Exercise 8.1

1. Find the ratio of the following.

(a) Speed of a cycle 15 km per hour to the speed of scooter 30 km per hour.

(b) 5 m to 10 km (c) 50 paise to Rs 5

Answer: The ratio is defined as the comparison between two quantities of the same units that indicates how much of one quantity is present in the other quantity.

(a) Speed of a cycle 15 km per hour to the speed of scooter 30 km per hour.

Speed of a cycle = 15 km/hr

Speed of a scooter = 30 km/hr

Speed of cycle: Speed of scooter = $\frac{15}{30} = \frac{1}{2}$

Thus, the ratio is 1:2.

(b) 5 m to 10 km

Quantities can be compared only when the units are same.

1 km = 1000 m

Therefore, 10 km = 10 × 1000 = 10000 m

5 m to 10 km = 5 m to 10000 m = $\frac{5}{10000} = \frac{1}{2000}$

Thus, the ratio is 1:2000

(c) 50 paise to Rs 5

Quantities can be compared only when the units are the same.

Rs 1 = 100 paise

Rs 5 = 5 × 100 paise = 500 paise

50 paise to Rs 5 = 50 paise to 500 paise = $\frac{50}{500} = \frac{1}{10}$

Thus, the ratio is 1:10

2. Convert the following ratios to percentages

(a) 3:4 (b) 2:3

Answer:

The ratio is defined as the comparison between two quantities of the same units that indicates how much of one quantity is present in the other quantity.

Percentage is defined as a given part or amount in every hundred. It is a fraction with 100 as the denominator and is represented by the symbol "%".

(a) 3:4

= $\frac{3}{4}$ x 100 = 3 x 25 = 75%

(b) 2:3

= $\frac{2}{3}$ x 100 = $\frac{200}{3}$ = 66.67 % or 66 $\frac{2}{3}$ %

3. 72% of 25 students are interested in mathematics. How many are not interested in mathematics?

Answer: Given: 72% of 25 students are interested in mathematics.

Therefore, Percentage of students who are not interested in Mathematics = (100 – 72)% = 28%

Therefore, the number of students who are not interested in mathematics

= 28% of the total number of students

= 28% of 25

= $\frac{28}{100}$ × 25

= $\frac{28}{4}$ = 7

Hence, the number of students who are not interested in mathematics is 7.

4. A football team won 10 matches out of the total number of matches they played. If their win percentage was 40, then how many matches did they play in all?

Answer: Assuming the total number of matches played as x, equating 40% of x to 10, the value of x can be found

Let the total number of matches played = x

Thus, 40% of x = 10

➔ $\frac{40}{100}$ × x = 10

➔ x = $\frac{10\ x\ 100}{40}$

➔ x = 25

Hence, the total number of matches played = 25

5. If Chameli had ₹ 600 left after spending 75% of her money, how much did she have in the beginning?

Answer: Given that Cameli spends 75% of her money

Since the whole is considered as 100%, the percentage of the amount left with Chameli is (100 - 75)% = 25%

Let the total amount Chameli had with her, in the beginning be x

Percentage of the amount left with Chameli = (100 - 75) % = 25%

And, 25% of x = 600

➔ $\frac{25}{100}$ × x = 600

➔ x = $\frac{600\ x\ 100}{25}$

➔ x = 2400

6. If 60% people in a city like Cricket, 30% like Football and the remaining like other games, then what percent of the people like other games? If the total number of people is 50 lakh, find the exact number who like each type of game.

Answer:

Percentage of people who like cricket = 60%

Percentage of people who like football = 30%

Total number of people = 50 lakhs

Since the whole is considered as 100%, percentage of people who like other games is

100% - (60 + 30) % = 10%

Number of people who like cricket = 60% of 50 lakhs

= $\frac{60}{100}$ × 5000000

= 3000000

= 30 lakhs

Number of people who like football = 30% of 50 lakhs

= $\frac{30}{100}$ × 5000000

= 1500000

= 15 lakhs

Number of people who like other games = 10% of 50 lakhs

= $\frac{10}{100}$ × 5000000

= 500000

= 5 lakhs

Exercise 8.2

1. A man got a 10% increase in his salary. If his new salary is ₹ 1,54,000, find his original salary.

Answer:

Given: Percentage of increase in salary = 10%

New Salary = Rs 1,54,000

Let the original salary be x

The percentage increase is 10%

Therefore, Original salary + Increment in salary = New Salary

x + 10% of x = 154000

x + $\frac{10}{100}$ × x = 154000

$\frac{110}{100}$ × x = 154000

x = $\frac{154000\ x\ 100}{110}$

x = 140000

2. On Sunday 845 people went to the Zoo. On Monday only 169 people went. What is the percent decrease in the people visiting the Zoo on Monday?

