Exercise 7.1

In each of the following, write the correct answer:

Q1. Which of the following is not a criterion for congruence of triangles?

(A) SAS

(B) ASA

(C) SSA

(D) SSS

Answer:

(C) SSA

Explanation:

We know that,

Two triangles are congruent, if the side(S) and angles (A) of one triangle is equal to another.

And criterion for congruence of triangle are SAS, ASA, SSS, and RHS.

SSA is not the criterion for congruency of a triangle.

Hence, option C is the correct answer.

2. If AB = QR, BC = PR and CA = PQ, then

(A) Δ ABC ≅ ΔPQR

(B) ΔCBA ≅ ΔPRQ

(C) ΔBAC ≅ Δ RPQ

(D) Δ PQR ≅ Δ BCA

Answer:

(B) ΔCBA ≅ ΔPRQ

Explanation:

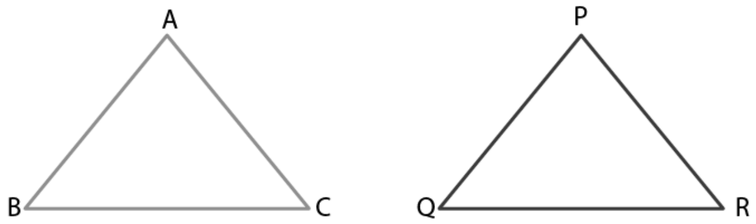
According to the question,

AB = QR, BC = PR and CA = PQ

Since, AB = QR, BC = PR and CA = PQ

We can say that,

A corresponds to Q, B corresponds to R, C corresponds to P.



Hence, (B) ΔCBA ≅ ΔPRQ

Hence, option B is the correct answer.

3. In Δ ABC, AB = AC and ∠B = 50°. Then ∠C is equal to

(A) 40° (B) 50° (C) 80° (D) 130°

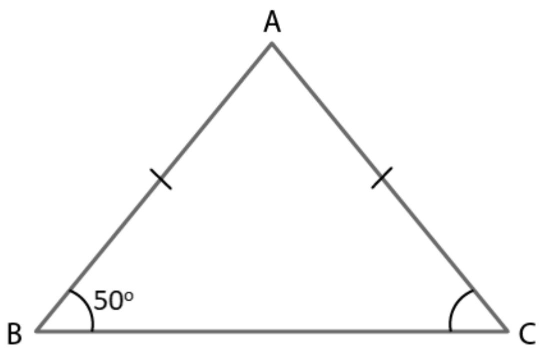
Answer:

(B) 50°

Explanation:

According to the question,

Δ ABC, AB = AC and ∠B = 50°.



Since, AB = AC

Δ ABC is an isosceles triangle.

Hence, ∠B = ∠C

∠B = 50° (given)

⇒ ∠C = 50°

Hence, option B is the correct option.

Q4. In Δ ABC, BC = AB and ∠B = 80°. Then ∠A is equal to

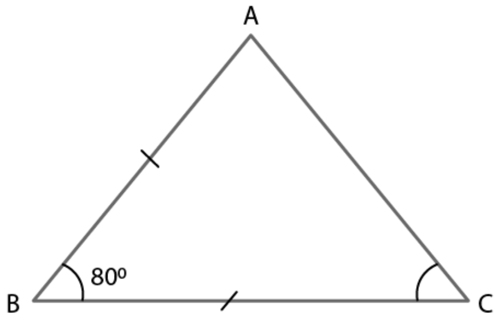
(A) 80° (B) 40° (C) 50° (D) 100°

Answer:

(C) 50°

Explanation:

Given: Δ ABC, BC = AB and ∠B = 80°



Since, BC = AB

Δ ABC is an isosceles triangle.

Let, ∠C = ∠A = x

∠B = 80° (given)

We know that,

Using angle sum property,

Sum of interior angles of a triangle should be = 180º

∠A + ∠B + ∠C = 180°

⇒ x + 80° + x = 180°

⇒ 2x = 180° – 80°

⇒ 2x = 100°

⇒ x = 50°

Therefore, ∠C = ∠A = 50°

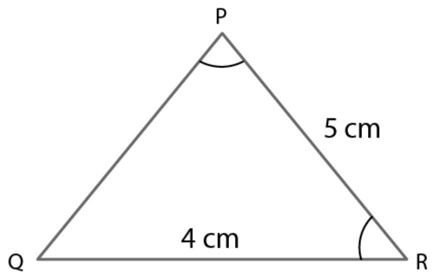
Hence, option C is the correct answer.

Q5. In Δ PQR, ∠R = ∠P and QR = 4 cm and PR = 5 cm. Then the length of PQ is

(A) 4 cm (B) 5 cm (C) 2 cm (D) 2.5 cm

Answer:

Given: ΔPQR, ∠R = ∠P and QR = 4 cm and PR = 5 cm



Since, ∠R = ∠P

Δ PQR is an isosceles triangle.

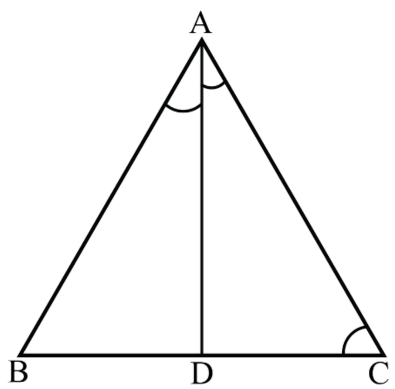
Hence, PQ = QR

⇒ PQ = 4cm

Hence, option A is the correct answer.

Q6. If D is a point on the side BC of a ΔABC such that AD bisects ∠BAC. Then,

(a) BD = CD (b) BA > BD (c) BD > BA (d)CD > CA



Answer:

It is given that

In Δ ABC, AD is the angular bisector

AD meets BC at the point D

We know that

AD is the bisector of ∠BAC

∠BAD = ∠CAD

In Δ ACD, an external angle is ∠ADC

As the external angle of a triangle is greater than each internal opposite angle of the same triangle

∠ADB = ∠DAC + ∠DCA

Here ∠DAC = ∠BAD

∠ADB = ∠BAD + ∠DCA

i.e. ∠ADC > ∠BAD

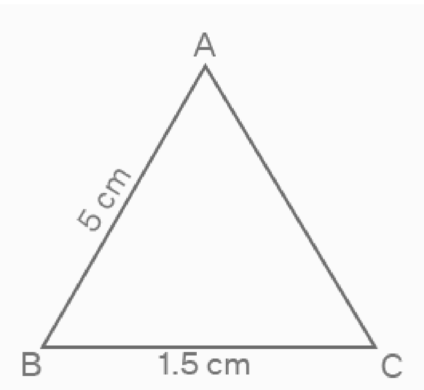
BA > BD

Therefore, if D is a point on the side BC of an Δ ABC such that AD bisects ∠BAC then BA > BD.

Q8. If two sides of a triangle are of lengths 5 cm and 1.5 cm, then the length of the third side of the triangle cannot be

(a) 3.6 cm (b) 4.1 cm (c) 3.8 cm (d) 3.4 cm

Answer:



From the figure

AB = 5 cm

BC = 1.5 cm

So the third side is AC

We know that

AB - BC < AC < AB + BC

Substituting the values

5 - 1.5 < AC < 5 + 1.5

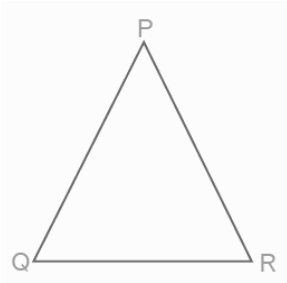
3.5 < AC < 6.5

It cannot be 3.4 as it is less than 3.5

Therefore, the length of the third side of the triangle cannot be 3.4 cm.

Q9: In ΔPQR, if ∠R > ∠Q, then

(a) QR > PR (b) PQ > PR (c) PQ < PR (d) QR < PR



Answer:

It is given that

∠R > ∠Q

As the sides opposite to the greater angle are greater

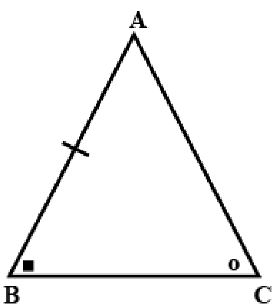
PQ > PR

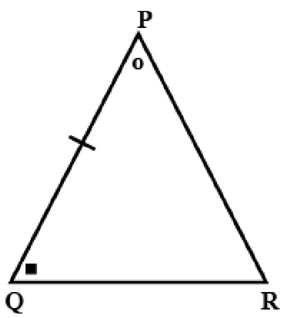
Therefore, PQ > PR.

