∠QCM

Given, AP = CQ

Also, the alternate interior angles ∠MPA and ∠MQC are equal.

So, ∠MPA = ∠MQC

SAS criterion states that if two sides of one triangle are proportional to two sides of another triangle and their included angles are congruent, then the triangles are congruent.

By SAS criteria, the triangles ANP and CMQ are congruent.

The Corresponding Parts of Congruent Triangles are Congruent (CPCTC) theorem states that when two triangles are congruent, then their corresponding sides and angles are also congruent or equal in measurements.

By CPCTC,

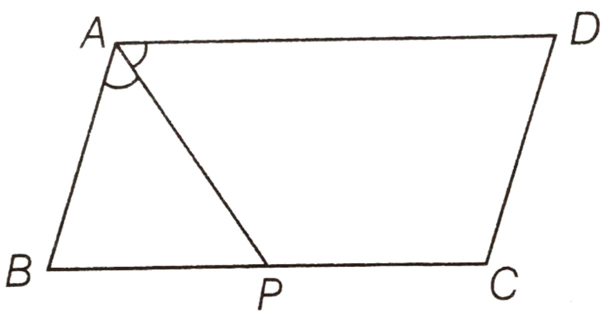
AM = CM

PM = MQ

This implies AC and BQ bisect each other at M.

Therefore, AC and PQ bisect each other.

10. In Fig. 8.7, P is the mid-point of side BC of a parallelogram ABCD such that ∠BAP = ∠DAP. Prove that AD = 2CD.



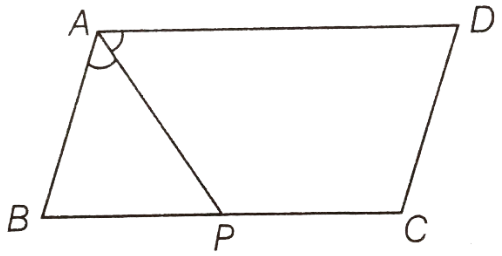
Answer:

Given, ABCD is a parallelogram

P is the midpoint of the side BC of the parallelogram

Given, ∠BAP = ∠DAP

We have to prove that AD = 2CD



We know that opposite sides of a parallelogram are parallel and congruent.

So, AD is parallel to BC

i.e., AD || BC ---------------------- (1)

Considering the two parallel lines AD and BC cut by a transversal AP,

We know that the sum of interior angles lying on the same side of the transversal is always supplementary.

∠A + ∠B = 180°

∠B = 180° - ∠A -------------- (2)

Considering triangle ABP,

By angle sum property,

∠BAP + ∠B + ∠BPA = 180°

Since, ∠BAP = ∠DAP

∠BAP = ∠DAP = 1/2 ∠A

1/2 ∠A + ∠B + ∠BPA = 180°

From (2),

1/2 ∠A + 180° - ∠A + ∠BPA = 180°

∠BPA - 1/2 ∠A = 0

∠BPA = 1/2 ∠A

So, ∠BPA = ∠BAP

We know that the sides opposite to equal angles are equal.

So, AB = BP

Multiplying by 2 on both sides,

2AB = 2BP

Since P is the midpoint of BC

BP = CP

BC = BP + PC

BC = BP + BP

BC = 2BP

So, 2AB = BC

From (1),

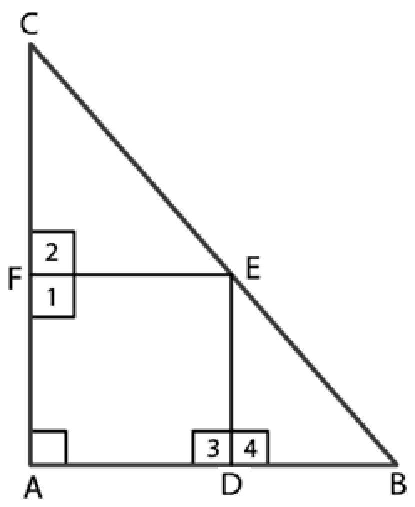
2CD = AD

Therefore, it is proven that 2CD = AD

Exercise 8.4:

1. A square is inscribed in an isosceles right triangle so that the square and the triangle have one angle in common. Show that the vertex of the square opposite the vertex of the common angle bisects the hypotenuse.

Answer:



Given, a square is inscribed in an isosceles right triangle.

The square and the triangle have one angle in common.

We have to show that the vertex of the square opposite the vertex of the common angle bisects the hypotenuse.

Consider an isosceles right triangle ABC right angled at A.

A square DEF is inscribed in the triangle.

Given, ∠A = 90°

We know that in an isosceles triangle two sides have equal length.

So, AB=AC ------------------------ (1)

We know that all sides of a square are equal

So, AD = AF ---------------------- (2)

On subtracting (1) and (2), we get

AB - AD = AC - AF

BD = CF ------------------------ (3)

Considering triangles CFE and BDE,

The sides of a square DE = EF

From (3), BD = CF

∠CFE = ∠EDB = 90°

SAS criterion states that if two sides of one triangle are proportional to two sides of another triangle and their included angles are congruent, then the triangles are similar.

By SAS criteria, the triangles CFE and BDE are similar.

The Corresponding Parts of Congruent Triangles are Congruent (CPCTC) theorem states that when two triangles are similar, then their corresponding sides and angles are also congruent or equal in measurements.