Answer:

Given: Number of people who visited the zoo on Sunday = 845

Number of people who visited the zoo on Monday = 169

Decrease in the number of people = 845 - 169 = 676

Percentage decrease in the number of people = $\frac{\text{Decrease in the number of people}}{\text{(Number of people who visited the zoo on Sunday)}}$ x 100

$= \frac{676}{845} \times 100$

$= \frac{4}{5} \times 100$

= 80 %

3. A shopkeeper buys 80 articles for ₹ 2,400 and sells them for a profit of 16%. Find the selling price of one article

Answer:

Given that the shopkeeper buys 80 articles for ₹ 2,400

Cost of one article = $\frac{2400}{80}$ = ₹ 30

Profit percentage = 16%

Profit percentage = $\frac{Profit}{C.P}$. x 100

16 = $\frac{Profit}{30}$ x 100

Profit = $\frac{16\ x\ 30}{100}$

= ₹ 4.8

Therefore, the selling price of one article = C.P. + Profit

= ₹ (30 + 4.80)

= ₹ 34.80

4. The cost of an article was ₹ 15,500. ₹ 450 was spent on its repairs. If it is sold for a profit of 15%, find the selling price of the article

Answer:

Cost Price of the article = ₹ 15,500

Overhead charges = ₹ 450

Profit percentage = 15%

New Cost Price = Cost Price + Overhead charges

New C.P. = 15500 + 450 = 15950

Selling Price = Cost Price + Profit

where, Profit = 15% of C.P.

S.P. = C.P. + Profit

= C.P. + 15% of C.P.

= C.P. + $\frac{15}{100}$ × C.P.

= C.P $\left(1 + \frac{15}{100}\right)$

= 115% C.P.

S.P. = $\frac{115}{100}$ × 15950 = 18342.50

Thus, the selling price is ₹ 18342.50

5. A VCR and TV were bought for ₹ 8,000 each. The shopkeeper made a loss of 4% on the VCR and a profit of 8% on the TV. Find the gain or loss percent on the whole transaction

Answer:

C.P of VCR and TV each = ₹ 8000

Loss percentage on VCR = 4%

Profit percentage on TV = 8%

Profit% or Gain% = $\frac{Gain}{C.P}$ × 100

Loss% = $\frac{Loss}{C.P.}$ × 100

S.P. of VCR = C.P. - Loss% × C.P.

= 8000 - 4% × 8000

= 8000 - $\frac{4}{100}$ × 8000

= 8000 – 320 = ₹ 7680

S.P. of TV = C.P. + Profit% × C.P.

= 8000 + $\frac{8}{100}$ × 8000

= 8000 + 640 = ₹ 8640

S.P. of VCR + TV = 7680 + 8640 = ₹ 16320

C.P. of VCR + TV = 8000 + 8000 = ₹ 16000

Overall Gain = ₹ (16320 - 16000) = ₹ 320

Gain% = $\frac{Gain}{C.P} \times 100 \times 100$

= Gain% = $\frac{320}{16000} \times 100 \times 100$

= 2%

6. During a sale, a shop offered a discount of 10% on the marked price of all the items. What would a customer have to pay for a pair of jeans marked at ₹ 1450 and two shirts marked at ₹ 850 each?

Answer:

Discount percentage = 10%

The marked price of a pair of jeans = ₹ 1450

Marked Price of a shirt = ₹ 850

(i) M.P. of a pair of jeans = C.P. of two jeans - Discount

(ii) M.P. of a shirt = C.P. of two shirts - Discount

Adding (i) and (ii) will give the amount he has to pay.

M.P of a pair of jeans = 1450

M.P of 2 shirts = 850 × 2 = 1700

Total M.P. = 1450 + 1700 = 3150

Discount on Total M.P =10% of 3150

= $\frac{10}{100} \times 3150 = 315$

The amount customer has to pay is = Total M.P - Discount on total M.P

= 3150 - 315 = ₹ 2835

7. A milkman sold two of his buffaloes for ₹ 20,000 each. On one he made a gain of 5% and on the other a loss of 10%. Find his overall gain or loss. (Hint: Find CP of each)

Answer:

S.P of each buffalo is ₹ 20,000

Number of buffaloes sold = 2

On selling Buffalo 1, gain % made is = 5%

S.P = $\left[\frac{100 + Gain\ \%}{100}\right] \times$ C.P.

