Exercise 14.1

1. The marks obtained by 17 students in a mathematics test (out of 100) are given below:

91, 82, 100, 100, 96, 65, 82, 76, 79, 90, 46, 64, 72, 68, 66, 48, 49.

The range of the data is:

(A) 46 (B) 54 (C) 90 (D) 100

Answer: (B) 54

The range of data is the difference of the highest value and the lowest value.

In the given question, the highest value is 100 and the lowest value is 46.

Therefore, the range = 100 - 46 = 54

2. The class-mark of the class 130-150 is:

(A) 130 (B) 135 (C) 140 (D) 145

Answer: (C) 140

Class-mark = $\frac{Lower\ limit+Higher\ limit}{2}$

= $\frac{130+150}{2}$ = $\frac{280}{2}$ = 140

3. A die is thrown 1000 times and the outcomes were recorded as follows:

Outcome	**1**	**2**	**3**	**4**	**5**	**6**
Frequency	**180**	**150**	**160**	**170**	**150**	**190**

If the die is thrown once more, then the probability that it shows 5 is:

(A) $\frac{9}{50}$ (B) $\frac{3}{20}$ (C) $\frac{4}{25}$ (C) $\frac{7}{25}$

Answer: (B) $\frac{3}{20}$

Probability of an event is the ratio of the number of possible outcomes to the number of total outcomes. Let A be the event that the outcome is 5. Therefore,

P(A) = $\frac{Number\ of\ possible\ outcomes}{Total\ number\ of\ outcomes}$

P(A) = $\frac{150}{1000} = \frac{3}{20}$

3. The class mark of the class 90-120 is:

(A) 90 (B) 105 (C) 115 (D) 120

Answer: (B) 105

Class-mark = $\frac{Lower\ limit + Higher\ limit}{2}$

= $\frac{90+120}{2}$ = $\frac{210}{2}$ = 105

4. The range of the data: 25, 18, 20, 22, 16, 6, 17, 15, 12, 30, 32, 10, 19, 8, 11, 20 is

(A) 10 (B) 15 (C) 18 (D) 26

Answer: (D) 26

The range of data is the difference of the highest value and the lowest value. In the given question, the highest value is 32 and the lowest value is 6.

Therefore, the range = 32 - 6 = 26

3. In a frequency distribution, the mid value of a class is 10 and the width of the class is 6. The lower limit of the class is:

(A) 6 (B) 7 (C) 8 (D) 12

Answer: (B) 7

The Mid-Value of the class is 10.

Width of the interval is 6.

Lower limit = mid value - $\frac{Width}{2}$

= 10 - $\frac{6}{2}$ = 10 – 3 = 7

4. The width of each of five continuous classes in a frequency distribution is 5 and the lower class-limit of the lowest class is 10. The upper class-limit of the highest class is:

(A) 15 (B) 25 (C) 35 (D) 40

Answer: (C) 35

Lower class limit is 10.

Width of each of five continuous classes is 5.

Total Width till upper class limit = 5 × 5 = 25

Therefore, the upper-class limit of the highest class is the sum of the lower-class limit and the total width till upper class limit = 10 + 25 = 35

5. Let m be the mid-point and l be the upper-class limit of a class in a continuous frequency distribution. The lower-class limit of the class is:

(A) 2m+1 (B) 2m−1 (C) m−1 (D) m−2

Answer: (B) 2m−1

Let the lower-class limit be z.

The upper-class limit is given as l.

Mid-point = $\frac{Lower\ Class\ limit + Higher\ Class\ limit}{2}$

Therefore, m = $\frac{z+l}{2}$

Hence, z = 2m − l

6. The class marks of a frequency distribution are given as follows:

15, 20, 25, ...

The class corresponding to the class mark 20 is:

(A) 12.5 – 17.5 (B) 17.5 – 22.5

(C) 18.5 – 21.5 (D) 19.5 – 20.5

Answer: (B) 17.5 – 22.5

Mean of 15 and 20 = $\frac{15+20}{2}$ = 17.5

Mean of 20 and 25 = $\frac{20+25}{2}$ = 22.5

Here, the lower-class limit is mean of 15 and 20.

Similarly, the upper-class limit is the mean of 20 and 25.

So, the class interval for class mark 20 is 17.5 - 22.5.

7. In the class intervals 10-20, 20-30, the number 20 is included in:

(A) 10 – 20 (B) 20 - 30

(C) both the intervals (D) none of these intervals

Answer: (B) 20 - 30

The number 20 is included in the class interval 20 – 30 because the lower-limit of the class interval is always counted first.