Q10. In ΔABC and ΔPQR, if AB = AC, ∠C =∠P, and ∠B = ∠Q, then the two triangles are

(a) isosceles but not congruent (b) isosceles and congruent

(c) congruent but not isosceles (d) Neither congruent nor isosceles





Answer:

The two triangles ABC and PQR are

We know that

∠A + ∠B + ∠C = ∠P + ∠Q + ∠R

It is given that

∠C = ∠P and ∠B = ∠Q … (1)

We get

∠A + ∠B + ∠C = ∠C + ∠B + ∠R

Here ∠A = ∠R

Using the AAA criterion

Δ ABC ≅ Δ PQR

As AB = AC, Δ ABC is an isosceles triangle

∠B = ∠C [opposite angles of equal sides]

From (1), ∠P = ∠Q

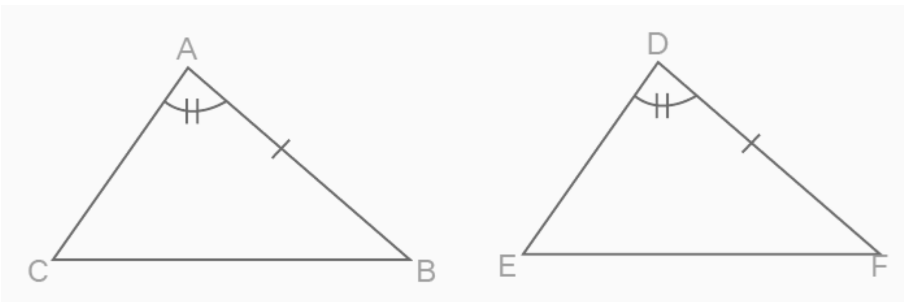
So Δ PQR is an isosceles triangle

As the relation between sides of two triangles is unknown, congruency cannot be proved using SAS or ASA

Therefore, the two triangles are isosceles but not congruent.

Q11. In ΔABC and ΔDEF, AB = FD and ∠A = ∠D. The two triangles will be congruent by SAS axiom, if (a) BC = EF (b) AC = DE (c)AC=EF (d) BC = DE

Answer:



The triangles ABC and DEF are

It is given that

AB = FD and ∠A = ∠D

For a triangle to be congruent from the SAS axiom two sides and an included angle must be equal

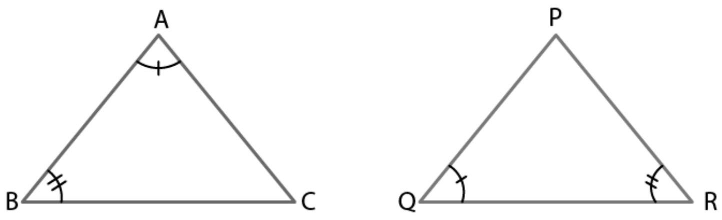
For Δ ABC ≅ Δ DFE from the SAS axiom we require AC = DE

Therefore, the two triangles will be congruent by the SAS axiom if AC = DE.

Exercise 7.2

Q1. In triangles ABC and PQR, ∠A = ∠Q and ∠B = ∠R. Which side of Δ PQR should be equal to side AB of Δ ABC so that the two triangles are congruent? Give a reason for your answer.

Answer:



In triangle ABC and PQR, we have

∠A = ∠Q [Given]

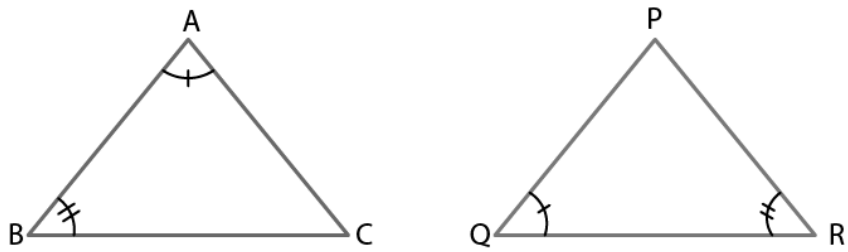
∠B = ∠R [Given]

For the triangle to be congruent, AB should be equal to QR.

Hence, triangles ABC and PQR can be congruent by ASA congruence rule.

Q2. In triangles ABC and PQR, ∠A = ∠Q and ∠B = ∠R. Which side of Δ PQR should be equal to side BC of Δ ABC so that the two triangles are congruent? Give a reason for your answer.

Answer:



In triangle ABC and PQR, we have

∠*A* = ∠*Q* and ∠*B* = ∠*R* [Given]

For the triangles to be congruent, we must have

BC = RP

Hence, triangles ABC and PQR will be congruent by the AAS congruence rule.

Q3. "If two sides and an angle of one triangle are equal to two sides and an angle of another triangle, then the two triangles must be congruent." Is the statement true? Why?

Answer: No, the statement, "If two sides and an angle of one triangle are equal to two sides and an angle of another triangle, then the two triangles must be congruent" is false.

Justification:

Because by the congruent rule,

The two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle, i.e., the SAS rule.

Q4. "If two angles and a side of one triangle are equal to two angles and a side of another triangle, then the two triangles must be congruent." Is the statement true? Why?

Answer:

The statement, "If two angles and a side of one triangle are equal to two angles and a side of another triangle, then the two triangles must be congruent." is true.

Justification:

The statement is true because the triangles will be congruent either by the ASA rule or the AAS rule. This is because two angles and one side are enough to construct two congruent triangles.

Q5. Is it possible to construct a triangle with lengths of sides 4 cm, 3 cm, and 7 cm? Give a reason for your answer.

Answer:

No, it is not possible to construct a triangle with lengths of sides 4 cm, 3 cm, and 7 cm.

Justification:

We know that,

Sum of any two sides of a triangle is always greater than the third side.

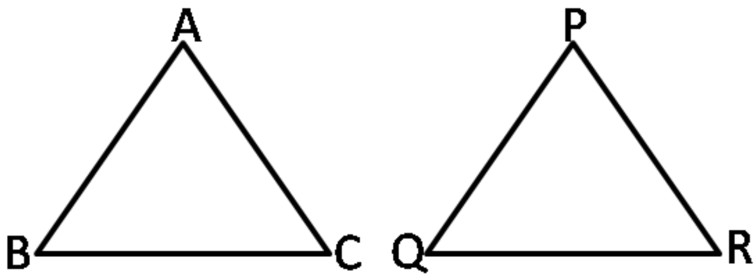
Here, the sum of two sides whose lengths are 4 cm and 3 cm = 4 cm + 3 cm = 7 cm,

Which is equal to the length of the third side, i.e., 7 cm.

Hence, it is not possible to construct a triangle with lengths of sides 4 cm, 3 cm, and 7 cm.

Q6. It is given that Δ ABC ≅ Δ RPQ. Is it true to say that BC = QR? Why?

Answer:



We know that

AB = RP

BC = PQ

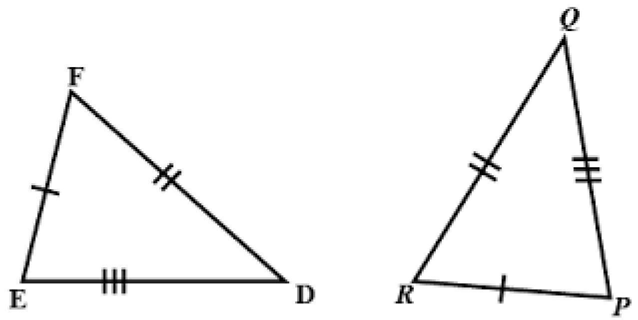
AC = RQ

Here BC cannot be equal to QR

Therefore, the statement is false.

Q7. If ΔPQR ≅ ΔEOF, then is it true to say that PR = EF? Give a reason for your answer.

Answer:



It is given that

Δ PQR ≅ Δ EDF

From the figure

PQ = ED

QR = DF

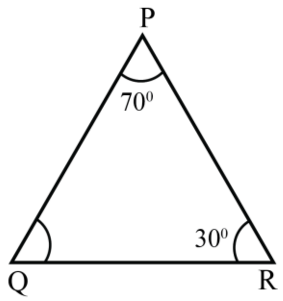
PR = EF

So PR = EF is true.

Therefore, the statement is true.

Q8. In ΔPQR, ∠P = 70° and ∠R = 30°. Which side of this triangle is the longest? Give a reason for your answer.

Answer:



It is given that

∠P = 70°

∠R = 30°

From the angle sum property

∠P + ∠Q + ∠R = 180°

Substituting the values

70° + ∠Q + 30° = 180°

∠Q + 100° = 180°

So we get

∠Q = 180° - 100°

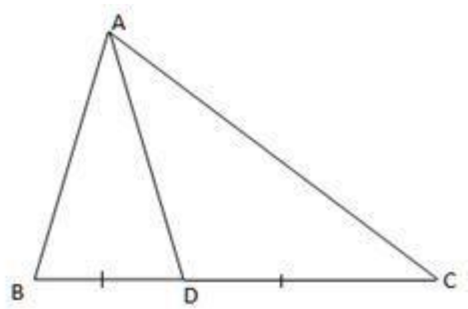
∠Q = 80° is the greatest angle

The opposite side PR is the longest side.

Therefore, the side PR of this triangle is the longest.

Q9. AD is a median of the ΔABC. Is it true that AB + BC +CA > 2AD? Give a reason for your answer.