Thus, on substituting the values,

20000 = $\frac{105}{100}$ × C.P.

C.P. = 20000 × $\frac{100}{105}$

C.P. = 20000 × $\frac{20}{21}$

C.P. = ₹ 19,047.62

Thus, C.P. of buffalo 1 = ₹ 19,047.62

Now, on selling Buffalo 2, loss % made is = 10%

S.P = $\left[\frac{100 + Loss\ \%}{100}\right]$ × C.P.

On substituting the values,

20000 = $\frac{90}{100}$× C.P.

20000 = $\frac{9}{10}$ × C.P.

C.P. = 20000 × $\frac{10}{9}$

C.P. = ₹ 22,222.22

Thus, C.P. of buffalo 2 = ₹ 22,222.22

Total Cost Price of two Buffaloes = 19,047.62 + 22,222.22

= ₹ 41,269.84

We know that, total Selling Price of two buffaloes = ₹ 20,000 + ₹ 20,000 = ₹ 40,000

Therefore,

Loss = Cost Price - Selling Price

= 41,269.84 – 40000 = 1269.84

Loss = ₹ 1269.84

Alternative method:

S.P. of each buffalo = ₹ 20,000

The milkman made a gain of 5% while selling one buffalo

This means if C.P. is ₹ 100, then S.P. is ₹ 105.

C.P. of one buffalo = 100/105 × 20000

= ₹ 19,047.62

Also, the second buffalo was sold at a loss of 10%

This means if C.P. is ₹ 100, then S.P. is ₹ 90

∴ C.P. of other buffalo = 100/90 × 20000

= ₹ 22222.22

Total C.P. = ₹ 19047.62 + ₹ 22222.22 = ₹ 41269.84

Total S.P. = ₹ 20000 + ₹ 20000 = ₹ 40000

Loss = ₹ 41269.84 − ₹ 40000 = ₹ 1269.84

Therefore, the overall loss of milkman was ₹ 1,269.84

8. The price of a TV is ₹ 13,000. The sales tax charged on it is at the rate of 12%. Find the amount that Vinod will have to pay if he buys it

Answer:

Price of the TV = ₹ 13,000

Sales tax on the TV = 12%

We know that,

Tax = Tax % of Selling Price

Hence, Sales Tax Amount = $\frac{12}{100}$ × 13000 = ₹ 1560

Therefore, Amount paid by Vinod = Price of the TV + Sales tax

= 13,000 + 1560 = ₹ 14,560

9. Arun bought a pair of skates at a sale where the discount given was 20%. If the amount he pays is ₹ 1,600 find the marked price

Answer:

Amount paid by Arun to buy skates is ₹ 1,600

Let the Marked price of a pair of skates be x

Discount percentage is 20%

Therefore, M.P. of skates - Discount = 1600 ------ (1)

where, Discount = Discount % × M.P.

Discount = 20% of x ------- (2)

From equation (1) and (2) we get,

x - $\frac{20x}{100}$ × (x) = 1600

x - $\frac{20x}{100}$ = 1600

$\frac{80x}{100}$ = 1600

x = 1600 × $\frac{100}{80}$

x = 2000

Marked price = ₹ 2000

Alternate Method:

Let the marked price be x

Discount percent = $\frac{Discount}{Marked\ Price}$ x 100

20 = $\frac{Discount}{x}$ × 100

Discount = $\frac{20}{100}$ × x

= $\frac{x}{5}$

Also,

Discount = Marked price – Sale price

$\frac{x}{5}$ = x – ₹ 1600

x – $\frac{x}{5}$ = 1600

$\frac{4x}{5}$ = 1600

x = 1600 x $\frac{5}{4}$ = 2000

Therefore, the marked price was ₹ 2000.