8. A grouped frequency table with class intervals of equal sizes using 250-270 (270 not included in this interval) as one of the class-interval is constructed for the following data:

268, 220, 368, 258, 242, 310, 272, 342, 310, 290, 300, 320, 319, 304, 402, 318, 406, 292, 354, 278, 210, 240, 330, 316, 406, 215, 258, 236.

The frequency of the class 310-330 is:

(A) 4 (B) 5 (C) 6 (D) 7

Answer: (C) 6

Given data is:

268, 220, 368, 258, 242, 310, 272, 342, 310, 290, 300, 320, 319, 304, 402, 318, 406, 292, 354, 278, 210, 240, 330, 316, 406, 215, 258, 236.

Out of the given data, the numbers that lie in the class 310-330 are:

310, 310, 320, 319, 318, 316.

(The presence of 310 does not matter for the interval 310 – 330 because it will not be counted in this interval)

The frequency of numbers that lie in class 310 – 330 = 6.

9. A grouped frequency distribution table with classes of equal sizes using 63 – 72 (72 included) as one of the classes is constructed for the following data:

30, 32, 45, 54, 74, 78, 108, 112, 66, 76, 88, 40, 14, 20, 15, 35, 44, 66, 75, 84, 95, 96, 102, 110, 88, 74, 112, 14, 34, 44.

The number of classes in the distribution will be:

(A) 9 (B) 10 (C) 11 (D) 12

Answer: (A) 9

Given data points are:

30, 32, 45, 54, 74, 78, 108, 112, 66, 76, 88, 40, 14, 20, 15, 35, 44, 66, 75, 84, 95, 96, 102, 110, 88, 74, 112, 14, 34, 44.

Given class is 63 – 72 (where 72 is included)

Range of the given class 63 – 72 = 10.

Range of the given data points is 112 – 14 = 98

Number of classes in the given distribution is $\frac{Range\ of\ data\ points}{Range\ of\ given\ class} = \frac{98}{10} = 9.8$

Since, the number of classes is always a whole number, then the number of classes for this distribution is 10.

10. To draw a histogram to represent the following frequency distribution

Class interval	5-10	10-15	15-25	25-45	45-77
Frequency	6	12	10	8	15

the adjusted frequency for the class 25-45 is:

(A) 6 **(B) 5** **(C) 3** **(D) 2**

Answer: (D) 2

C.I	Frequency	Width of Class	Adjusted Frequency
5 – 10	6	5	$\frac{6}{5}\ x\ 5 = 6$
10 – 15	12	5	$\frac{12}{5}\ x\ 5 = 12$
15 – 25	10	10	$\frac{10}{10}\ x\ 5 = 5$
25 – 45	8	20	$\frac{8}{20}\ x\ 5 = 2$
45 – 75	15	30	$\frac{15}{30}\ x\ 5 = 2.5$

So, the adjusted frequency for the class 25 – 45 is 2.

11. The mean of five numbers is 30. If one number is excluded, their mean becomes 28. The excluded number is:

(A) 28 **(B) 30** **(C) 35** **(D) 38**

Answer: (D) 38

The mean of 5 numbers is = 30

Let the number which is excluded be x.

After excluding x, the mean is = 28

Let the other numbers be a_1, a_2, a_3 and a_4

Therefore, 30 = $\frac{a_1 + a_2 + a_3 + a_4 + x}{5}$ ....equation 1

And 28 = $\frac{a_1 + a_2 + a_3 + a_4}{4}$

So, $a_1 + a_2 + a_3 + a_4$ = 28 x 4 = 112 .... equation 2

Putting the value of equation 2 in equation 1.

30 = $\frac{112+x}{5}$

150 = 112 + x

X = 38

12. If the mean of the observations:

x, x+3, x+5, x+7, x+10x, x+3, x+5, x+7, x+10 is 9, the mean of the last three observations is

(A) $10\frac{1}{3}$ (B) $10\frac{2}{3}$ (C) $11\frac{1}{3}$ (D) $11\frac{2}{3}$

Answer: (C) $11\frac{1}{3}$

Mean of given observations = 9

Observations are:

x, x+3, x+5, x+7, x+10x, x+3, x+5, x+7, x+10

Therefore,

9 = $\frac{x+x+3+x+5+x+7+x+10}{5}$

9 = $\frac{5x+25}{5}$

9 = x + 5

x = 4

So, the mean of last three observations is

= $\frac{9+11+14}{3} = \frac{34}{3} = 11\frac{1}{3}$

13. If $\bar{x}$ represents the mean of n **observations** x , x_1 , x_2,…………, x_n **then value of**

$\sum_{i=1}^{n} (x_i - \bar{x})$ **is :**

(A) -1 (B) 0 (C) 1 (D) n - 1

Answer: (B) 0

Mean of n observation x , x_1 , x_2,…………, x_n = $\bar{x}$

Therefore, $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$

➔ $\sum_{i=1}^{n} x_i = n\,\bar{x}$ then,

→ $\sum_{i=1}^{n} (x_i - \bar{x}) = \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \bar{x}$