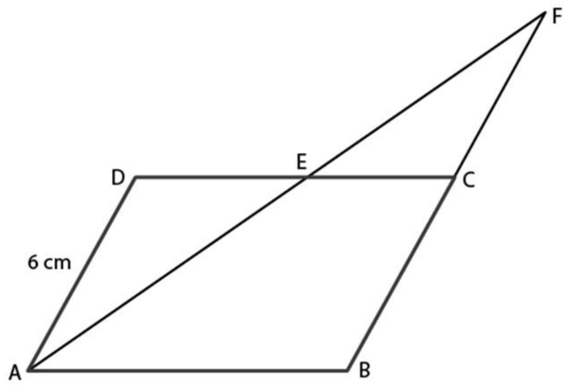
By CPCTC,

CE = BE

Therefore, vertex E of the square bisects the hypotenuse BC.

2. In a parallelogram ABCD, AB = 10 cm and AD = 6 cm. The bisector of ∠A meets DC in E. AE and BC produced meet at F. Find the length of CF.

Answer:



Given, ABCD is a parallelogram

AB = 10 cm

AD = 6 cm

The bisector of ∠A meets DC in E.

AE and BC are produced to meet at F.

We have to find the length of CF.

Now extend AD to H and join HF.

So, ABFH is a parallelogram

We know that the opposite sides of a parallelogram are parallel and congruent.

AB || HF and HF = AB

We know that the alternate interior angles are equal.

∠AFH = ∠FAB ------------------------- (1)

Since AF is the bisector of ∠A

∠HAF = ∠FAB ------------------------ (2)

From (1) and (2),

∠AFH = ∠HAF

We know that the sides opposite to equal angles are equal.

HF = AH

Since, HF = AB

HF = 10 cm

Since HF = AH, AH = 10 cm

From the figure,

AH = AD + DH

10 = 6 + DH

DH = 10 - 6

DH = 4 cm

CFHD is a parallelogram

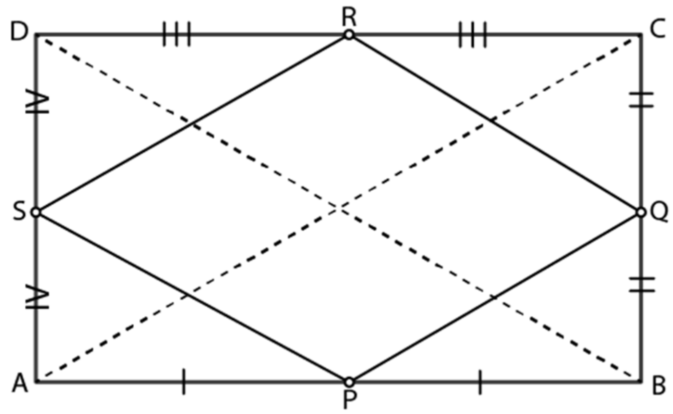
We know that the opposite sides of a parallelogram are parallel and congruent.

So, DH = CF

Therefore, CF = 4cm

3. P, Q, R and S are, respectively, the mid-points of the sides AB, BC, CD and DA of a quadrilateral ABCD in which AC = BD. Prove that PQRS is a rhombus.

Answer:



Given, ABCD is a quadrilateral

The points P, Q, R and S are the midpoints of the sides AB, BC, CD and AD.

AC = BD

We have to prove that PQRS is a rhombus.

The midpoint theorem states that “The line segment in a triangle joining the midpoint of two sides of the triangle is said to be parallel to its third side and is also half of the length of the third side.”

Considering triangle ADC,

S and R are the midpoints of AD and DC

By midpoint theorem,

SR || AC

SR = 1/2 AC --------------------- (1)

Considering triangle ABC,

P and Q are the midpoints of AB and BC

By midpoint theorem,

PQ || AC

PQ = 1/2 AC -------------------- (2)

Comparing (1) and (2),

SR = PQ = 1/2 AC ----------- (3)

Considering triangle BCD,

By midpoint theorem,

RQ || BD

RQ = 1/2 BD ------------------ (4)

Considering triangle BAD,

SP || BD

By midpoint theorem,

SP = 1/2 BD ----------------- (5)

Comparing (4) and (5),

SP = RQ = 1/2 BD ----------- (6)

Given, AC = BD

Dividing by 2 on both sides,

1/2 AC = 1/2 BD

From (3) and (6),

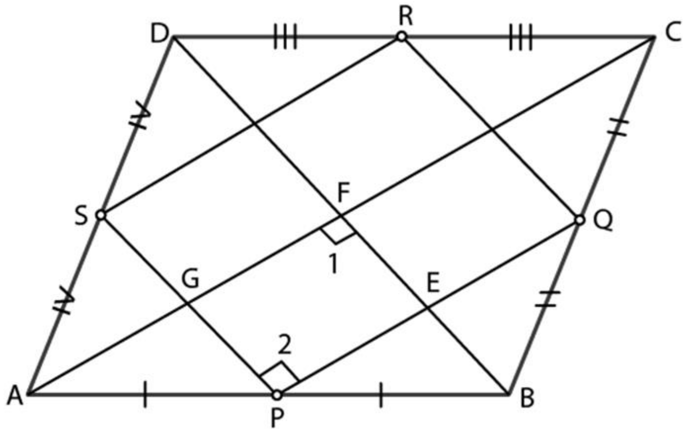
SR = PQ = SP = RQ

This implies all the sides of the quadrilateral are equal.

Therefore, PQRS is a rhombus.

4. P, Q, R and S are, respectively, the mid-points of the sides AB, BC, CD and DA of a quadrilateral ABCD such that AC ⊥ BD. Prove that PQRS is a rectangle.

Answer:



Given, ABCD is a quadrilateral

P, Q, R and S are the midpoints of the sides AB, BC, CD and AD

AC ⊥ BD

We have to prove that PQRS is a rectangle.

Since AC ⊥ BD

∠AOD = ∠AOB = ∠BOC = ∠COD = 90°

The midpoint theorem states that "The line segment in a triangle joining the midpoint of two sides of the triangle is said to be parallel to its third side and is also half of the length of the third side."