10. I purchased a hair-dryer for ₹ 5,400 including 8% VAT. Find the price before VAT was added.

Answer: Price of a hairdryer with VAT = ₹ 5400

VAT% = 8%

Let the initial price of the hairdryer be ₹ x

Price of hairdryer with VAT is ₹ 5400

VAT % = 8%

VAT = 8% of the initial price of hairdryer

So, the initial price of hairdryer + VAT = 5400

x + 8% × x = 5400

$x + \frac{8x}{100} = 5400$

$\frac{108x}{100} = 5400$

$x = 5400 \times \frac{100}{108}$

x = ₹ 5000

Alternate method:

The price includes VAT

So, 8% VAT means that if the price without VAT is ₹ 100,

Then, the price including VAT will be ₹ 108

When price including VAT is ₹ 108, original price = ₹ 100

When price including VAT is ₹ 5400, original price = ₹ (100/108 × 5400)

= ₹ 5000

Therefore, the price of the hair dryer before the addition of VAT was ₹ 5,000.

11. An article was purchased for ₹ 1239 including GST of 18%. Find the price of the article before GST was added?

Answer:

Price of an article with GST = Rs 1239

GST% = 18%

Let the initial price of the article be Rs x

Price of article with GST is Rs 1239

GST % = 18%

GST = 18% of the initial price of article

So, the initial price of article + GST = 1239

x + 18% × x = 1239

$x + \frac{18x}{100} = 1239$

$\frac{118x}{100} = 1239$

$x = 1239 \times \frac{100}{118}$

x = Rs 1050

Exercise 8.3

1. Calculate the amount and compound interest on
(a) ₹ 10,800 for 3 years at 12(1/2)% per annum compounded annually
(b) ₹ 18,000 for 2(1/2) years at 10% per annum compounded annually
(c) ₹ 62,500 for 1(1/2)years at 8% per annum compounded half yearly
(d) ₹ 8,000 for 1 year at 9% per annum compounded half yearly. (You could use the year by year calculation using SI formula to verify)
(e) ₹ 10,000 for 1 year at 8% per annum compounded half yearly

Answer:

What is known: Principal, Time Period and Rate of Interest

What is unknown: Amount and Compound Interest (C.I)

Reasoning:

$A = P[1+(r/100)]^n$

$A = P\left[1 + \left(\frac{r}{100}\right)\right]^n$

(a) ₹ 10,800 for 3 years at 12(1/2)% per annum compounded annually
P = ₹ 10800

N = 3 years

R = $12\frac{1}{2}$ % = $\frac{25}{2}$ % compounded annually

A = P $\left[1 + \left(\frac{r}{100}\right)\right]^n$

A = 10800 $\left[1 + \left(\frac{\frac{25}{2}}{100}\right)\right]^3$

A = 10800 $\left[1 + \left(\frac{25}{200}\right)\right]^3$

A = 10800 $\left[\left(\frac{225}{200}\right)\right]^3$

A = 10800 x $\left(\frac{225}{200}\right)^3$

A = 10800 x $\frac{225}{200}$ x $\frac{225}{200}$ x $\frac{225}{200}$

A = 15377.34

C.I. = A - P

= 15377.34 - 10800

= 4577.34

Amount = ₹ 15377.34

Compound Interest = ₹ 4577.34

(b) ₹ 18,000 for 2(1/2) years at 10% per annum compounded annually

P = ₹ 18000

N = 2 $\frac{1}{2}$ years

R = 10% compounded annually

A = P $\left[1 + \left(\frac{r}{100}\right)\right]^n$

Since 'n' is 2 $\frac{1}{2}$ years, amount can be calculated for 2 years and having amount as principal Simple Interest (S.I.) can be calculated for $\frac{1}{2}$ years because C.I. is only annually.

A = P $\left[1 + \left(\frac{r}{100}\right)\right]^n$

A = 18000 $\left[1 + \left(\frac{10}{100}\right)\right]^2$ $\frac{10}{100} = \frac{1}{10}$

A = 18000 x $\frac{11}{10}$ x $\frac{11}{10}$

A = ₹ 21780

Amount after 2 years = ₹ 21870

S.I for $\frac{1}{2}$ years = $\frac{1}{2}$ X 21780 = ₹ 1089

Amount for $2\frac{1}{2}$ years = ₹ 21870 + 1089 = ₹ 22869

C.I for $2\frac{1}{2}$ years = 22869 – 18000 = ₹ 4869

Amount = ₹ 22869

Compound Interest = ₹ 4869

(c) ₹ 62,500 for 1(1/2) years at 8% per annum compounded half yearly.

P = ₹ 62,500

N = $1\frac{1}{2}$ years

R = 8% compounded half yearly

A = P $\left[1 + \left(\frac{r}{100}\right)\right]^n$

There are 3 half years in 1(1/2) years. Therefore, compounding has to be done 3 times and rate of interest will be 4%.

