Exercise 1.1

Using appropriate properties find:
(i) -2/3 × 3/5 + 5/2 - 3/5 × 1/6
(ii) 2/5 × (-3/7) -1/6 × 3/2 + 1/14 × 2/5

Answer:

Rational numbers numbers are in the form of p/q, where p and q can be any integer and q ≠ 0.

By using commutative property of multiplication and addition we proceed with the questions.

(i) -2/3 × 3/5 + 5/2 - 3/5 × 1/6

Answer: $\frac{3}{5}$ X $\left(-\frac{2}{3}\right) + \frac{5}{2} - \frac{3}{5}$ X $\frac{1}{6}$ [By commutativity of multiplication]

= $\frac{3}{5}$ X $\left(-\frac{2}{3}\right) - \frac{3}{5}$ X $\frac{1}{6} + \frac{5}{2}$ [By commutativity of addition]

Rearranging to take 3/5 common

= $\frac{3}{5}$ X $\left(-\frac{2}{3} - \frac{1}{6}\right) + \frac{5}{2}$

= $\frac{3}{5}$ X $\left(-\frac{5}{6}\right) + \frac{5}{2}$

= $-\frac{1}{2} + \frac{5}{2}$

= 2

(ii) 2/5 × (-3/7) -1/6 × 3/2 + 1/14 × 2/5

Answer: Taking 2 / 5 as common

= $\frac{2}{5}$ X $\left[\left(-\frac{3}{7}\right) + \frac{1}{14}\right] - \frac{1}{6}$ X $\frac{3}{2}$ [By distributive property of multiplication]

= $\frac{2}{5}$ X $\left(\frac{-3\ x\ 2+1}{14}\right) - \frac{1}{6}$ X $\frac{3}{2}$

= $\frac{2}{5}$ X $\left(\frac{-5}{14}\right) - \frac{1}{6}$ X $\frac{3}{2}$

= $-\frac{1}{7} - \frac{1}{4}$

= $\frac{-(4+7)}{28} = \frac{-11}{28}$

2. Write the additive inverse of each of the following:

(i) 2/8 (ii) -5/9 (iii) -6/-5 (iv) 2/-9 (v) 19/-6

Answer:

(i) Additive Inverse of $\frac{2}{8}$ is - $\frac{2}{8} = \frac{-2}{8}$

We see that, $\frac{2}{8} + \left(\frac{-2}{8}\right) = 0$

(ii) Additive Inverse of $\frac{-5}{9}$ is - $\left(\frac{-5}{9}\right) = \frac{5}{9}$

We see that, $\frac{-5}{9} + \frac{5}{9} = 0$

(iii) Additive Inverse of $\frac{-6}{-5}$ is - $\left(\frac{-6}{-5}\right) = \left(-\frac{6}{5}\right) = \frac{-6}{5}$

We see that, $\frac{-6}{-5} + \left(\frac{-6}{5}\right) = \frac{6}{5} - \frac{6}{5} = 0$

(iv) Additive Inverse of $\frac{2}{-9}$ is - $\left(\frac{2}{-9}\right) = -\left(\frac{-2}{9}\right) = \frac{2}{9}$

We see that, $\frac{2}{-9} + \frac{2}{9} = \frac{-2}{9} + \frac{2}{9} = 0$

(v) Additive Inverse of $\frac{19}{-6} = -\left(\frac{19}{-6}\right) = -\left(\frac{-19}{6}\right) = \frac{19}{6}$

We see that, $\frac{19}{-6} + \frac{19}{6} = \frac{-19}{6} + \frac{19}{6} = 0$

3. Verify that: -(-x) = x for:

(i) x = 11/15

(ii) x = -13/17

Answer:

Rational numbers are in the form of p/q, where p and q can be any integer and q ≠ 0.

The negative of a negative rational number is the same rational number.

(i) x = $\frac{11}{15}$

$-(-x) = -\left(\frac{-11}{15}\right) = \frac{11}{15} = x$

Hence proved.