Considering triangle ADC,

S and R are the midpoints of AD and DC

By midpoint theorem,

SR || AC

SR = 1/2 AC --------------- (1)

Considering triangle ABC,

P and Q are the midpoints of AB and BC

By midpoint theorem,

PQ || AC

PQ = 1/2 AC ----------------- (2)

Comparing (1) and (2),

SR = PQ = 1/2 AC ------------ (3)

Considering triangle BAD,

SP || BD

By midpoint theorem,

SP = 1/2 BD ----------------- (5)

Comparing (4) and (5),

SP = RQ = 1/2 BD ----------- (6)

Considering quadrilateral EOFR,

OE || FR

OF || ER

∠EOF = ∠ERF = 90°

Therefore, PQRS is a rectangle.

5. P, Q, R and S are, respectively, the mid-points of sides AB, BC, CD and DA of quadrilateral ABCD in which AC = BD and AC ⊥ BD. Prove that PQRS is a square.

Answer:

Given, ABCD is a quadrilateral

P, Q, R and S are the midpoints of the sides AB, BC, CD and AD

AC = BD

AC ⊥ BD

We have to prove that PQRS is a square.

The midpoint theorem states that "The line segment in a triangle joining the midpoint of two sides of the triangle is said to be parallel to its third side and is also half of the length of the third side."

Considering triangle ADC,

S and R are the midpoints of AD and DC

By midpoint theorem,

SR || AC

SR = 1/2 AC --------------- (1)

Considering triangle ABC,

P and Q are the midpoints of AB and BC

By midpoint theorem,

PQ || AC

PQ = 1/2 AC ----------------- (2)

Comparing (1) and (2),

SR = PQ = 1/2 AC ------------ (3)

Considering triangle BAD,

SP || BD

By midpoint theorem,

SP = 1/2 BD ----------------- (5)

Comparing (4) and (5),

SP = RQ = 1/2 BD ----------- (6)

Given, AC = BD

Dividing by 2 on both sides,

1/2 AC = 1/2 BD

From (3) and (6),

SR = PQ = SP = RQ

This implies all the sides of the quadrilateral are equal.

Considering quadrilateral OERF,

OE || FR

OF || ER

∠EOF = ∠ERF = 90°

Sinc AC ⊥ BD

∠DOC = ∠EOF = 90°

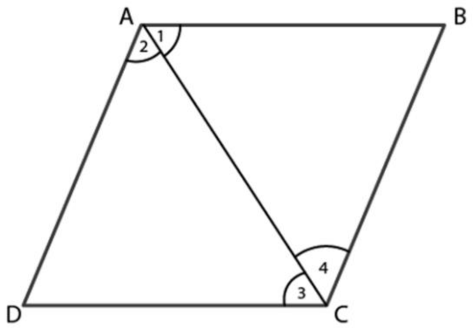
∠QRS = 90°

∠RQS = 90°

Therefore, PQRS is a square.

6. A diagonal of a parallelogram bisects one of its angles. Show that it is a rhombus.

Answer:



Consider a parallelogram ABCD

Join the diagonal AC

AC bisects angle A

We have to prove that ABCD is a rhombus.

Since AC bisects angle A

∠CAB = ∠CAD ------------------ (1)

We know that the opposite sides of a parallelogram are parallel and congruent.

So, AB||CD and AC is a transversal.

We know that the alternate interior angles are equal.

∠CAB = ∠ACD ----------------- (2)

Similarly, AD||BC and AC is a transversal.

∠DAC = ∠ACB ----------------- (3)

From (1), (2) and (3),

∠BCA = ∠DCA

We know that the opposite angles of a parallelogram are equal.

∠A = ∠C

Dividing by 2 on both sides,

1/2 ∠A = 1/2 ∠C

Comparing (1) and (2),

∠CAD = ∠ACD

We know that the sides opposite to the equal angles are equal.

CD = AD

We know that the opposite sides of a parallelogram are parallel and congruent.

AB = CD

AD = BC

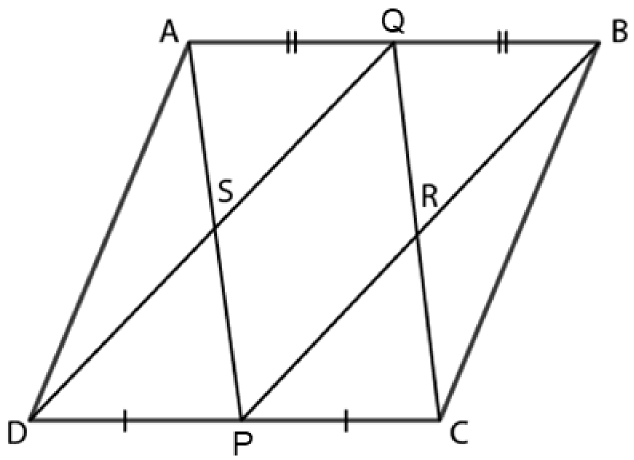
So, AB = BC = CD = AD

This implies all the sides are equal.

Therefore, ABCD is a rhombus.

7. P and Q are the mid-points of the opposite sides AB and CD of a parallelogram ABCD. AQ intersects DP at S and BQ intersects CP at R. Show that PRQS is a parallelogram.