Answer:



It is given that

In triangle ABC, AD is the median

We know that

The sum of any two sides of a triangle is greater than the third side

AB + BD > AD …. (1)

AC + DC > AD ….. (2)

It can be written as

AB + (BD + DC) + AC > 2AD

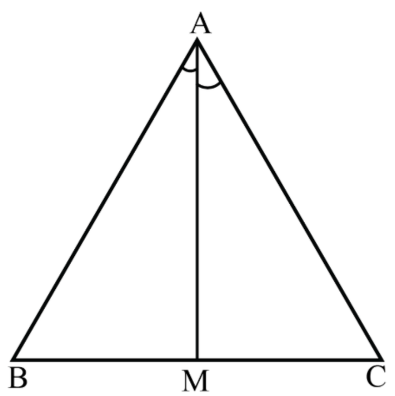
So we get

AB + BC + AC > 2AD

Therefore, AB + BC + CA > 2 AD is true.

Q10: M is a point on side BC of a triangle ABC such that AM is the bisector of ∠BAC. Is it true to say that the perimeter of the triangle is greater than 2 AM? Give a reason for your answer.

Answer:



From the question

We know that

The sum of two sides of a triangle is greater than the third side

In Δ ABM,

AB + BM > AM …. (1)

In Δ ACM

AC + CM > AM …. (2)

Adding equation (1) and (2)

AB + BM + AC + CM > AM + AM

AB + AC + BC > 2AM

Therefore, it is true that the perimeter of the triangle is greater than 2 AM.

Q11: Is it possible to construct a triangle with lengths of sides 9 cm, 7 cm, and 17 cm? Give a reason for your answer.

Answer: In a triangle, we know that the sum of two sides is greater than the third side

The lengths of the sides given are 9 cm, 7 cm, and 17 cm

We know that

9 + 7 = 16 cm < 17 cm

Here the sum of the two sides is less than the third side

Therefore, it is not possible to construct a triangle with lengths of sides 9 cm, 7 cm, and 17 cm.

Q12. Is it possible to construct a triangle with lengths of its sides 8 cm, 7 cm, and 4 cm? Give a reason for your answer.

Answer: In a triangle, we know that the sum of two sides is greater than the third side

The lengths of sides given are 8 cm, 7 cm, and 4 cm

We know that

8 + 7 = 15 cm > 4 cm

Here the sum of the two sides is greater than the third side

Therefore, it is possible to construct a triangle with lengths of its sides as 8 cm, 7 cm, and 4 cm.

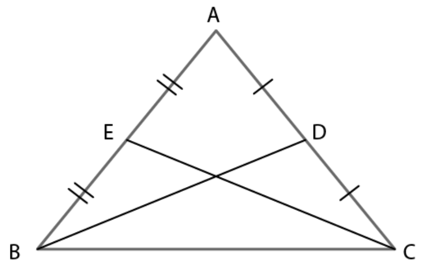
Exercise 7.3

1. ABC is an isosceles triangle with AB = AC and BD and CE are its two medians. Show that BD = CE.

Answer:

According to the question,

ΔABC is an isosceles triangle and AB = AC, BD and CE are two medians



From ΔABD and ΔACE,

AB = AC (given)

2 AE = 2 AD (as D and E are mid points)

So, AE = AD

∠A = ∠A (common)

Hence, ΔABD ≅ ΔACE (using SAS)

⇒ BD = CE (by CPCT)

Hence proved.

Q2. In Fig.7.4, D and E are points on side BC of a Δ ABC such that BD = CE and AD = AE.

Show that Δ ABD ≅ Δ ACE.

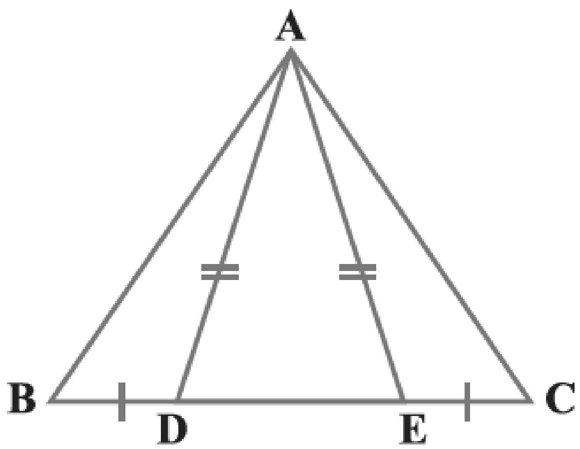


Fig. 7.4

Answer:

According to the question,

In ΔABC,

BD = CE and AD = AE.

In ΔADE,

AD = AE

Since opposite angles to equal sides are equal,

We have,

∠ADE = ∠AED … (1)

Now, ∠ADE + ∠ADB = 180° (linear pair)

∠ADB = 180° – ∠ADE … (2)

Also, ∠AED + ∠AEC = 180° (linear pair)

∠AEC = 180° – ∠AED

Since, ∠ADE = ∠AED

∠AEC = 180° – ∠ADE … (3)

From equation (2) and (3)

∠ADB = ∠AEC … (4)

Now, In ΔADB and ΔAEC,

AD = AE (given)

BD = EC (given)

∠ADB = ∠AEC (from (4)

Hence, ΔABD ≅ΔACE (by SAS)

Q3. CDE is an equilateral triangle formed on a side CD of a square ABCD (Fig.7.5). Show that Δ ADE ≅ Δ BCE.

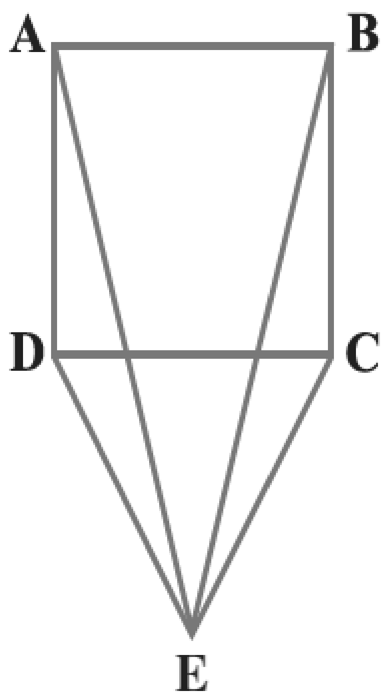


Fig. 7.5

Answer:

According to the question,

CDE is an equilateral triangle formed on a side CD of a square ABCD.

In ΔADE and ΔBCE,

DE = CE (sides of equilateral triangle)

Now,

∠ADC = ∠BCD = 90°

And, ∠EDC = ∠ECD = 60°

Hence, ∠ADE = ∠ADC + ∠CDE = 90° + 60° = 150°

And ∠BCE = ∠BCD + ∠ECD = 90° + 60° = 150°

⇒ ∠ADE = ∠BCE

AD = BC (sides of square)

Hence, ΔADE ≅ΔBCE (by SAS)

Q4. In Fig.7.6, BA ⊥ AC, DE ⊥ DF such that BA = DE and BF = EC. Show that Δ ABC ≅ Δ DEF.

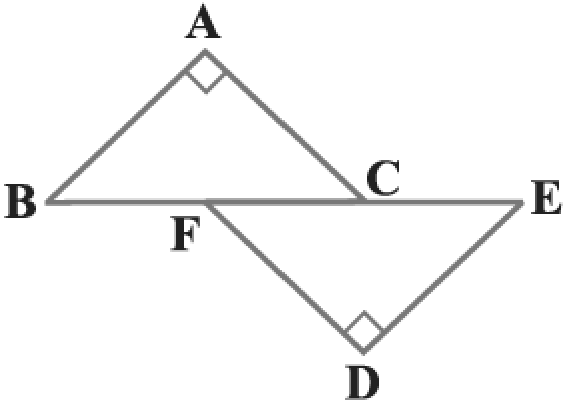


Fig. 7.6

Answer:

According to the question,

BA ⊥ AC, DE ⊥ DF such that BA = DE and BF = EC.

In ΔABC and ΔDEF

BA = DE (given)

BF = EC (given)

∠A = ∠D (both 90°)

BC = BF + FC

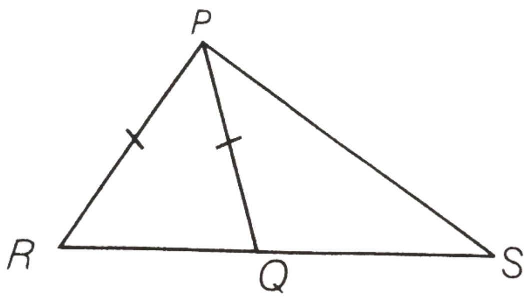
EF = EC + FC = BF + FC (∵ EC = BF)

⇒ EF = BC

Hence, ΔABC ≅ ΔDEF (by RHS)

Q5. Q is a point on the side SR of a Δ PSR such that PQ = PR. Prove that PS > PQ.

Answer:



Given, PSR is a triangle.

Q is a point on the side SR of the triangle

PQ = PR

We have to show that PS > PQ

We know that the angles opposite to the equal sides are equal.

∠PRQ = ∠PQR --------------- (1)

We know that the exterior angle of a triangle is greater than each of the opposite interior angles.

∠PQR > ∠S ------------------- (2)

From (1) and (2),

∠PRQ > ∠S

We know that in a triangle side opposite to the greater angle is longer.

So, PS > PR

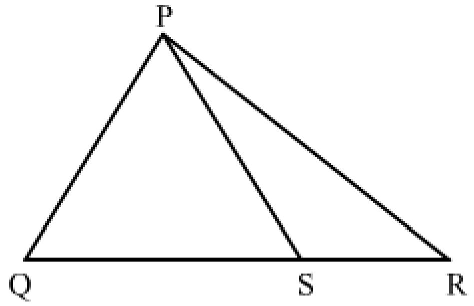
Given, PQ = PR

Therefore, PS > PQ

Q6. S is any point on side QR of a ΔPQR. Show that PQ + QR + RP > 2 PS.