(ii) x = -13/17

$- (- x) = - \left[-\left(\frac{-13}{17}\right)\right] = \frac{13}{17} = x$

Hence proved.

4. Find the multiplicative inverse of the following
(i) -13 (ii) -13/19 (iii) 1/5 (iv) -5/8 × -3/7 (v) -1 × -2/5 (vi) -1

Answer: The reciprocal of a given rational number is known as its multiplicative inverse. The product of a rational number and its multiplicative inverse is 1.

(i) The Multiplicative inverse of -13 is $\frac{-1}{13}$

$\because -13 \times \left(\frac{-1}{13}\right) = 1$

(ii) The Multiplicative inverse of $\frac{-13}{19}$ is $\frac{-19}{13}$

$\because \frac{-13}{19} \text{ x } \left(\frac{-19}{13}\right) = 1$

(iii) The Multiplicative inverse of $\frac{1}{5}$ is 5

$\because \frac{1}{5} \text{ x } 5 = 1$

(iv) The Multiplicative inverse of $\frac{-5}{8} \text{ x } \frac{-3}{7}$ is $\frac{56}{15}$

$\because \frac{-5}{8} \text{ x } \frac{-3}{7} = \frac{15}{56}$ and $\frac{15}{56} \text{ x } \frac{56}{15} = 1$

(v) The Multiplicative inverse of $-1 \text{ x } \frac{-2}{5}$ is $\frac{5}{2}$

$\because -1 \text{ x } \frac{-2}{5} = \frac{2}{5}$ and $\frac{2}{5} \text{ x } \frac{5}{2} = 1$

(vi) The Multiplicative inverse of -1 is -1

$\because -1 \times (-1) = 1$

5. Name the property under multiplication used in each of the following:

(i) - 4/5 × 1 = 1 × -4/5 = -4/5 **(ii) -13/17 × -2/7 = -2/7 × -13/17**

(iii) -19/29 × 29/-19 = 1

Answer:

(i) $\frac{-4}{5}$ x 1 = 1 x $\frac{-4}{5}$ = $\frac{-4}{5}$

∴ 1 is the multiplicative identity in the above expression.

Thus, the property of Multiplicative Identity is used here.

(ii) $\frac{-13}{17}$ x $\frac{-2}{7}$ = $\frac{-2}{7}$ x $\frac{-13}{17}$

In general, a × b = b × a for any two rational numbers.

Thus, the property of Commutativity of Multiplication is used here.

(iii) $\frac{-19}{29}$ x $\frac{29}{-19}$ = 1

For a rational number $\frac{a}{b}$, the multiplicative inverse is the reciprocal of that number which is $\frac{b}{a}$.

Thus, the property of Multiplicative Inverse is used here.

6. Multiply 6/13 by the reciprocal of -7/16

Answer: A rational number is a number that is of the form p/q where p and q are integers and q is not equal to 0.

The reciprocal of $\frac{-7}{16}$ is $\frac{-16}{7}$

We will use the multiplication operation of rational numbers to solve the given question.

So, we calculate the product of the expression as follows:

➔ $\frac{6}{13}$ x $\left(\frac{-16}{7}\right)$ = $\left[\frac{6\ x\ (-16)}{13\ x\ 7}\right]$ = $\frac{-96}{91}$

Thus, the product of $\frac{6}{13}$ and its reciprocal of $\frac{-7}{16}$ is $\frac{-96}{91}$

7. Tell what property allows you to compute 1/3 × (6 × 4/5) as (1/3 × 6) × 4/3

Answer: A rational number is a number that is of the form p/q where p and q are integers and q is not equal to 0.

According to the associative property of multiplication,

(a × b) × c = a × (b × c)

Thus, by using the associativity of multiplication we see that,

➔ $\frac{1}{3}$ x $\left(6 \ x \ \frac{4}{5}\right)$ = $\left(\frac{1}{3} \ x \ 6\right)$ x $\frac{4}{5}$

8. Is 8/9 the multiplicative inverse of $-1\frac{1}{8}$? Why or why not ?

Answer: The reciprocal of a given rational number is known as its multiplicative inverse. The product of a rational number and its multiplicative inverse is 1.