Answer:



Given, ABCD is a parallelogram

P and Q are the midpoints of the opposite sides AB and CD of the parallelogram

AQ intersects DP at S

BQ intersects CP at R

We have to show that PQRS is a parallelogram

Since P is the midpoint of AB

AP = PB

AB = AP + PB

AB = AP + AP

AB = 2AP

AP = 1/2 AB ----------- (1)

Since Q is the midpoint of CD

QC = QD

CD = QC + QD

CD = QC + QC

CD = 2QC

QC = 1/2 CD ------------ (2)

We know that the opposite sides of a parallelogram are parallel and congruent

AB || CD

Also, AB = CD

Dividing by 2 on both sides,

1/2 AB = 1/2 CD

From (1) and (2),

AP = QC

Also, AP || QC

Therefore, APCQ is a parallelogram

So, AQ || PC or SQ || PR

Similarly , AB || DC or BP || DQ

So, AB = DC

Dividing by 2 on both sides,

1/2 AB = 1/2 DC

Since P is the midpoint of AB

AP = PB

AB = AP + PB

AB = BP + BP

AB = 2BP

BP = 1/2 AB ----------- (3)

Since Q is the midpoint of CD

QC = QD

CD = QC + QD

CD = QD + QD

CD = 2QD

QD = 1/2 CD ------------ (4)

From (3) and (4),

BP = QD

Therefore, BPDQ is a parallelogram

We know that the opposite sides of a parallelogram are parallel and congruent

PD || BQ

Similarly, PS || QR

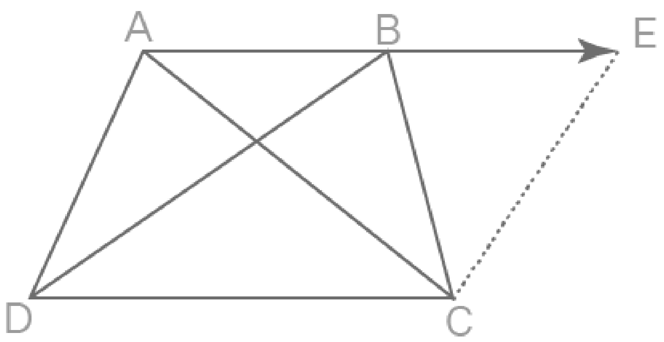
So, SQ || RP

PS || QR

Therefore, PQRS is a parallelogram.

8. ABCD is a quadrilateral in which AB || DC and AD = BC. Prove that ∠A = ∠B and ∠C = ∠D.

Answer:



Given, ABCD is a quadrilateral

AB || DC

AD = BC

We have to prove that ∠A = ∠B and ∠C = ∠D.

Extend AB to E and join CE such that AD || CE

So, AECD is a parallelogram.

We know that the opposite sides of a parallelogram are parallel and congruent.

AD = EC

Given, AD = BC

So, BC = EC

We know that the angles opposite to equal sides are equal.

∠CBE = ∠CEB ----------- (1)

We know that the linear pair of angles is always supplementary.

∠B + ∠CBE = 180° -------- (2)

Since AD || EC and cut by transversal AE,

We know that two parallel lines are cut by a transversal, the sum of interior angles lying on the same side of the transversal is always supplementary.

∠A + ∠CEB = 180° --------- (3)

From (1),

∠A + ∠CEB = 180° ------------- (4)

Comparing (2) and (4),

∠A = ∠B

We know that the sum of supplementary angles of a parallelogram is always equal to 180 degrees.

∠A + ∠D = 180°

∠B + ∠C = 180°

On comparing,

∠A + ∠D = ∠B + ∠C

since , ∠A = ∠B

∠A + ∠D = ∠A + ∠C

∠C = ∠D

Therefore, it is proven that ∠A = ∠B and ∠C and ∠D

9. In Fig. 8.11, AB || DE, AB = DE, AC || DF and AC = DF. Prove that BC || EF and BC = EF.

Answer:

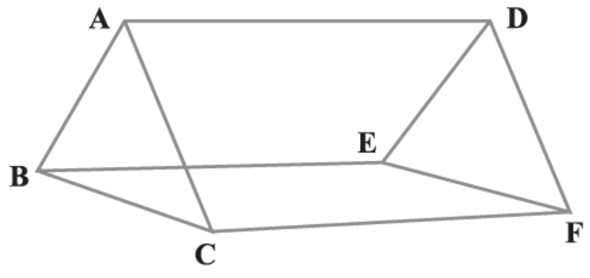
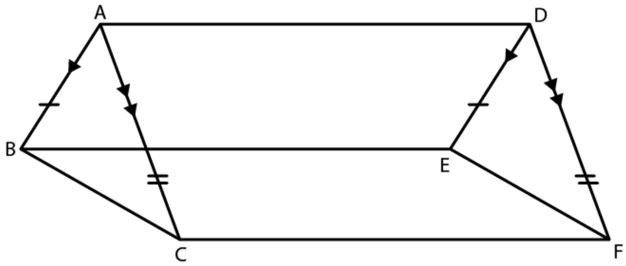


Fig. 8.11



Given, AB || DE and AB = DE

AC || DF and AC = DF

We have to prove that BC || EF and BC = EF.

Considering quadrilateral ABED,

Given, AB || DE

AB = DE

So, ABED is a parallelogram

Now, AD || BE

AD = BE ------------------------- (1)

Considering quadrilateral ACFD,

AC || FD

AC = FD

So, ACFD is a parallelogram

Now, AD || FC

AD = FC -------------------------- (2)

From (1) and (2),

AD = BE = FC

CF || BE

So, BCFE is a parallelogram.

We know that the opposite sides of a parallelogram are parallel and equal.

Now, BC = EF

BC || EF

Therefore, it is proven that BC = EF and BC || EF.

Q10. E is the mid-point of a median AD of ΔABC and BE is produced to meet AC at F. Show that AF = 1/3 AC.