Answer:

Thinking Process: Use the inequality of a triangle i.e., a sum of two sides of a triangle is greater than the third side. Further, show the required result.



Given, PQR is a triangle

S is any point on the side QR of the triangle

We have to show that PQ + QR + RP > 2 PS

We know that in a triangle the sum of two sides of a triangle is greater than the third side.

Considering triangle PQS,

PQ + QS > PS ------------ (1)

Considering triangle PRS,

SR + RP > PS ------------- (2)

On adding (1) and (2),

PQ + QS + SR + RP > PS + PS

PQ + (QS + SR) + RP > 2 PS

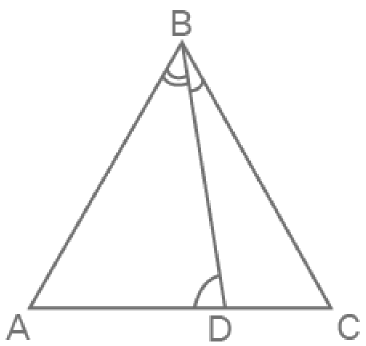
From the figure,

QR = QS + SR

Therefore, PQ + QR + RP > 2 PS

Q7. D is any point on side AC of a ΔABC with AB = AC. Show that CD < BD.

Answer:



We know that the angles opposite to the equal sides are equal.

∠ABC = ∠ACB ------------ (1)

Considering triangles ABC and DBC,

∠B = common angle

Since ∠DBC is an internal angle of ∠B

So, ∠ABC > ∠DBC

From (1), ∠ACB > ∠DBC

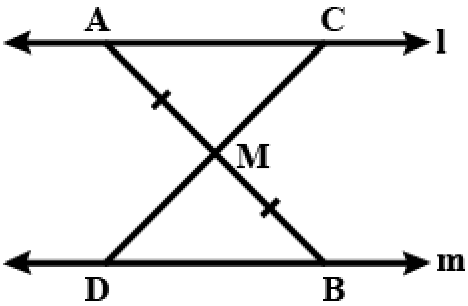
We know that in a triangle a side opposite to a greater angle is longer.

BD > CD

Therefore, CD < BD

Q8. In given figure l || m and M are the mid-points of a line segment AB. Show that M is also the mid-point of any line segment CD, having its endpoints on l and m, respectively.

Answer:



Given, l is parallel to m

M is the midpoint of a line segment AB

We have to show that M is also the mid-point of any line segment CD, having its end points on l and m, respectively.

Since M is the midpoint of AB

AB = AM + BM

AM = BM

We know that the alternate interior angles are equal

∠CAB = ∠ABD

We know that vertically opposite angles are equal.

∠AMC = ∠DMB

Considering triangles AMC and BMD,

∠CAB = ∠ABD

Given, AM = BM

∠AMC = ∠DMB

ASA criterion states that two triangles are congruent, if any two angles and the side included between them of one triangle are equal to the corresponding angles and the included side of the other triangle.

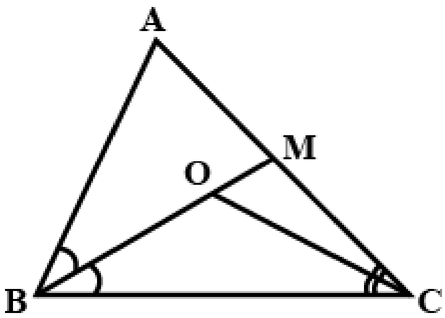
By ASA criterion, Δ AMC ≅ Δ BMD

By CPCTC, MC = MD

Therefore, M is the point of the line segment CD.

Q9: Bisectors of the angles B and C of an isosceles triangle with AB = AC intersect each other at O. BO is produced to a point M. Prove that ∠MOC = ∠ABC.

Answer:



Given, ABC is an isosceles triangle with AB = AC

Bisectors of the angles B and C intersect each other at O

BO is produced to a point M

We have to prove that ∠MOC = ∠ABC

Considering triangle ABC,

Given, AB = AC

We know that the angles opposite to equal sides are equal.

∠ACB = ∠ABC

Since OB is the bisector of angle B

∠ABO = ∠OBC

∠ABC = ∠ABO + ∠OBC

∠ABC = ∠OBC + ∠OBC

∠ABC = 2∠OBC ------------ (1)

Since OC is the bisector of angle C

∠ACO = ∠OCB

∠ACB = ∠ACO + ∠OCB

∠ACB = ∠OCB + ∠OCB

∠ACB = 2∠OCB ------------- (2)

So, 2∠OCB = 2∠OBC

∠OCB = ∠OBC --------------- (3)

We know that the exterior angle of a triangle is equal to the sum of two interior angles.

∠MOC = ∠OBC + ∠OCB

From (3),

∠MOC = ∠OBC + ∠OBC

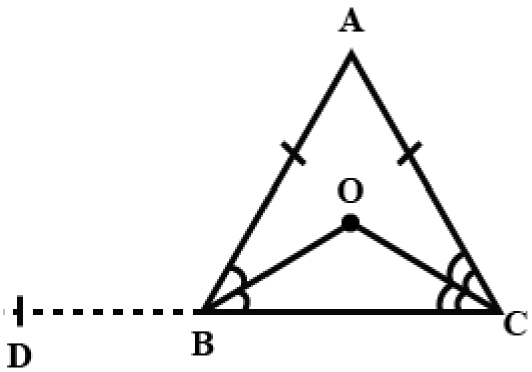
∠MOC = 2∠OBC

From (1),

Therefore, ∠MOC = ∠ABC

Q10. Bisectors of the angles B and C of an isosceles ΔABC with AB = AC intersect each other at O. Show that the external angle adjacent to ∠ABC is equal to ∠BOC.

Answer:



Given, ABC is an isosceles triangle with AB = AC

Bisectors of the angles B and C intersect each other at O.

We have to show that external angle adjacent to ∠ABC is equal to ∠BOC

Line segment CB is extended to a point D outside the triangle.

Considering triangle ABC,

Given, AB = AC

We know that the angles opposite to the equal sides are equal.

∠ACB = ∠ABC

Since OB is the bisector of angle B

∠ABO = ∠OBC

∠ABC = ∠ABO + ∠OBC

∠ABC = ∠OBC + ∠OBC

∠ABC = 2∠OBC ------------ (1)

Since OC is the bisector of angle C

∠ACO = ∠OCB

∠ACB = ∠ACO + ∠OCB

∠ACB = ∠OCB + ∠OCB

∠ACB = 2∠OCB ------------- (2)

So, 2∠OCB = 2∠OBC

∠OCB = ∠OBC --------------- (3)

Considering triangle BOC,

By angle sum property,

∠OBC + ∠OCB + ∠BOC = 180°

From (3),

∠OBC + ∠OBC + ∠BOC = 180°

2∠OBC + ∠BOC = 180°

From (2),

∠ABC + ∠BOC = 180°

We know that the linear pair of angles is equal to 180 degrees.

∠ABD + ∠ABC = 180°

∠ABC = 180° - ∠ABD

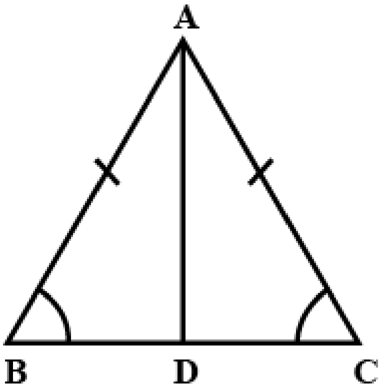
Now, 180° - ∠ABD + ∠BOC = 180°

180° - 180° + ∠BOC = ∠ABD

∠BOC = ∠ABD

Therefore, the external angle adjacent to ∠ABC is equal to ∠BOC.

Q11. In the following figure if AD is the bisector of ∠BAC, then prove that AB > BD.



Answer:

Given, ABC is a triangle

AD is the bisector of ∠BAC

We have to prove that AB > BD

Since AD is the bisector of ∠BAC

∠BAD = ∠CAD ----------------------------- (1)

We know that the exterior angle of a triangle is greater than each of the opposite interior angles.

∠ADB > ∠CAD

From (1),

∠ADB > ∠BAD

We know that in a triangle a side opposite to a greater angle is longer.

The side AB is greater than BD.

Therefore, AB > BD

Exercise 7.4

Q1. Find all the angles of an equilateral triangle.

Answer:

In equilateral triangle,

All the sides are equal.

Therefore, all angles are also equal

Let the angles of an equilateral triangle = x

According to the angle sum property,

We know that the sum of the interior angles is equal to 180^o.

$x + x + x = 180^o$

$3x = 180$

$x = 60°$

Therefore, all the angles of an equilateral triangle are 60°

Q2. The image of an object placed at point A before a plane mirror LM is seen at point B by an observer at D as shown in Fig. 7.12. Prove that the image is as far behind the mirror as the object is in front of the mirror.