We know that, $-1\frac{1}{8} = \frac{-9}{8}$

Now, $\frac{8}{9}$ x $\left(\frac{-9}{8}\right)$ = -1 ≠ 1

So, $\frac{8}{9}$ is not the multiplicative inverse of $-1\frac{1}{8}$ since the product of $\frac{8}{9}$ x $\left(\frac{-9}{8}\right)$ is not equal to 1.

Thus, by using the multiplicative identity property, we conclude that $\frac{8}{9}$ is not the multiplicative inverse of $-1\frac{1}{8}$.

9. Is 0.3 the multiplicative inverse of $3\frac{1}{3}$? Why or why not?

Answer: The reciprocal of a given rational number is known as its multiplicative inverse. The product of a rational number and its multiplicative inverse is 1.

0.3 can be written as $\frac{3}{10}$

We know that, $3\frac{1}{3}$ can be written as $\frac{10}{3}$

➔ $\frac{3}{10}$ x $\frac{10}{3}$ = 1

Yes, 0.3 is the multiplicative inverse of $3\frac{1}{3}$ since $\frac{3}{10}$ x $\frac{10}{3}$ = 1

Thus, by using the multiplicative identity property, we conclude that 0.3 is the multiplicative inverse of $3\frac{1}{3}$.

10. Write:

(i) The rational number that does not have a reciprocal.

(ii) The rational numbers that are equal to their reciprocals.

(iii) The rational number that is equal to its negative.

Answer:

(i) 0 (zero) is the rational number that does not have a reciprocal as it is undefined (∞).

(ii) The rational numbers 1 and -1 are equal to their respective reciprocals.

Reciprocal of 1 is 1/1 = 1, Reciprocal of -1 is $\frac{1}{-1}$ = -1

(iii) Rational number 0 is equal to its negative.

0 is neither a positive nor a negative number. Hence 0 is the same as -0.

11. Fill in the blanks

(i) Zero has ________ reciprocal.

(ii) The numbers ________ and ________ are their own reciprocals

(iii) The reciprocal of -5 is ________.

(iv) Reciprocal of 1/x, where x ≠ 0 is ________.

(v) The product of two rational numbers is always a _______.

(vi) The reciprocal of a positive rational number is ________.

Answer:

(i) Zero has **no** reciprocal

(ii) The numbers **1** and **-1** are their own reciprocals.

(iii) The reciprocal of -5 is $\mathbf{\frac{-1}{5}}$

(iv) Reciprocal of 1/x, where x ≠ 0 is **x**

(v) The product of two rational numbers is always a **rational number**

(vi) The reciprocal of a positive rational number is **positive**

Exercise 1.2

1. Represent these numbers on the number line. (i) 7/4 (ii) -5/6

Answer: The positive numbers are on the right of 0 and negative numbers are represented on the left of 0 on a number line.

The denominator of the rational number indicates the number of equal parts into which one unit of a number line has to be divided into whereas the numerator indicates how many of these parts are to be taken into consideration.

(i) $\frac{7}{4}$

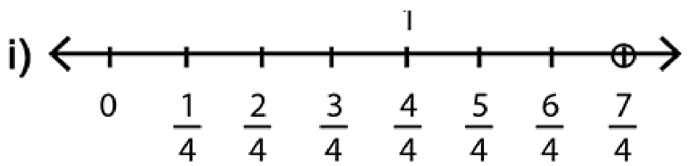
Since $\frac{7}{4}$ is an improper fraction, we will convert this to a mixed fraction.

$\frac{7}{4} = 1\frac{3}{4}$

The first unit has to be divided into 4 parts and we have to make 7 markers of distance towards the right of 0.