A = ₹ 62,500 $\left[1 + \left(\frac{4}{100}\right)\right]^3$

A = ₹ 62,500 $\left[\left(\frac{104}{100}\right)\right]^3$

A = ₹ 62,500 x $\frac{104}{100}$ x $\frac{104}{100}$ x $\frac{104}{100}$

A = 70304

C.I. = A - P

= 70304 – 62500 = 7804

Amount = ₹ 70304

Compound Interest = ₹ 7804

(d) ₹ 8,000 for 1 year at 9% per annum compounded half yearly. (You could use the year by year calculation using SI formula to verify)

P = ₹ 8000

n = 1 year

R = 9% p.a. compounded half yearly

A = P $\left[1 + \left(\frac{r}{100}\right)\right]^n$

S.I. for 1st 6 months = $\frac{1}{2}$ X 8000 X $\frac{9}{100}$

= 40 × 9 = 360

Amount after 1st 6 months including Simple Interest = 8000 + 360 = ₹ 8360

Principal for 2nd 6 months = ₹ 8360

S.I. for 2nd 6 months = $\frac{1}{2}$ X 8360 X $\frac{9}{100}$ = $\frac{418\ x\ 9}{100}$ = 376.20

C.I. after 1 year (9% p.a. interest half yearly) = 360 + 376.20 = 736.20

Amount after 1 year (9% p.a. interest half yearly) = 8000 + 736.20 = 8736.20

Amount = ₹ 8736.20

Compound Interest = ₹ 736.20

(e) ₹ 10,000 for 1 year at 8% per annum compounded half yearly

P = ₹ 10,000

n = 1 year

R = 8% p.a. compounded half yearly

A = P $\left[1 + \left(\frac{r}{100}\right)\right]^n$

There are 2 half years in 1 years. Therefore, compounding has to be done 2 times and rate of interest will be 4%

A = P $\left[1 + \left(\frac{r}{100}\right)\right]^n$

A = 10000 $\left[1 + \left(\frac{4}{100}\right)\right]^2$

A = 10000 x $\frac{104}{100}$ X $\frac{104}{100}$ = A = 10816

C.I. after 1 year (8% p.a. interest half yearly) = 10816 – 10000 = 816

Amount after 1 year (8% p.a. interest half yearly) 10816 = 10816

Amount after 1 year = ₹ 10816

Compound Interest after 1 year = ₹ 816

2. Kamala borrowed ₹ 26,400 from a Bank to buy a scooter at a rate of 15% p.a. compounded yearly. What amount will she pay at the end of 2 years and 4 months to clear the loan? (Hint: Find A for 2 years with interest is compounded yearly and then find SI on the 2nd year amount for 4/12 years)

Answer:

$A = P\left[1 + \left(\frac{r}{100}\right)\right]^n$

Principal (P) = ₹ 26400

Time period (n) = 2 years 4 months

Rate % (R) = 15% compounded annually

First, we will calculate Compound Interest (C.I) for the period of 2 years

$A = P\left[1 + \left(\frac{r}{100}\right)\right]^n$

$A = 26400\left[1 + \left(\frac{15}{100}\right)\right]^2$

$A = 26400 \times \frac{115}{100} \times \frac{115}{100}$

$A = 26400 \times \frac{23}{20} \times \frac{23}{20}$

A = 26400 × 1.3225

A = 34914

C.I. = A - P

C.I. = 34914 – 26400 = 8514

Now, we will find Simple Interest (S.I) for the period of 4 months

Principal for 4 months after C.I. for 2 years = ₹ 34,914

We know that,

$SI = \frac{PRT}{100}$

Here T = 4 months = $\frac{4}{12}$ years = $\frac{1}{3}$ years

S.I. for 4 months = $\frac{1}{3} \times 34914 \times \frac{15}{100}$

$= \frac{34914}{20} = 1745.70$

Total interest for 2 years 4 months = 8514 + 1745.70 = 10259.70

Total amount for 2 years 4 months = 26400 + 10259.70 = ₹ 36659.70

3. Fabina borrows ₹ 12,500 at 12% per annum for 3 years at simple interest and Radha borrows the same amount for the same time period at 10% per annum, compounded annually. Who pays more interest and by how much?