[Hint: CN is normal to the mirror. Also, angle of incidence = angle of reflection].

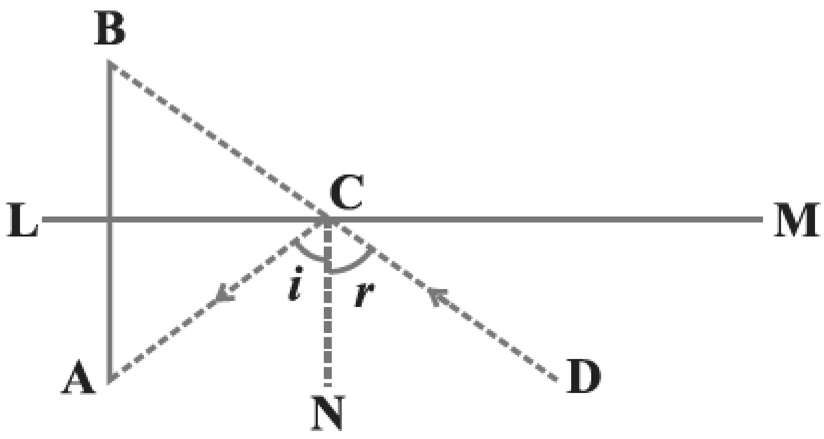


Fig. 7.12

Answer:

Let AB intersect LM at O. We have to prove that AO = BO.

Now, ∠i = ∠r …(1)

[∵Angle of incidence = Angle of reflection]

∠B = ∠i [Corresponding angles] …(2)

And ∠A = ∠r [Alternate interior angles] …(3)

From (1), (2) and (3), we get

∠B = ∠A

⇒ ∠BCO = ∠ACO

In ΔBOC and ΔAOC we have

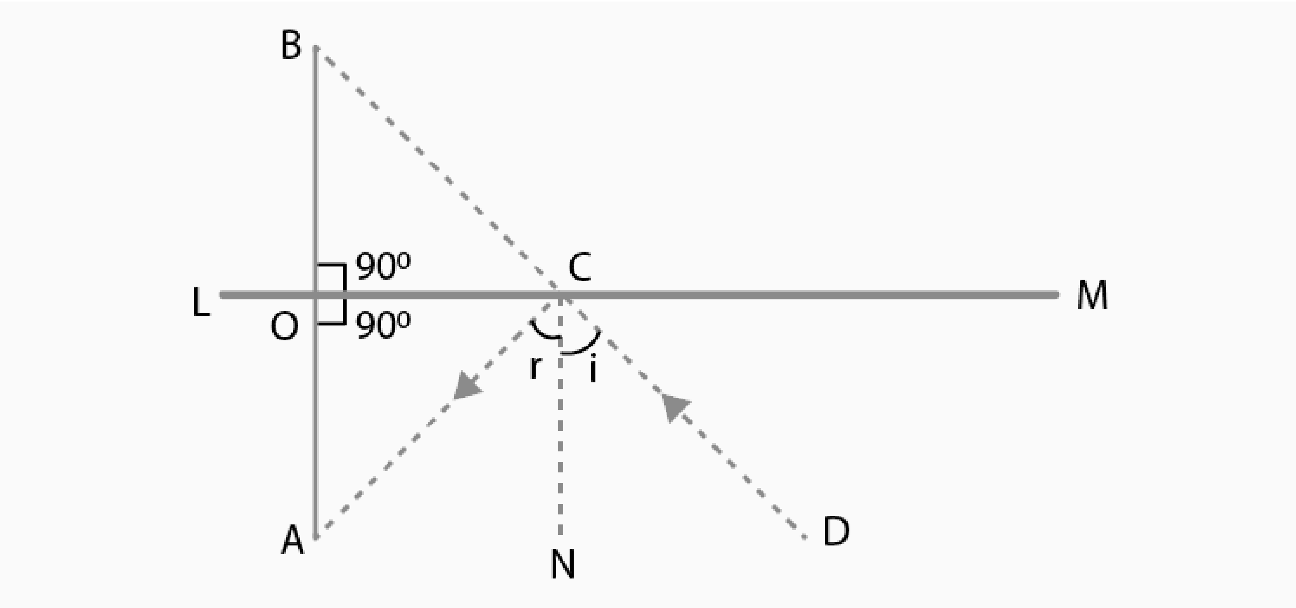
∠1 = ∠2 [Each = 90^o]

OC = OC [Common side]

And ∠BCO = ∠ACO [Proved above]

ΔBOC ≅ ΔAOC [ASA congruence rule]

Hence, AO = BO [CPCT]



From (1), (2) and (3), we get

∠B = ∠A

⇒ ∠BCO = ∠ACO

In ΔBOC and ΔAOC we have

∠1 = ∠2 [Each = 90º]

OC = OC [Common side]

And ∠BCO = ∠ACO [Proved above]

ΔBOC ≅ ΔAOC [ASA congruence rule]

Hence, AO = BO [CPCT]

Q3. ABC is an isosceles triangle with AB = AC and D is a point on BC such that AD ⊥ BC (Fig. 7.13).

To prove that ∠BAD = ∠CAD, a student proceeded as follows:

In Δ ABD and Δ ACD,

AB = AC (Given)

∠B = ∠C (because AB = AC)

And ∠ADB = ∠ADC

Therefore, Δ ABD Δ Δ ACD (AAS)

So, ∠BAD = ∠CAD (CPCT)

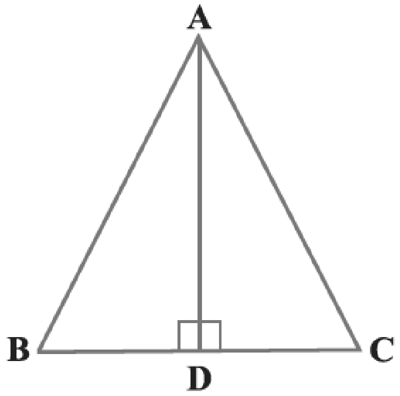
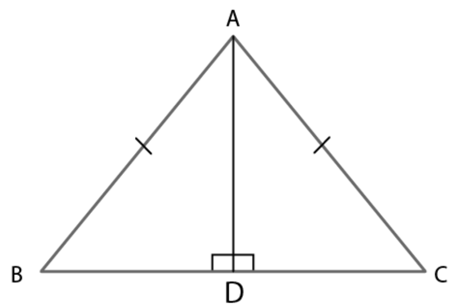


Fig. 7.13

What is the defect in the above arguments?

[Hint: Recall how ∠B = ∠C is proved when AB = AC].

Answer:



In Δ ABD and Δ ADC, we have

∠ADB = ∠ADC

According to the question,

AB = BC

AD = AD [Common side]

By RHS criterion of congruence,

We have,

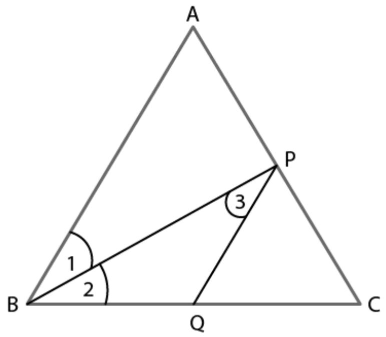
Δ ABD ≅ Δ ACD

∠BAD = ∠CAD [CPCT]

Hence Proved.

Q4. P is a point on the bisector of ∠ABC. If the line through P, parallel to BA meets BC at Q, prove that BPQ is an isosceles triangle.

Answer:



To prove: BPQ is an isosceles triangle.

According to the question,

Since, BP is the bisector of ∠ABC,

∠1 = ∠2 … (1)

Now, PQ is parallel to BA and BP cuts them

∠1 = ∠3 [Alternate angles] … (2)

From equations, (1) and (2),

We get

∠2 = ∠3

In Δ BPQ,

We have

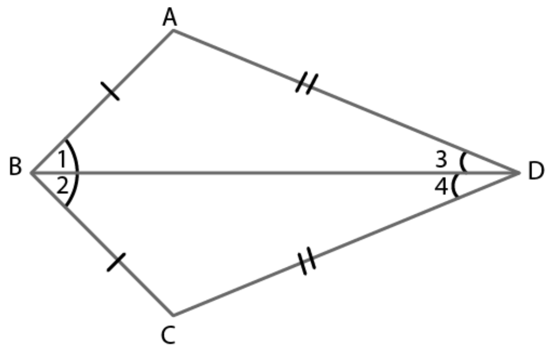
∠2 = ∠3

PQ = BQ

Hence, BPQ is an isosceles triangle.

Q5. ABCD is a quadrilateral in which AB = BC and AD = CD. Show that BD bisects both the angles ABC and ADC.

Answer:



According to the question,

In ΔABC and ΔCBD,

We have

AB = BC

AD = CD

BD = BD [Common side]

ΔABC ≅ ΔCBD [By SSS congruence rule]

⇒ ∠1 = ∠2 [CPCT]

And ∠3 = ∠4

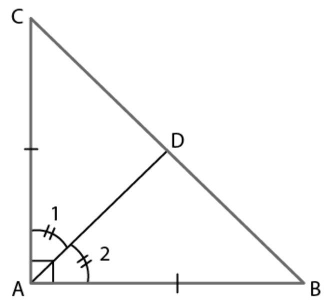
Hence, BD bisects both ∠ABC and ∠ADC.