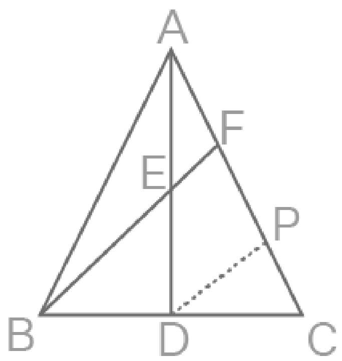
Answer:

Given, ABC is a triangle

E is the midpoint of a median AD

BE is produced to meet AC at F

We have to show that AF = 1/3 AC



Draw DP parallel to EF

Considering triangle ADP,

E is the midpoint of AD

EF || DP

By converse of midpoint theorem,

F is the midpoint of AP.

Considering triangle FBC,

D is the midpoint of BC

DP || BF

By converse of midpoint theorem,

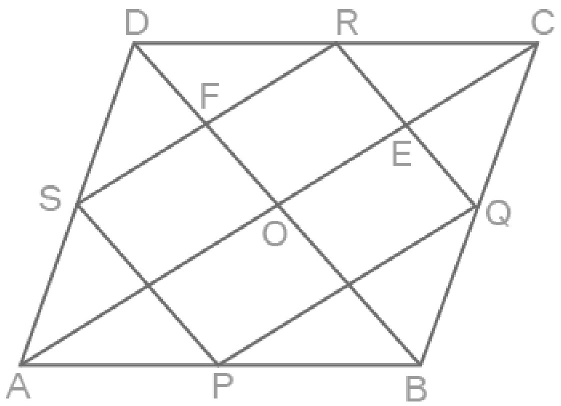
P is the midpoint of FC

So, AF = FP = PC

Therefore, AF = 1/3 AC

11. Show that the quadrilateral formed by joining the mid-points of the consecutive sides of a square is also a square.

Answer:



Consider a square ABCD

P, Q, R and S are the midpoints AB, BC, CD and DA

We have to show that PQRS is a square.

Join the diagonals AC and BD of the square ABCD

We know that all the sides of the square are equal in length

So, AB = BC = CD = AD

The midpoint theorem states that "The line segment in a triangle joining the midpoint of two sides of the triangle is said to be parallel to its third side and is also half of the length of the third side."

Considering triangle ADC,

S and R are the midpoints of AD and DC

By midpoint theorem,

SR || AC

SR = 1/2 AC --------------- (1)

Considering triangle ABC,

P and Q are the midpoints of AB and BC

By midpoint theorem,

PQ || AC

PQ = 1/2 AC ----------------- (2)

Comparing (1) and (2),

SR = PQ = 1/2 AC ------------ (3)

Considering triangle BAD,

SP || BD

By midpoint theorem,

SP = 1/2 BD ----------------- (5)

Comparing (4) and (5),

SP = RQ = 1/2 BD ----------- (6)

We know that the diagonals of a square bisect each other at right angle.

So, AC = BD

Dividing by 2 on both sides,

1/2 AC = 1/2 BD

From (3) and (6),

SR = PQ = SP = RQ

Considering quadrilateral OERF,

OE || FR

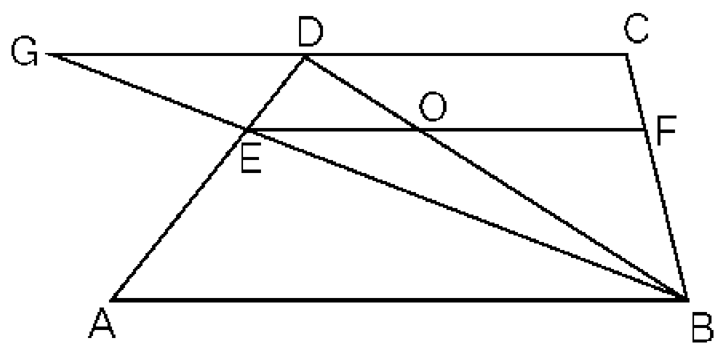
OF || ER

∠FOE = ∠FRE = 90°

Therefore, PQRS is a square.

12. E and F are respectively the mid-points of the non-parallel sides AD and BC of a trapezium ABCD. Prove that EF || AB and EF = 1/2 (AB + CD) [Hint: Join BE and produce it to meet CD produced at G.]

Answer:



Given, ABCD is a trapezium in which AB || CD

E and F are the midpoints of the non-parallel sides AD and BC

We have to prove that EF || AB and EF = 1/2 (AB + CD)

Join BE and extend it to meet CD produced at G

Draw BD which intersects EF at O.

Consider triangle GCB,

E and F are the midpoints of BG and BC.

The midpoint theorem states that "The line segment in a triangle joining the midpoint of two sides of the triangle is said to be parallel to its third side and is also half of the length of the third side."

By Midpoint theorem,

EF || GC

Given, AB || GC or CD || AB

So, EF || AB

Considering triangle ABD,

AB || EO

E is the midpoint of AD

The converse of the midpoint theorem states that if a line is drawn through the midpoint of one side of a triangle, and parallel to the other side, it bisects the third side.

By converse of midpoint theorem,

O is the midpoint of BD

EO = 1/2 AB -------------- (1)

Considering triangle BDC,

OF || CD

O is the midpoint of BD

By converse of midpoint theorem,

OF = 1/2 CD ------------- (2)

Adding (1) and (2),

EO + OF = 1/2 AB + 1/2 CD

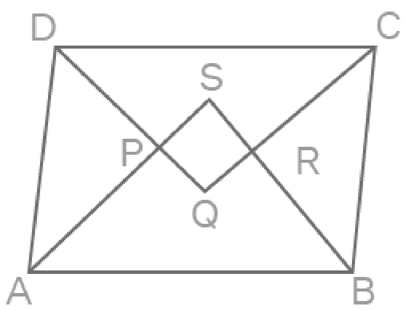
From the figure,

EF = EO + OF

Therefore, EF = 1/2 (AB + CD)

13. Prove that the quadrilateral formed by the bisectors of the angles of a parallelogram is a rectangle.

Answer:



Consider a parallelogram ABCD

AP, BR and CR be the bisectors of ∠A, ∠B, ∠C and ∠D

We have to prove that PQRS is a rectangle.