Answer:

For Simple Interest: $\frac{P\ x\ R\ x\ T}{100}$

P = ₹ 12,500

T = 3 years

R = 12% simple interest

where , A = Amount, P = Principal, T = Time period in years and R = Rate percent

Thus, Simple Interest paid by Fabina for 3 years at the rate of 12% per annum

S.I. for 3 years = $\frac{12500\ x\ 12\ x\ 3}{100}$ = 4500

For Compound Interest:

A = P $\left[1 + \left(\frac{r}{100}\right)\right]^n$

P = ₹ 12,500

n = 3 years

R = 10% compounded annually

Thus, Amount paid by Radha for 3 years at the rate of 10% p.a. compounded annually

A = 12500 $\left[1 + \left(\frac{10}{100}\right)\right]^3$

A = 12500 x $\frac{11}{10}$ x $\frac{11}{10}$ x $\frac{11}{10}$ = 12500 x $\frac{1331}{1000}$

A = 12500 x 1.331

A = 16637.50

Compound Interest = A - P

= 16637.50 – 12500 = 4137.50

Since 4500 > 4137.50, Fabina paid more interest than Radha.

Additional Interest paid by Fabina = 4500 – 4137.50 = ₹ 362.50

4. I borrowed ₹ 12,000 from Jamshed at 6% per annum simple interest for 2 years. Had I borrowed this sum at 6% per annum compound interest, what extra amount would I have to pay?

Answer:

For Simple Interest:

A = $\frac{P\ x\ R\ x\ T}{100}$

P = ₹ 12,000

T = 2 years

R = 6% simple interest

where , A = Amount, P = Principal, T = Time period in years and R = Rate percent\

Simple Interest to be paid for 2 years at the rate of 6% per annum

S.I. for 2 years = 2 × 12000 × $\frac{6}{100}$

= 2 × 120 × 6 = 1440

For Compound Interest:

A = P $\left[1 + \left(\frac{r}{100}\right)\right]^n$

P = ₹ 12,000

n = 2 years

R = 6% compounded annually

Compound Interest to be paid for 2 years at the rate of 6% per annum

A = P $\left[1 + \left(\frac{r}{100}\right)\right]^n$

A = 12000 $\left[1 + \left(\frac{6}{100}\right)\right]^2$

A = 12000 x $\frac{106}{100}$ x $\frac{106}{100}$

A = 12000 × 1.1236

A = 13483.20

Compound Interest = A - P

= 13483.20 – 12000 = ₹ 1483.20

Compound Interest - Simple Interest = 1483.20 – 1440 = ₹ 43.20

5. Vasudevan invested ₹ 60,000 at an interest rate of 12% per annum compounded half-yearly. What amount would he get

(i) after 6 months? (ii) after 1 year?

Answer: Given that, Vasudevan invested ₹ 60,000

For Compound Interest (C.I.)

$A = P\left[1 + \left(\frac{r}{100}\right)\right]^n$

P = ₹ 60,000

n = 6 months and 1 year

R = 12% p.a. compounded half-yearly

where , A = Amount, P = Principal, n = Time period and R = Rate percent

(i) For easy calculation of compound interest, we will put Interest Rate as 6% half-yearly and n = 1.

$A = P\left[1 + \left(\frac{r}{100}\right)\right]^n$

$A = 60000\left[1 + \left(\frac{6}{100}\right)\right]^1$

$A = 60000 \times \frac{106}{100}$

A = 60000 x 1.06

A = ₹ 63600

(ii) Compound Interest to be paid for 12 months (1 year) compounded half yearly.

So, assume n = 2, r = 6%

$A = P\left[1 + \left(\frac{r}{100}\right)\right]^n$

$A = 60000\left[1 + \left(\frac{6}{100}\right)\right]^2$

$A = 60000 \times \frac{106}{100} \times \frac{106}{100}$

$A = 60000 \times \frac{11236}{10000} = 60000 \times 1.1236 = ₹ 67416$

6. Arif took a loan of ₹ 80,000 from a bank. If the rate of interest is 10% per annum, find the difference in amounts he would be paying after $1\frac{1}{2}$ years if the interest is

(i) compounded annually (ii) compounded half-yearly

Answer:

Given that, Arif took a loan of ₹ 80,000 from a bank

For Amount and Compound Interest (C.I.)

$A = P\left[1 + \left(\frac{r}{100}\right)\right]^n$

P = ₹ 80,000

n = $\mathbf{1\frac{1}{2}}$ years

R = 10% p.a. compounded half-yearly and 10% p.a. compounded yearly

(i) For calculation of Compound Interest (C.I.) compounded annually:

Since 'n' is $\mathbf{1\frac{1}{2}}$ years, the amount can be calculated for 1 year, and having that amount as principal, S.I. can be calculated for the remaining $\frac{1}{2}$ year because C.I. is always calculated annually.