Q6. ABC is a right triangle with AB = AC. Bisector of ∠A meets BC at D. Prove that BC = 2 AD.

Answer:

Given: A right angles triangle with AB = AC bisector of ∠A meets BC at D.

To prove: BC = 2AD



Proof:

According to the question,

In right Δ ABC,

AB = AC

Since, hypotenuse is the longest side,

BC is hypotenuse

∠BAC = 90^o

Now,

In Δ CAD and Δ BAD,

We have,

AC = AB

Since, AD is the bisector of ∠A,

∠1 = ∠2

AD = AD [Common side]

Now,

By SAS criterion of congruence,

We get,

Δ CAD ≅ Δ BAD

CD = BD [CPCT]

Since the mid-point of the hypotenuse of a right triangle is equidistant from the 3 vertices of a triangle.

AB = BD = CD …(1)

Now, BC = BD + CD

⇒ BC = AD + AD [Using eq.(1)]

⇒ BC = 2AD

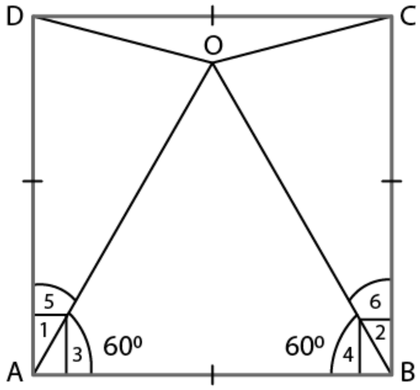
Hence, proved.

Q7. O is a point in the interior of a square ABCD such that OAB is an equilateral triangle. Show that Δ OCD is an isosceles triangle.

Answer:

According to the question,

It is given that: A square ABCD and OA = OB = AB.



To prove: Δ OCD is an isosceles triangle.

Proof:

In square ABCD,

Since ∠1 and ∠2 is equal to 90°

∠1 = ∠2 …(1)

Now, in Δ OAB , we have

Since ∠3 and ∠4 is equal to 60°

∠3 = ∠4 …(2)

Subtracting equations (2) from (1),

We get

∠1−∠3 = ∠2 −∠4

⇒ ∠5 = ∠6

Now,

In Δ DAO and Δ CBO,

AD = BC [Given]

∠5 = ∠6 [Proved above]

OA = OB [Given]

By SAS criterion of congruence,

We have

Δ DAO ≅ Δ CBO

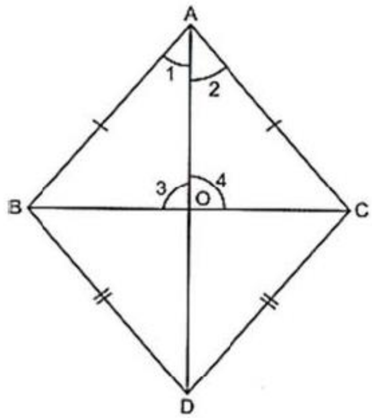
OD = OC

⇒ Δ OCD is an isosceles triangle.

Hence, proved.

Q8. ABC and DBC are two triangles on the same base BC such that A and D lie on the opposite sides of BC, AB = AC and DB = DC. Show that AD is the perpendicular bisector of BC.

Answer:



Given: Δ ABC and Δ DBC on the same base BC. Also, AB = AC and BD = DC.

To prove: AD is the perpendicular bisector of BC i.e., OB = OC

Proof: In Δ BAD and Δ CAD, we have

AB = AC [Given]

BD = CD [Given]

AD = AD [common side]

So, by SSS criterion of congruence, we have

Δ BAD ≅ Δ CAD

∠1 = ∠2 [CPCT]

Now, in Δ BAO and Δ CAO, we have

AB = AC [Given]

∠1 = ∠2 [Proved above]

AO = AO [Common side]

So, by SAS criterion of congruence, we have

Δ BAO ≅ Δ CAO

BO = CO [CPCT]

And, ∠3 = ∠4 [CPCT]

But, ∠3+∠4 =180^o [Linear pair axiom]

⇒ ∠3+∠3 =180

⇒ 2∠3 =180

⇒ ∠3 =180/2

⇒ ∠3 = 90°

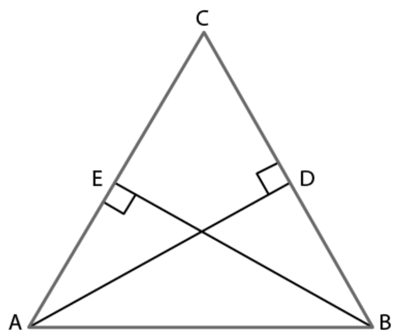
Since BO = CO and ∠3 = 90°,

AD is a perpendicular bisector of BC.

Hence proved.

9. ABC is an isosceles triangle in which AC = BC. AD and BE are respectively two altitudes to sides BC and AC. Prove that AE = BD.

Answer:



According to the question,

In Δ ADC and Δ BEC,

We have

AC = BC [Given] …(1)

Since ∠ADC and ∠BEC = 90°

∠ADC = ∠BEC

∠ACD = ∠BCE [Common angle]

Δ ADC ≅ Δ BEC [By ASA congruence rule]

CE = CD … (2) [CPCT]

Subtracting equation (2) from (1),

We get

AC – CE = BC – CD

⇒ AE = BD

Hence proved.

Q10. Prove that the sum of any two sides of a triangle is greater than twice the median with respect to the third side.

Answer:

According to the question,

We have, Δ ABC with median AD.

To prove:

AB + AC > 2AD

AB + BC > 2AD

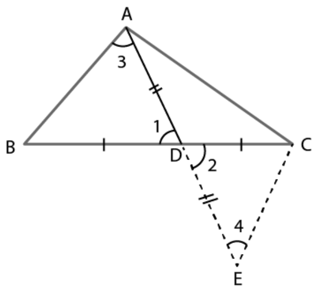
BC + AC > 2AD

Construction:

Extend AD to E such that DE = AD

Join EC.

Proof:



In Δ ADB and Δ EDC,

AD = ED [By construction]

∠1 = ∠2 [Vertically opposite angles are equal]

DB = DC [Given]

So, by the SAS criterion of congruence, we have

Δ ADB ≅ Δ EDC

AB = EC [CPCT]

And ∠3 = ∠4 [CPCT]

Now, in Δ AEC,

Since the sum of the lengths of any two sides of a triangle must be greater than the third side,

We have

AC + CE > AE

⇒ AC + CE > AD + DE

⇒ AC +CE > AD+ AD [\AD = DE]

⇒ AC + CE > 2AD

⇒ AC + AB > 2AD [∵AB = CE]

Similarly,

We get,

AB + BC > 2AD and BC + AC >2AD.

Hence proved.

Q11: Show that in a quadrilateral ABCD, AB + BC + CD + DA< 2 (BD + AC)

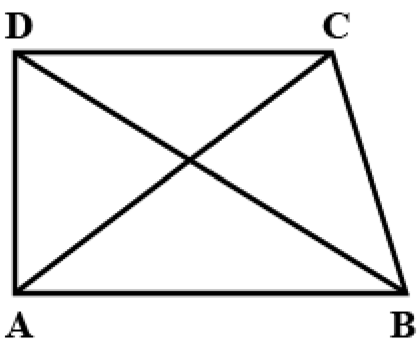
Answer:

Thinking Process:

Firstly, draw a quadrilateral ABCD. Further, use the property of a triangle that the sum of two sides of a triangle is greater than the third side and show the required result.

This question is based on the triangle inequality theorem that the sum of lengths of two sides of a triangle is always greater than the third side.

Now visually identify that the quadrilateral ABCD is divided by diagonals AC and BD into four triangles.



O is the point where the two diagonals are intersecting.

Now, take each triangle separately that is the triangle AOB, COD, BOC, and AOD and apply the above property and then add L.H.S and R.H.S of the equation formed

In triangle AOB,

AB < OA + OB.........(1)

In triangle COD,

CD < OC + OD........(2)

In triangle AOD,

DA < OD + OA.........(3)

In triangle COB,

BC < OC + OB........(4)

Adding equation (1), (2), (3) and (4) we get,

AB + BC + CD + DA < OA + OB + OC + OD + OD + OA + OC + OB

= AB + BC + CD + DA < 2OA + 2OB + 2OC + 2OD

= AB + BC + CD + DA < 2(OA + OB) + 2(OC + OD)

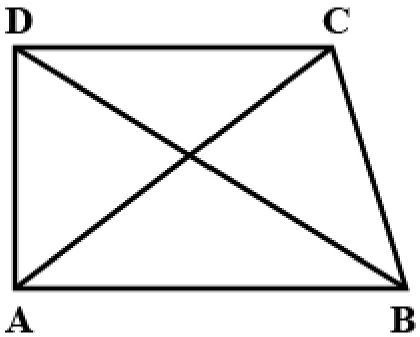
= AB + BC + CD + DA < 2(AC + BD) [From diagram OA + OB = AC and OC + OD = BD]

Yes, AB + BC + CD + DA < 2(AC + BD) is true.