→ n $\bar{x}$ = $\bar{x} \sum_{i=1}^{n} 1$ $\lfloor and \ \sum_{i=1}^{n} 1 = n \rfloor$

➔ n $\bar{x}$ - $\bar{x}$ n

= 0

Hence the value of $\sum_{i=1}^{n} (x_i - \bar{x}) = 0$

14. If each observation of the data is increased by 55**, then their mean**

(A) remains the same

(B) becomes 5 times the original mean

(C) is decreased by 5

(D) is increased by 5

Answer: (D) is increased by 5

Assume mean of n observations be $x_1, x_2, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ x_n = \bar{x}$

Let's represent the older mean by $\bar{x}_{old}$ then,

$\bar{x}_{old} = \frac{\sum_{i=1}^{n} x_i}{n}$

Now, new mean by adding 5 in each observation, becomes

Representing new mean by $\bar{x}_{new}$ therefore,

➔ $\bar{x}_{new} = \frac{(x_1+5) + (x_2+5)..................+ (x_n+5)}{n}$

➔ $\frac{(x_1+(x_2+\cdots......+ x_n)+ 5_n}{n}$

➔ $\frac{\sum_{i=1}^{n} x_i}{n} + 5 = \bar{x}_{old} + 5 \left[\because \frac{\sum_{i=1}^{n} x_i}{n} = \bar{x}_{old}\right]$

➔ $\bar{x}_{new} = \bar{x}_{old} + 5$

Hence, the new mean is increased by 5.

15. Let $\bar{x}$ be the mean of $x_1, x_2,$.................... x_n and $\bar{y}$ the mean of $y_1, y_2,$ .... y_n, then $\bar{z}$ is mean of $x_1, x_2,$...... $x_n, y_1, y_2,$ .... y_n, then $\bar{z}$ is equal to

(A) $\bar{x} + \bar{y}$

(B) $\frac{\bar{x} + \bar{y}}{2}$

© $\frac{\bar{x} + \bar{y}}{n}$

(D) $\frac{\bar{x} + \bar{y}}{2n}$

Answer: (B) $\frac{\bar{x} + \bar{y}}{2}$

If $\bar{x} = \frac{\sum x}{n}$ where, $\bar{x}$ is the mean of $x_1, x_2,$...... x_n

Therefore, n $\bar{x} = \sum x$

Similarly, $\bar{y} = \frac{\sum y}{n}$ = n $\bar{y} = \sum y$

Given that $\bar{z}$ is mean of $x_1, x_2,$...... $x_n, y_1, y_2,$ .... y_n then

➔ $\bar{z} = \frac{\sum y + \sum y}{n+n}$

➔ $\frac{n\bar{x} + n\bar{y}}{2n}$

➔ $\frac{n(\bar{x} + \bar{y})}{2n}$

➔ $\frac{(\bar{x} + \bar{y})}{2}$

Hence, $\bar{z} = \frac{(\bar{x} + \bar{y})}{2}$

16. If $\bar{x}$ is the mean of $x_1, x_2, \ldots\ldots x_n$, then for $a \neq 0$, the mean of $ax_1, ax_2 \ldots\ldots\ldots ax_n, \frac{x_1}{a}, \frac{x_2}{a}, \ldots\ldots\ldots\ldots \frac{x_n}{a}$ is

(A) $\left(a + \frac{1}{a}\right)\bar{x}$

(B) $\left(a + \frac{1}{a}\right)\frac{\bar{x}}{2}$

© $\left(a + \frac{1}{a}\right)\frac{\bar{x}}{n}$

(D) $\left(\frac{\left(a + \frac{1}{a}\right)\bar{x}}{2n}\right)$

Answer: (B) $\left(a + \frac{1}{a}\right)\frac{\bar{x}}{2}$

Given that $\bar{x}$ is the mean of $x_1, x_2, \ldots\ldots x_n$ then

➔ $\frac{ax_1, ax_2 \ldots\ldots\ldots ax_n}{n} = \bar{x}$

Similarly, the mean of $ax_1, ax_2 \ldots\ldots\ldots ax_n$ is

➔ $\frac{ax_1, ax_2, \ldots\ldots ax_n}{an} = a\bar{x}$

Also, the mean of

$$\rightarrow \frac{\frac{1}{a}x_1 + \frac{1}{a}x_2 + \dots \frac{1}{a}x_n}{\frac{1}{a}n}$$

$$\rightarrow \frac{\frac{1}{a}x_1 + \frac{1}{a}x_2 + \dots \frac{1}{a}x_n}{n} = \frac{\bar{x}}{a}$$