$A = P\left[1 + \left(\frac{r}{100}\right)\right]^n$

$A = 80000\left[1 + \left(\frac{10}{100}\right)\right]^1$

$A = 80000 \times \frac{11}{10} = 80000 \times 1.1 = 88000$

Amount after 1 year = ₹ 88,000

Therefore, the principal for the next $\frac{1}{2}$ year = ₹ 88,000

Simple interest = $\frac{P\ x\ R\ x\ T}{100}$

Simple interest for $\frac{1}{2}$ year = $88000 \times \frac{1}{2} \times \frac{10}{100} = \frac{88000}{2} = 4400$

Therefore, amount after $1\frac{1}{2}$ years = 88000 + 4400 = ₹ 92400

(ii) For calculation of Compound Interest (C.I.) compounded half-yearly, we will consider rate as 5% p.a. and 'n' as 3

$A = P\left[1 + \left(\frac{r}{100}\right)\right]^n$

$A = 80000\left[1 + \left(\frac{5}{100}\right)\right]^3$

$A = 80000 \times \frac{105}{100} \times \frac{105}{100} \times \frac{105}{100}$

$A = 80000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}$

$A = 80000 \times \frac{9261}{8000}$

A = 10 × 9261 = ₹ 92610

Therefore, difference in the amount = ₹ 92610 - ₹ 92400 = ₹ 210

7. Maria invested ₹ 8,000 in a business. She would be paid interest at 5% per annum compounded annually. Find:

(i) The amount credited against her name at the end of the second year. (ii) The interest for the 3rd year.

Answer: Given that, Maria invested ₹ 8,000 in a business

We know that,

$A = P\left[1 + \left(\frac{r}{100}\right)\right]^n$

P = ₹ 8,000

n = 2

R = 5% p.a. compounded annually

(i) The amount credited against her name at the end of the second year.

$A = 8000\left[1 + \left(\frac{5}{100}\right)\right]^2$

$A = 8000 \times \frac{105}{100} \times \frac{105}{100}$

$A = 8000 \times \frac{21}{20} \times \frac{21}{20}$

$A = 8000 \times \frac{441}{400}$

A = 20 × 441

A = 8820

(ii) The interest for the 3rd year.

For calculating interest for the 3rd year, the principal = 8820

Simple interest = $\frac{P\ x\ R\ x\ T}{100}$

Simple interest = $\frac{8820\ x\ 5\ x\ 1}{100}$ = 441

8. Find the amount and the compound interest on ₹ 10,000 for $1\frac{1}{2}$ years at 10% per annum, compounded half yearly. Would this interest be more than the interest he would get if it was compounded annually?

Answer: What is known: Principal, Time Period, and Rate of Interest

What is unknown: Amount and Compound Interest (C.I.)

A = P $\left[1 + \left(\frac{r}{100}\right)\right]^n$

P = ₹ 10,000

n = $1\frac{1}{2}$ years

R = 10% p.a. compounded annually and half-yearly

where , A = Amount, P = Principal, n = Time period and R = Rate percent

For calculation of C.I. compounded half-yearly, we will take the Interest rate as 5% and n = 3

A = P $\left[1 + \left(\frac{r}{100}\right)\right]^n$

A = 10000 x $\frac{105}{100}$ X $\frac{105}{100}$ X $\frac{105}{100}$

A = 10000 x $\frac{21}{20}$ X $\frac{21}{20}$ X $\frac{21}{20}$

A = 10000 x $\frac{9261}{8000}$

A = 5 x $\frac{9261}{4}$

A = 11576.25

Interest earned at 10% p.a. compounded half-yearly = A – P

= ₹ 11576.25 - ₹ 10000 = ₹ 1576.25

Now, let's find the interest when compounded annually at the same rate of interest.

Hence, for 1 year R = 10% and n = 1

$A = P\left[1 + \left(\frac{r}{100}\right)\right]^n$

$A = 10000\left[1 + \left(\frac{10}{100}\right)\right]^1$

$A = 10000 \times \frac{11}{10} = 11000$

Now, for the remaining $\frac{1}{2}$ year P = 11000, R = 5%

$A = P\left[1 + \left(\frac{r}{100}\right)\right]^n$

$A = 11000\left[1 + \left(\frac{5}{100}\right)\right]^1$

$A = 11000 \times \frac{105}{100}$

$A = 11000 \times 1.05 = 11550$

Thus, amount at the end of $1\frac{1}{2}$ when compounded annually = ₹ 11550

Thus, compound interest = ₹ 11550 - ₹ 10000 = ₹ 1550

Therefore, the interest will be less when compounded annually at the same rate.