Useful Tip : Whenever you encounter problems of this kind, it is best to think of the property based on the sum of lengths of any two sides of a triangle is always greater than the third side.

Q12. Show that in a quadrilateral ABCD, AB + BC + CD + DA > AC + BD.

Answer:



This question is based on the triangle inequality theorem that the sum of lengths of two sides of a triangle is always greater than the third side.

Now visually identify that the quadrilateral ABCD. It is divided by diagonals AC and BD into four triangles.

Now, take each triangle separately and apply the above property and then add L.H.S and R.H.S of the equation formed.

In triangle ABC,

AB + BC > AC ……………(1)

In triangle ADC,

AD + CD > AC …………. (2)

In triangle ADB,

AD + AB > DB --------------(3)

In triangle DCB,

DC+ CB > DB..................(4)

Adding equations (1), (2), (3) and (4) we get,

AB + BC + AD + CD + AD + AB + DC+ CB > AC + AC + DB + DB

AB + AB + BC + BC + CD + CD + AD + AD > 2AC + 2DB

2AB + 2BC + 2CD + 2AD > 2AC + 2DB

AB + BC + CD + AD > AC + DB

Hence, AB + BC + CD + DA > AC + BD is true

Useful Tip:

Whenever you encounter problems of this kind, it is best to think of the property based on the sum of lengths of any two sides of a triangle is always greater than the third side.

Q13. In a ABC, D is the mid-point of side AC such that BD = ½ AC. Show that ∠ABC is a right angle.

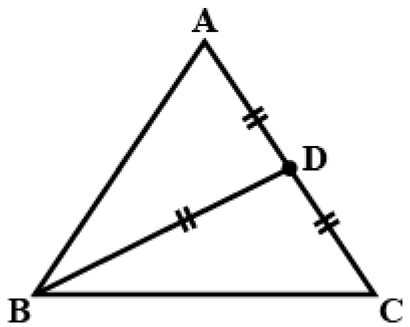
Answer:

Given, ABC is a triangle.

D is the midpoint of side AC

BD = 1/2 AC ------------------------- (1)

We have to show that ∠ABC is a right angle.



D is the midpoint of AC.

AD = CD

AC = AD + CD

Now, AC = AD + AD or CD + CD

AC = 2AD or 2CD

So, AD = CD = 1/2 AC ----------- (2)

Comparing (1) and (2),

AD = CD = BD --------------------- (3)

Considering triangle DAB,

From (3), AD = BD

We know that the angles opposite to the equal sides are equal.

∠ABD = ∠BAD ---------------------------- (4)

Considering triangle DBC,

From (3), BD = CD

We know that the angles opposite to the equal sides are equal.

∠BCD = ∠CBD ---------------------------- (5)

Considering triangle ABC,

By angle sum property,

∠ABC + ∠BAC + ∠ACB = 180°

From the figure, ∠BAC = ∠BAD

∠ACB = ∠DCB

Now, ∠ABC + ∠BAD + ∠BCD = 180°

From (4) and (5),

∠ABC + ∠ABD + ∠CBD = 180°

Given, BD = 1/2 AC

∠ABC = ∠ABD + ∠CBD

Now, ∠ABC + ∠ABC = 180°

2∠ABC = 180°

∠ABC = 180°/2

∠ABC = 90°

Therefore, it is proven that ∠ABC = 90°

Q14. In a right triangle, prove that the line segment joining the mid-point of the hypotenuse to the opposite vertex is half the hypotenuse.

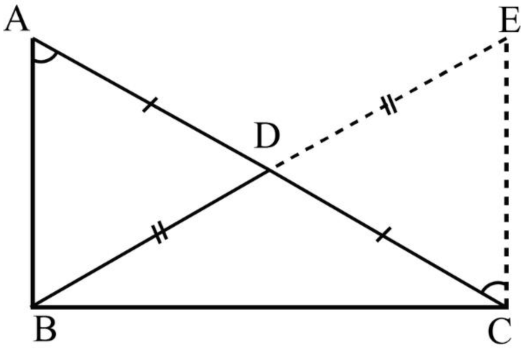
Answer:

Consider a right triangle ABC right angled at B.

D is the midpoint of the hypotenuse AC

Extend BD upto E so that BD = DE and join CE

We have to prove that the line segment joining the midpoint of the hypotenuse to the opposite vertex is half the hypotenuse i.e.,BD = 1/2 AC



Considering triangles ADB and CDE,

Given, D is the midpoint of AC

So, AC = CD

Also, BD = DE

We know that the vertically opposite angles are equal.

∠ADB = ∠CDE

SAS criterion states that if two sides of one triangle are proportional to two sides of another triangle and their included angles are congruent, then the triangles are congruent.

By SAS criterion, the triangles ADB and CDE are congruent.

By CPCTC,

AB = EC

∠BAD = ∠DCE

Where ∠BAD and ∠DCE are alternate angles.

This implies that EC is parallel to AB and BC is a transversal.

We know that if two lines are parallel and cut by a transversal, the sum of interior angles lying on the same side of the transversal is always supplementary.

$\angle ABC + \angle BCE = 180°$

$90 + \angle BCE = 180°$

$\angle BCE = 180° - 90°$

$\angle BCE = 90°$

Considering triangles ABC and ECB,

We know AB = EC

Common side = BC

$\angle ABC = \angle ECB = 90°$

By SAS criterion, the triangles ABC and ECB are congruent

By CPCT,

AC = EB

Dividing by 2 on both sides,

AC/2 = EB/2

We know BD = DE

So, BE = BD + DE

BE = BD + BD

BE = 2BD

BD = BE/2

Now, AC/2 = BD

Therefore, BD = 1/2 AC

Q15. Two lines l and m intersect at point 0 and P is a point on a line n passing through point 0 such that P is equidistant from l and m. Prove that n is the bisector of the angle formed by l and m.

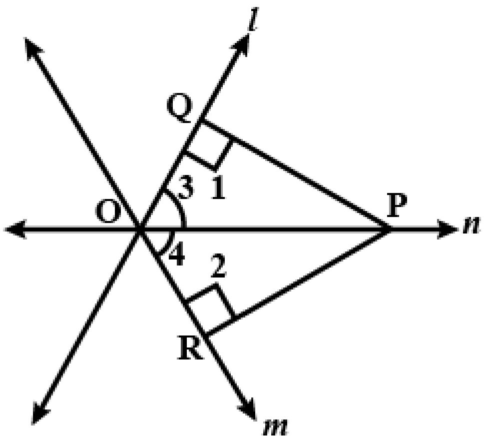
Answer:

Given, two lines l and m intersect at point O

P is a point on a line n passing through the point O such that P is equidistant from l and m.

P meets the line l at Q and the line m at R

Given, PQ = PR



The angle formed by l and m is ∠QOR.

We have to prove that n is the bisector of ∠QOR.

Considering triangles OQP and ORP,

Given, P is equidistant from the lines l and m.

So, PQ and PR should be perpendicular to the lines l and m.

∠PQO = ∠PRO = 90°

Common side = OP

Given, PQ = PR

RHS Congruence Rule states that two right triangles are congruent if the hypotenuse and one side of one triangle are equal to the corresponding hypotenuse and one side of the other triangle.

By RHS criterion, the triangles OQP and ORP are congruent

By CPCTC,

∠POQ = ∠POR

Therefore, n is the bisector of ∠QOR.

Q16. The line segment joining the mid-points M and N of parallel sides AB and DC, respectively of a trapezium ABCD is perpendicular to both the sides AB and DC. Prove that AD = BC.

Answer:

Given, ABCD is a trapezium

M and N are the midpoints of parallel sides AB and DC of the trapezium.

The line segment joining M and N is perpendicular to both the sides AB and DC.

We have to prove that AD = BC

Since M is the midpoint of AB

AM = MB

Considering triangles AMN and BMN,

AM = MB

∠3 = ∠4 = 90

Common side = MN

By SAS criterion, the triangles AMN and BMN are congruent

By CPCTC, ∠1 = ∠2

Multiplying by -1 on both sides,

(-1)∠1 = (-1)∠2

Adding 90 on both sides,

90° - ∠1 = 90° - ∠2

From the figure,

∠AND = ∠BNC

Considering triangle ADN and BCN,

Since triangles AMN and BMN are congruent, AN = BN

∠AND = ∠BNC

Since N is the midpoint of CD

CN = DN

By SAS criterion, the triangles AMN and BMN are congruent.

The Corresponding Parts of Congruent Triangles are Congruent (CPCTC) theorem states that when two triangles are congruent, then their corresponding sides and angles are also congruent or equal in measurements.

By CPCTC,

AD = BC

Therefore, it is proved that AD = BC

Q17. If ABCD is a quadrilateral such that diagonal AC bisects the angles A and C, then prove that AB = AD and CB = CD.

Answer:

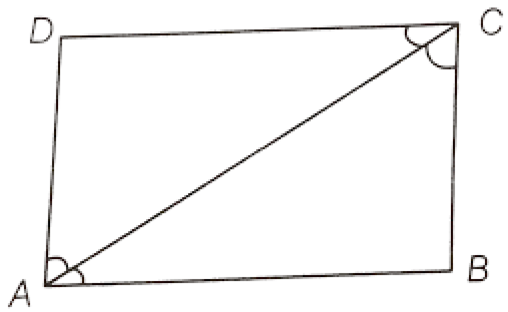
Given, ABCD is a quadrilateral

The diagonal AC bisects the angles A and C.