We know that the opposite sides of a parallelogram are parallel and congruent.

So, DC || AB

Now, DC || AB and DA is a transversal

We know that if two parallel lines are cut by a transversal, the sum of interior angles lying on the same side of the transversal is always supplementary.

∠A + ∠D = 180° ------------------ (1)

Dividing by 2 on both sides,

1/2 ∠A + 1/2 ∠D = 180°/2

=> 1/2 ∠A+ 1/2 ∠D = 90°

∠PAD + ∠PDA = 90° ----------- (2)

Considering triangle PDA,

By angle sum property,

∠APD + ∠PAD + ∠PDA = 180°

From (2),

∠APD + 90° = 180°

∠APD = 180° - 90°

∠APD = 90°

We know that the vertically opposite angles are equal.

So, ∠APD = ∠SPQ

∠SPQ = 90°

Similarly, ∠PQR = 90°

∠QRS = 90°

∠PSR = 90°

PQRS is a quadrilateral with each angle equal to 90°.

Therefore, PQRS is a rectangle.

14. P and Q are points on opposite sides AD and BC of a parallelogram ABCD such that PQ passes through the point of intersection O of its diagonals AC and BD. Show that PQ is bisected at O.

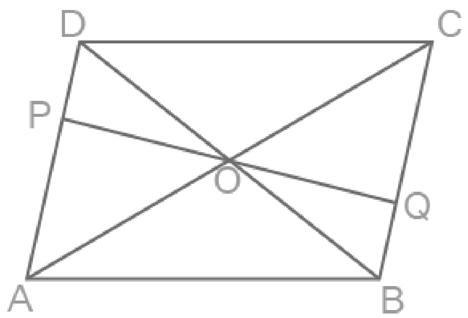
Answer: Given, ABCD is a parallelogram

P and Q are the points on opposite sides AD and BC

The diagonals AC and BD intersect at O

PQ passes through the point of intersection O of its diagonals.

We have to show that PQ is bisected at O.



Considering triangles ODP and OBQ,

We know that the vertically opposite angles are equal.

∠BOQ = ∠POD

We know that the alternate interior angles are equal.

∠OBQ = ∠ODP

Given, OB = OD

ASA criterion states that two triangles are congruent if any two angles and the side included between them of one triangle are equal to the corresponding angles and the included side of the other triangle.

By ASA criteria, the triangles ODP and OBQ are congruent.

The Corresponding Parts of Congruent Triangles are Congruent (CPCTC) theorem states that when two triangles are congruent, then their corresponding sides and angles are also congruent or equal in measurements.

By CPCTC rule,

OP = OQ

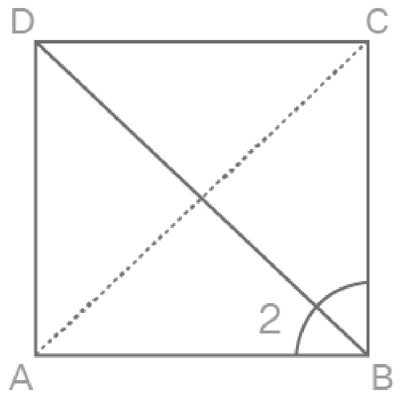
Therefore, PQ is bisected at O.

Q15. ABCD is a rectangle in which diagonal BD bisects ∠B. Show that ABCD is a square.

Answer: Given, ABCD is a rectangle

The diagonal BD bisects ∠B.

We have to show that ABCD is a square.



Join the diagonal AC

Considering triangles BAD and BCD,

Since BD is the bisector of ∠B

∠ABD = ∠CBD

∠A = ∠C = 90°

Common side = BD

The ASA congruence rule states that if any two consecutive angles of a triangle along with a non-included side are equal to the corresponding consecutive angles and the non-included side of another triangle, the two triangles are said to be congruent.

By ASA criteria, the triangles BAD and BCD are congruent.

The Corresponding Parts of Congruent Triangles are Congruent (CPCTC) theorem states that when two triangles are congruent, then their corresponding sides and angles are also congruent or equal in measurements.

By CPCTC,

AB = BC

AD = CD

We know that the opposite sides of a rectangle are equal.

AB = CD

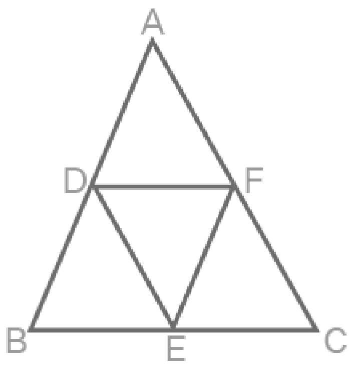
BC = AD

So, AB = BC = CD = AD

Therefore, ABCD is a square.

Q16. D, E and F are respectively the mid-points of the sides AB, BC and CA of a triangle ABC. Prove that by joining these mid-points D, E and F, the triangles ABC is divided into four congruent triangles.

Answer:



Given, ABC is a triangle

D, E and F are the midpoints of the sides AB, BC and CA

We have to prove that by joining the midpoints D, E and F, the triangle ABC is divided into four congruent triangles.