9. Find the amount which Ram will get on ₹ 4096 if he gave it for 18 months at $12\frac{1}{2}$ % per annum, interest being compounded half yearly.

Answer:

$A = P\left[1 + \left(\frac{r}{100}\right)\right]^n$

P = ₹ 4096

n = 18 months = $1\frac{1}{2}$ years

R = $12\frac{1}{2}$ % p.a. compounded half-yearly

For calculation of Compound Interest (C.I.) compounded half-yearly, we will take the interest rate as half of $12\frac{1}{2}$ % i.e, $\frac{25}{2} \div 2 = \frac{25}{4}$ %

and 'n' = 3 (Since, 18 ÷ 6 = 3)

$A = P\left[1 + \left(\frac{r}{100}\right)\right]^n$

$A = 4096\left[1 + \left(\frac{\frac{25}{4}}{100}\right)\right]^3$

$A = 4096\left[1 + \left(\frac{25}{4\,x\,100}\right)\right]^3$

$A = 4096 \times \frac{425}{400} \times \frac{425}{400} \times \frac{425}{400}$

$A = 4096 \times \frac{17}{16} \times \frac{17}{16} \times \frac{17}{16}$

$A = 4096 \times \frac{4193}{4096}$

A = 4193

10. The population of a place increased to 54,000 in 2003 at a rate of 5% per annum (i) find the population in 2001 (ii) what would be its population in 2005?

Answer:

(i) find the population in 2001

Let the population in the year 2001 be 'P' and the population in 2003 is 'A' = 54000

Also, R = 5%, n = 2

$A = P\left[1 + \left(\frac{r}{100}\right)\right]^n$

$54000 = P\left[1 + \left(\frac{5}{100}\right)\right]^2$

$54000 = P \times \frac{105}{100} \times \frac{105}{100}$

$54000 = P \times \frac{21}{20} \times \frac{21}{20}$

$54000 = P \times \frac{441}{400}$

P = $\frac{400}{441}$ x 54000 = 48979.6

The population in 2001 = 48980 (approx.)

(ii) what would be its population in 2005?

Now, the population in 2003 is considered as 'P' = 540000 and the population in 2005 is 'A'

R = 5%, n = 2

A = P $\left[1 + \left(\frac{r}{100}\right)\right]^n$

A = 540000 $\left[1 + \left(\frac{5}{100}\right)\right]^2$

A = 540000 x $\frac{21}{20}$ x $\frac{21}{20}$

A = 540000 x $\frac{441}{400}$

A = 135 × 441

A = 59535

The population in 2005 = 59535

11. In a Laboratory, the count of bacteria in a certain experiment was increasing at the rate of 2.5% per hour. Find the bacteria at the end of 2 hours if the count was initially 5,06,000

Answer: Let the initial count of the bacteria be 'P' = 506000, n = 2, R = 2.5%

The count of the bacteria after 2 hours is assumed as 'A' and calculated as follows:

A = P $\left[1 + \left(\frac{r}{100}\right)\right]^n$

A = 506000 $\left[1 + \left(\frac{\frac{25}{10}}{100}\right)\right]^2$

A = 506000 $\left[1 + \left(\frac{25}{1000}\right)\right]^2$

A = 506000 $\left[1 + \left(\frac{1}{40}\right)\right]^2$

A = 506000 $\left[\left(\frac{41}{40}\right)\right]^2$

A = 506000 x $\frac{41}{40}$ X $\frac{41}{40}$

A = 506000 x $\frac{1681}{1600}$

A = 506000 × 1.050625

A = 531616(approx.)

12. A scooter was bought at ₹ 42,000. Its value depreciated at the rate of 8% per annum. Find its value after one year

Answer:

Original value of the scooter = ₹ 42,000

Rate of depreciation = 8%

The value of the scooter after 1 year = 42000 - $\left[42000\ x\ \frac{8}{100}\right]$

= 42000 - $\left[42000\ x\ \frac{2}{25}\right]$

= 42000 - (1680 × 2)

= 42000 - 3360

= 38640

Thus, the value of the scooter after 1 year is ₹ 38640.