We have to prove that AB = AD and CB = CD.

Considering triangles ADC and ABC,

AC is the bisector of angle A



So, ∠DAC = ∠BAC

AC is the bisector of angle C

So, ∠DCA = ∠BCA

Common side = AC

SAS criterion states that if two sides of one triangle are proportional to two sides of another triangle and their included angles are congruent, then the triangles are congruent

By the SAS criterion, the triangles ADC and ABC are congruent

By CPCTC,

AD = AB

CD = CB

Therefore, it is proven that AD = AB and CD = CB.

Q18. If ABC is a right-angled triangle such that AB = AC and the bisector of angle C intersects the side AB at D, then prove that AC + AD = BC.

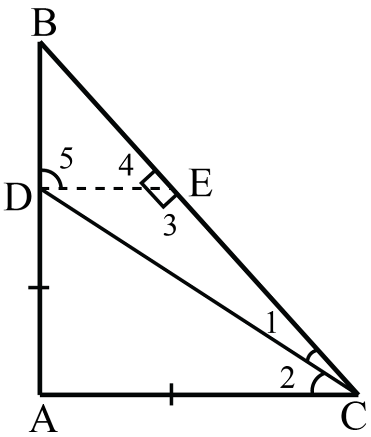
Answer:

Given, that ABC is the right triangle

AB = AC

The bisector of angle C intersects the side AB at D.

We have to prove that AC + AD = BC



In right triangle ABC,

AB = AC

BC is the hypotenuse

∠A = 90°

Draw DE perpendicular to BC

∠E = 90°

∠3 = ∠4 = 90°

Considering triangles DAC and DEC,

∠A = ∠3 = 90°

CD is the bisector of angle C.

So, ∠1 = ∠2

Common side = CD

AAS criterion (Angle-angle-side) states that when two angles and a non-included side of a triangle are equal to the corresponding angles and sides of another triangle, then the triangles are said to be congruent.

By AAS criterion, the triangles DAC and DEC are congruent.

The Corresponding Parts of Congruent Triangles are Congruent (CPCTC) theorem states that when two triangles are congruent, then their corresponding sides and angles are also congruent or equal in measurements.

By CPCTC,

DA = DE ------------ (1)

AC = EC ------------ (2)

In triangle ABC,

AB = AC

We know that the angles opposite to equal sides are equal.

So, $\angle C = \angle B$ ------------- (3)

By angle sum property,

$\angle A + \angle B + \angle C = 180°$

From (3), $\angle A + \angle B + \angle B = 180°$

$\angle A + 2\angle B = 180°$

$90° + 2\angle B = 180°$

$2\angle B = 180° - 90°$

$\angle B = 90°/2$

$\angle B = 45°$

In triangle BED,

$\angle B + \angle E + \angle D = 180°$

$\angle B + \angle 4 + \angle 5 = 180°$

$90° + \angle 5 = 180° - \angle B$

$\angle 5 = 180° - 45° - 90°$

$\angle 5 = 45°$

So, $\angle B = \angle 5$

We know that the sides opposite to equal angles are equal.

DE = BE -------------- (4)

From (1) and (4),

DA = DE = BE ----------- (5)

From the figure,

BC = BE + CE

From (2) and (5),

BC = CA + DA

Therefore, BC = AC + AD

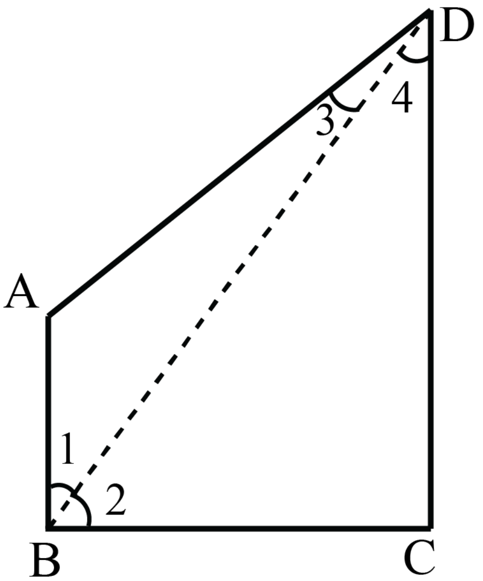
Q19. If AB and CD are the smallest and largest sides of a quadrilateral ABCD, out of ∠B and ∠D decide which is greater.

Answer:

Given, ABCD is a quadrilateral.

AB and CD are the smallest and largest sides of a quadrilateral.

We have to find the greater angle between ∠B and ∠D.



Join the diagonal BD of the quadrilateral.

In triangle ABD,

Given, AB is the smallest side of ABCD.

So, AD > AB

We know that in a triangle angle opposite to the larger side is greater.

So, ∠1 > ∠3 ------------ (1)

Given, CD is the largest side of ABCD

So, CD > BC

We know that in a triangle angle opposite to the larger side is greater.

So, ∠2 > ∠4 ----------------- (2)

Adding (1) and (2),

∠1 + ∠2 > ∠3 + ∠4

From the figure,

∠B = ∠1 + ∠2

∠D = ∠3 + ∠4

Now, ∠B > ∠D

Therefore, it is proven that angle B is greater than angle D.

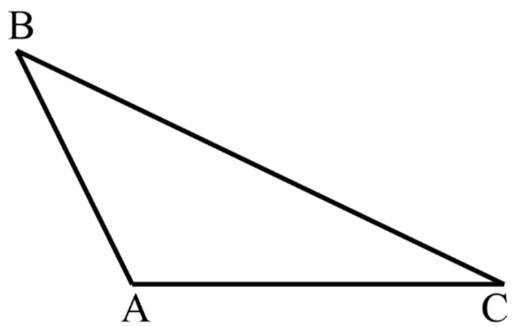
Q20. Prove that in a triangle, other than an equilateral triangle, the angle opposite the longest side is greater than 2/3 of a right angle.

Answer:

Consider a triangle ABC with BC as the longest side.

The angle opposite to the longest side is angle A

We have to prove that the angle opposite to the longest side is greater than 2/3 of a right angle.



In triangle ABC,

Since BC is the largest side

So, BC > AB

We know that the angle opposite to the longest side is greater.

∠A > ∠C -------------------- (1)

Similarly, BC > AC

So, ∠A > ∠B --------------- (2)

Adding (1) and (2),

∠A + ∠A > ∠C + ∠B

2∠A > ∠C + ∠B

Adding ∠A on both sides,

∠A + 2∠A > ∠C + ∠B + ∠A

Angle sum property states that the sum of all three interior angles of a triangle is always equal to 180 degrees.

So, ∠A + ∠B + ∠C = 180°

Now, 3∠A > 180°

∠A > 180°/3

∠A > 2/3 (90°)

Therefore, angle A is greater than 2/3 of a right angle.

Q21. If ABCD is quadrilateral such that AB = AD and CB = CD, then prove that AC is the perpendicular bisector of BD.

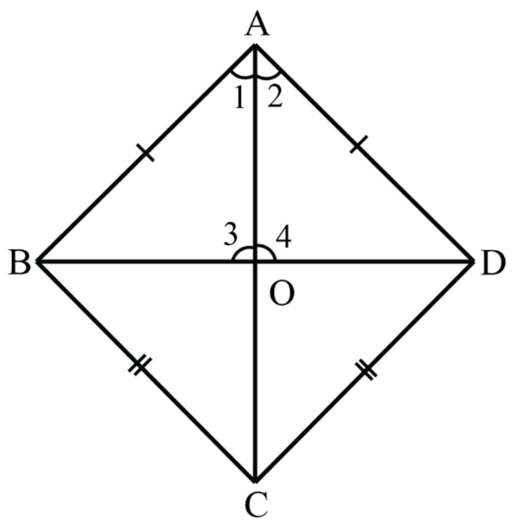
Answer:

Given, ABCD is a quadrilateral

AB = AD

CB = CD

We have to prove that AC is the perpendicular bisector of BD.



Considering triangles ABC and ADC,

Common side = AC

Given, AB = AD

BC = BD

SSS Criterion (Side-Side-Side) states that if all three sides of one triangle are equal to the three corresponding sides of another triangle, the two triangles are said to be congruent.

By SSS criterion, the triangles ABC and ADC are congruent

The Corresponding Parts of Congruent Triangles are Congruent (CPCTC) theorem states that when two triangles are congruent, then their corresponding sides and angles are also congruent or equal in measurements.

By CPCTC,

$\angle 1 = \angle 2$

Considering triangles AOB and AOD,

Common side = AO

Given, AB = AD

$\angle 1 = \angle 2$

By SSS criterion, the triangles AOB and AOD are congruent

By CPCTC,

BO = DO

$\angle 3 = \angle 4$

We know that the linear pair of angles is always supplementary.

So, $\angle 3 + \angle 4 = 180°$

$\angle 3 + \angle 3 = 180°$

$2\angle 3 = 180°$

$\angle 3 = 180°/2$

$\angle 3 = 90°$

This implies that AC is perpendicular to BD

Therefore, AC is the perpendicular bisector of BD.