Since D is the midpoint of AB

AD = DB = 1/2 AB

Since E is the midpoint of BC

BE = EC = 1/2 BC

Since F is the midpoint of AC

AF = FC = 1/2 AC

By midpoint theorem,

EF || AB

EF = 1/2 AB

So, EF = AD = BD

Also, ED || AC

ED = 1/2 AC

So, ED = AF = FC

Similarly, DF || BC

DF = 1/2 BC

So, DF = BE = CE

Considering triangles ADF and EFD,

AD = EF

AF = DE

Common side = FD

The Side-Side-Side congruence rule states that, if all the three sides of a triangle are equal to the three sides of another triangle then the triangles are congruent.

By SSS criterion, the triangles ADF and EFD are congruent.

Similarly,

The triangles DEF and EDB are congruent.

The triangles DEF and CFE are congruent.

Therefore, the triangle ABC is divided into four congruent triangles.

Q17. Prove that the line joining the mid-points of the diagonals of a trapezium is parallel to the parallel sides of the trapezium.

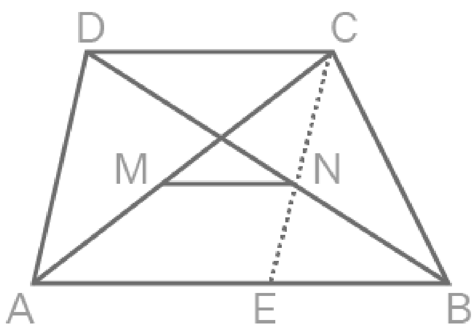
Answer:

Given, ABCD is a trapezium

AC and BD are the diagonals of the trapezium

M and N are the midpoints of the diagonals AC and BD

We have to prove that MN || AB || CD



Join CN and extend it to meet AB at E.

Considering triangles CDN and EBN,

Since N is the midpoint of BD

DN = BN

We know that the alternate interior angles are equal.

∠DCN = ∠NEB

∠CDN = ∠NBE

The ASA congruence rule states that if any two consecutive angles of a triangle along with a non-included side are equal to the corresponding consecutive angles and the non-included side of another triangle, the two triangles are said to be congruent.

By ASA criterion, the triangles CDN and EBN are congruent.

By CPCTC,

DC = EB

CN = NE

Considering triangle CAE,

M and N are the midpoints of AC and CE

MN || AE

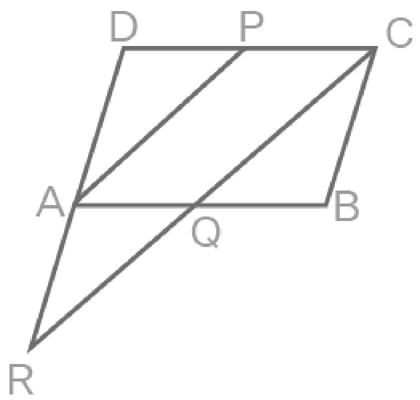
By midpoint theorem,

MN || AB || CD

Therefore, it is proved that the line joining the mid-points of the diagonals of a trapezium is parallel to the parallel sides of the trapezium.

Q18. P is the mid-point of the side CD of a parallelogram ABCD. A line through C parallel to PA intersects AB at Q and DA produced at R. Prove that DA = AR and CQ = QR.

Answer:



Given, ABCD is a parallelogram

P is the midpoint of the side CD

A line through C parallel to PA intersects AB at Q and DA produced at R.

We have to prove that DA = AR and CQ = QR

We know that the opposite sides of a parallelogram are parallel and congruent.

BC || AD and BC = AD

AB || CD and AB = CD

Since P is the midpoint of DC.

DP = PC = 1/2 CD ----------- (1)

Given, QC || AP

PC || AQ

Therefore, APCQ is a parallelogram

So, AQ = PC

From (1),

AQ = PC = 1/2 CD

Since AB = CD

PC = 1/2 AB = BQ

Considering triangles AQR and BQC,

AQ = BQ

We know that the vertically opposite angles are equal

∠AQR = ∠BQC

We know that the alternate interior angles are equal

∠ARQ = ∠BCQ

By ASA criterion, the triangles AQR and BQC

The Corresponding Parts of Congruent Triangles are Congruent (CPCTC) theorem states that when two triangles are congruent, then their corresponding sides and angles are also congruent or equal in measurements.

By CPCTC,

AR = BC

Given, BC = AD

So, AR = AD

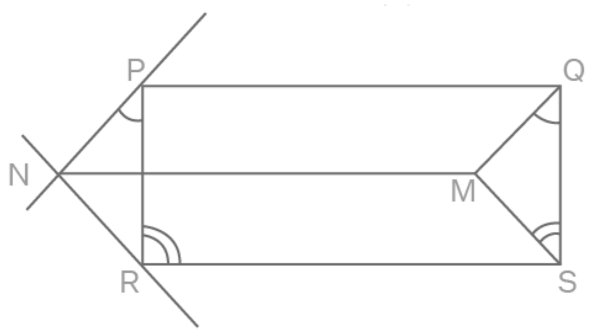
By CPCTC,

CQ =QR

Therefore, it is proved that AR = AD and CQ = QR.

(E) Long Answer Questions

1. PQ and RS are two equal and parallel line-segments. Any point M not lying on PQ or RS is joined to Q and S and lines through P parallel to QM and through R parallel to SM meet at N. Prove that line segments MN and PQ are equal and parallel to each other.



Answer: Given, PQ and RS are two equal and parallel line-segments

Any point M not lying on PQ or RS is joined to Q and S.

The lines through P and R parallel to QM and SM meet at N.

We have to prove that the line segments MN and PQ are equal and parallel to each other.

We know that the opposite sides of a parallelogram are parallel and congruent.

PQ = RS and PQ|| RS --------- (1)

PQRS is a parallelogram.

We know that the sum of interior angles lying on the same side of the transversal is always supplementary.