These markings will begin with $\frac{1}{4}$ up to $\frac{7}{4}$

➔ $\frac{4}{4}$ = 1 on the number line, thus, to plot $1\frac{3}{4}$ we will move three positions towards the right from $\frac{4}{4}$ which is $\frac{7}{4}$

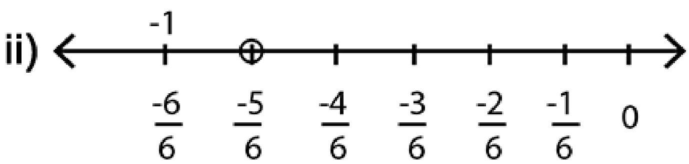


(i) $\frac{-5}{6}$

The first unit has to be divided into 6 parts and we have to make 5 markers of distance towards the left of 0 since it is negative.

These markings will begin with $\frac{-1}{6}$ up to $\frac{-5}{6}$

Hence, we will move by five positions towards the left from 0 which is $\frac{-5}{6}$

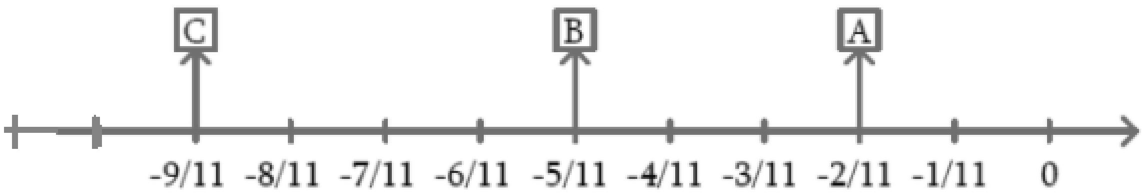


3. Represent -2/11, -5/11, -9/11 on the number line

Answer: The positive numbers are on the right of 0 and negative numbers are represented on the left of 0 on a number line.

The denominator of the rational number indicates the number of equal parts into which one unit of a number line has to be divided into whereas the numerator indicates how many of these parts are to be taken into consideration.

We draw 11 markers to the left of 0 on the number line since the numbers given $\frac{-2}{11}$, $\frac{-5}{11}$, $\frac{-9}{11}$ are negative.



The 2nd marker represents -$\frac{-2}{11}$ (A)

The 5th marker represents $\frac{-5}{11}$ (B)

The 9th marker represents $\frac{-9}{11}$ (C)

4. Find ten rational numbers between $\frac{-2}{5}$ and $\frac{1}{2}$

Answer: We can find infinitely many rational numbers between any two given rational numbers by taking the mean of the two rational numbers.

Alternative method: We can make the denominators same for the two given rational numbers.

The given numbers are $\frac{-2}{5}$ and $\frac{1}{2}$

The LCM of both denominators is 10.

So we will multiply the denominators by such a number which gives us a multiple of 10.

Multiplying both the numerator and denominator of $\frac{-2}{5}$ by 4, we get

➔ $\frac{-2 \; x \; 4}{5 \; x \; 4}$ = $\frac{-8}{20}$

Multiplying both the numerator and denominator of $\frac{1}{2}$ by 10, we get

➔ $\frac{1 \; x \; 10}{2 \; x \; 10}$ = $\frac{10}{20}$

Thus, we will now find ten rational numbers between $\frac{-8}{20}$ and $\frac{10}{20}$

The numbers will be $\frac{-7}{20}$, $\frac{-6}{20}$, $\frac{-5}{20}$, $\frac{-4}{20}$, $\frac{-3}{20}$, $\frac{-2}{20}$, $\frac{-1}{20}$, 0, $\frac{1}{20}$, $\frac{2}{20}$

5. Find five rational numbers between

(i) 2/3 and 4/5

(ii) -3/2 and 5/3

(iii) 1/4 and 1/2

Answer:

We can find infinitely many rational numbers between any two given rational numbers by taking the mean of the two rational numbers.

Alternative method: We can make the denominator same for the two given rational numbers.