∠RPQ + ∠PQS = 180°

Now, ∠RPQ + ∠PQM + ∠MQS = 180° --------- (2)

Also, PN || QM

So, ∠NPQ + ∠PQM = 180°

Now, ∠NPR + ∠RPQ + ∠PQM = 180° ----------- (3)

Comparing (2) and (3),

∠MQS = ∠NPR ------------- (4)

Similarly, ∠MSQ = ∠NRP ---------- (5)

From (1), (4) and (5)

By ASA criteria, the triangles PNR and QMS are congruent.

By CPCTC,

NR = MS

PN = QM

So, PN || QM

Therefore, PQMN is a parallelogram

We know that the opposite sides of a parallelogram are parallel and congruent.

MN = PQ

NM || PQ

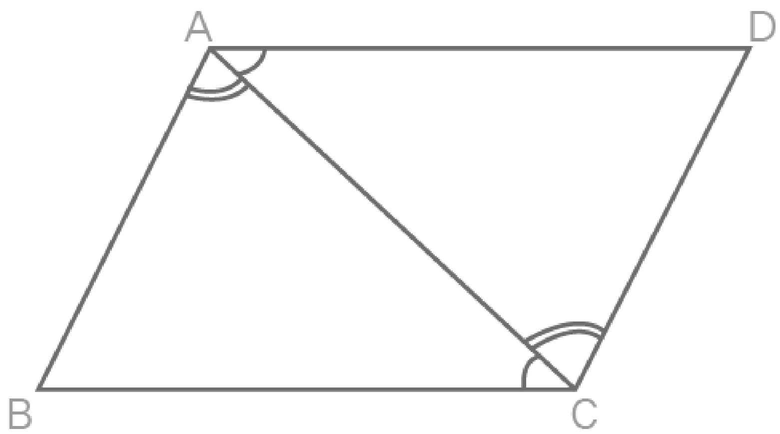
Therefore, it is proven that the line segments MN and PQ are equal and parallel to each other.

2. Prove that a diagonal of a parallelogram divides it into two congruent triangles.

Answer: Consider a parallelogram ABCD

Join the diagonal AC of the parallelogram

We have to prove that a diagonal of a parallelogram divides it into two congruent triangles.



The diagonal AC divides the parallelogram ABCD into two triangles ABC and ADC.

Considering triangles ABC and ADC,

We know that the opposite sides of a parallelogram are parallel and congruent.

So, AD = BC

AD||BC

We know that the alternate interior angles are equal.

∠BAC = ∠DCA

∠BCA = ∠DAC

Common side = AC

We observe that one side and two angles made on this side are equal.

By ASA criteria, the triangles ABC and ADC are similar.

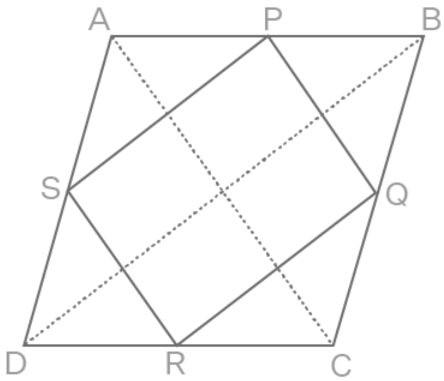
Therefore, △ABC ≅ △ADC.

3. Show that the quadrilateral formed by joining the mid-points the sides of a rhombus, taken in order, form a rectangle.

Answer: Consider a rhombus ABCD

The points P, Q, R and S are the midpoints of the sides AB, BC, CD and AD.

We have to show that PQRS is a rectangle.



Join the diagonals AC and BD of the rhombus ABCD.

Considering the triangle ABD,

Since S and P are the midpoints of the sides AD and AB.

SP || BD ----------------- (1)

SP = 1/2 BD --------------- (2)

Similarly, RQ || BD

RQ = 1/2 BD -------------- (3)

From (2) and (3),

SP = RQ

Also, SP || RQ

Therefore, PQRS is a parallelogram

We know that the diagonals of a rhombus are perpendicular.

So, AC⊥ BD -------------- (4)

Considering triangle BAC,

PQ || AC --------------- (5)

From (1), (4) and (5),

SP ⊥ PQ

i.e.,∠SPQ = 90°

We know that a rectangle is a quadrilateral with four right angles. The opposite sides are parallel and equal to each other.

Therefore, PQRS is a rectangle.

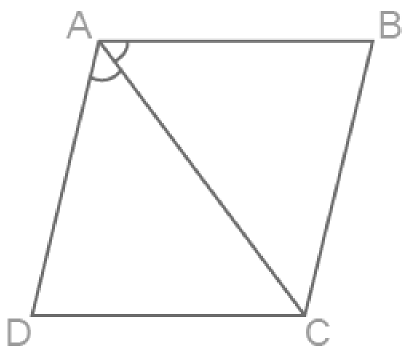
4. A diagonal of a parallelogram bisects one of its angle. Prove that it will bisect its opposite angle also.

Answer: Consider a parallelogram ABCD

Join the diagonal AC

AC bisects angle A

We have to prove that AC will bisect its opposite angle also.



Since AC bisects angle A

∠CAB = ∠CAD ------------------- (1)

We know that the opposite sides of a parallelogram are parallel and congruent.

So, AB||CD and AC is a transversal.

We know that the alternate interior angles are equal.

∠CAB = ∠ACD ----------------- (2)

Similarly, AD||BC and AC is transversal.

∠DAC = ∠ACB ----------------- (3)

From (1), (2) and (3),

∠BCA = ∠DCA

This implies AC bisects the opposite angle C.

Therefore, it is proven that AC bisects its opposite angle also.