(i) 2/3 and 4/5

The LCM of both denominators is 15.

We shall multiply the numbers to get the denominator as a multiple of 15

Multiplying both the numerator and denominator of $\frac{2}{3}$ by 20, we get

➔ $\frac{2 x 20}{3 x 20}$ = $\frac{40}{60}$

Multiplying both the numerator and denominator of $\frac{4}{5}$ by 12, we get

➔ $\frac{4 x 12}{5 x 12}$ = $\frac{48}{60}$

The five rational numbers between $\frac{2}{3}$ and $\frac{4}{5}$ can be taken as:

➔ $\frac{41}{60}$, $\frac{42}{60}$, $\frac{43}{60}$, $\frac{44}{60}$, $\frac{45}{60}$

(ii) -3/2 and 5/3

The LCM of both denominators is 6.

So we shall multiply the numbers to get the denominator as a multiple of 6

Multiplying both the numerator and denominator of $\frac{-3}{2}$ by 3, we get

➔ $\frac{-3 \ x \ 3}{2 \ x \ 3} = \frac{-9}{6}$

Multiplying both the numerator and denominator of $\frac{5}{3}$ by 2, we get

➔ $\frac{5 \ x \ 2}{3 \ x \ 2} = \frac{10}{6}$

The five rational numbers between $\frac{-3}{2}$ and $\frac{5}{3}$ can be taken as:

➔ $\frac{-9}{6}, \frac{-8}{6}, \frac{-7}{6}, 1, \frac{3}{6}, \frac{5}{6}$

(iii) 1/4 and 1/2

The LCM of both numbers is 8.

So we shall multiply the numbers to get the denominator as a multiple of 8

Multiplying both the numerator and denominator of $\frac{1}{4}$ by 8, we get

➔ $\frac{1 \ x \ 8}{4 \ x \ 8} = \frac{8}{32}$

Multiplying both the numerator and denominator of $\frac{1}{2}$ by 16, we get

➔ $\frac{1 \ x \ 16}{2 \ x \ 16} = \frac{16}{32}$

The five rational numbers between $\frac{1}{4}$ and $\frac{1}{2}$ can be taken as

➔ $\frac{9}{32}, \frac{10}{32}, \frac{11}{32}, \frac{12}{32}, \frac{13}{32}$

6. Write five rational numbers greater than -2

Answer: A rational number is a number that is of the form p/q where p and q are integers and q is not equal to 0.

We know that -2 is a rational number as it can in the form of p/q which is $\frac{-2}{1}$

We can write infinitely many rational numbers greater than – 2.

Five rational numbers greater than -2 are -1, 0, $\frac{1}{4}, \frac{1}{5}$ and $\frac{1}{6}$

7. Find ten rational numbers between 3/5 and 3/4

Answer:

We can find infinitely many rational numbers between any two given rational numbers by taking the mean of the two rational numbers.

Alternative method: We can make the denominator same for the two given rational numbers.

There are infinite numbers between any two rational numbers

Given numbers are $\frac{3}{5}$ and $\frac{3}{4}$

The LCM of both denominators is 20.

So we shall multiply the numbers to get the denominator as a multiple of 20

Multiplying both the numerator and denominator of $\frac{3}{5}$ by 40, we get

➔ $\frac{3 x 40}{5 x 40} = \frac{120}{200}$

Multiplying both the numerator and denominator of $\frac{3}{4}$ by 50, we get

➔ $\frac{3 x 50}{4 x 50} = \frac{150}{200}$

The ten rational numbers between $\frac{120}{200}$ and $\frac{150}{200}$ or $\frac{3}{5}$ and $\frac{3}{4}$ can be taken as:

➔ $\frac{121}{200}, \frac{122}{200}, \frac{123}{200}, \frac{124}{200}, \frac{125}{200}, \frac{126}{200}, \frac{127}{200}, \frac{128}{200}, \frac{129}{200}, \frac{130}{200}$