Then the mean of $ax_1, ax_2 \dots\dots\dots ax_n, \frac{x_1}{a},, \frac{x_2}{a}, \dots\dots\dots \frac{x_n}{a}$ is given by

$$\rightarrow \frac{a\bar{x} + \frac{\bar{x}}{a}}{2} = \frac{\bar{x}}{2}\left(\boldsymbol{a} + \frac{\mathbf{1}}{\boldsymbol{a}}\right)$$

Hence the mean of $ax_1, ax_2 \dots\dots\dots ax_n, \frac{x_1}{a},, \frac{x_2}{a}, \dots\dots\dots \frac{x_n}{a} = \frac{\bar{x}}{2}\left(\boldsymbol{a} + \frac{\mathbf{1}}{\boldsymbol{a}}\right)$

17. **if $\bar{x}_1, \bar{x}_2, \bar{x}_3 \dots\dots\dots \bar{x}_n$ are the means of n groups with $n_1, n_2, \dots\dots n_n$ number of observations respectively, then the mean $\bar{x}$ of all the groups taken together is given by:**

(A) $\sum_{i=1}^{n} n_i \, \bar{x}_i$

(B) $\frac{\sum_{i=1}^{n} n_i \bar{x}_i}{\boldsymbol{n^2}}$

(C) $\frac{\sum_{i=1}^{n} n_i \bar{x}_i}{\sum_{i=1}^{n} n_i}$

(D) $\frac{\sum_{i=1}^{n} n_i \bar{x}_i}{2n}$

Answer: $\frac{\sum_{i=1}^{n} n_i \bar{x}_i}{\sum_{i=1}^{n} n_i}$

Mean of all the groups together is given by total sum divided by total count.

i.e = $\frac{total\ sum}{total\ count}$

If $\bar{x}$ is the mean of one group then mean of n group is given by n$\bar{x}$

Therefore, mean is given by

Mean = $\frac{\sum_{i=1}^{n} n_i \overline{x}_i}{\sum_{i=1}^{n} n_i}$

18. The mean of 100 observations is 50. If one of the observations which was 50 is replaced by 150, the resulting mean will be:

(A) 50.5 (B) 51 (C) 51.55 (D) 52

Answer: (B) 51

Given that, number of observations i.e., $\bar{x}$ = 100

Also n = 100

And mean = 50

We know that, mean = $\frac{\sum_{i=1}^{n} x_i}{n}$

Substituting the value

➔ $\frac{1}{100}$ X $\sum_{i=1}^{100} x_i$ = 50

➔ $\sum_{i=1}^{100} x_i$ = 5000

If observation 50 is replaced by 150 then

➔ $\sum_{i=1}^{100} x_i$ = 5000 – 50 + 150

➔ $\sum_{i=1}^{100} x_i$ = 5100

Required mean is given by = $\frac{\sum_{i=1}^{100} x_i}{100} = \frac{5100}{100}$ = 51

19. There are 50 numbers. Each number is subtracted from 53 and the mean of the numbers so obtained is found to be −3.5. The mean of the given numbers is:

(A) 46.5 (B) 49.5 (C) 53.5 (D) 56.5

Answer: **(D) 56.5**

Given number of observations are =50 i.e., n = 50

$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$

$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{50}$

➔ $\sum_{i=1}^{50} x_i = 50\bar{x}$

If each number is subtracted from 53 and there are 50 numbers then its new mean is given by $\bar{x}_{new}$

➔ $\bar{x}_{new} = \frac{(53 - x_1) + (53 - x_2) + \cdots (53 - x_n)}{50}$

➔ $\frac{(53 - x_1) + (53 - x_2) + \cdots (53 - x_n)}{50} = -3.5$

➔ - 3.5 x 50 = $-(x_1 + x_2 \ldots \ldots .. x_{50})$ + 53 x 50

➔ $\sum_{i=1}^{50} x_i$ = 2650 + 175 = 2825

Therefore, mean of 50 observation is

➔ $\frac{1}{50} \sum_{i=1}^{50} x_i$

➔ $\frac{1}{50}$ x 2825 = 56.5

Hence, the mean is 56.5

20. The mean of 25 observations is 36. Out of these observations if the mean of first 13 observations is 32 and that of the last 1313 observations is 40, the 13th observation is:

(A) 23 (B) 36 (C) 38 (D) 40

Answer: (B) 36

According to the question, the mean of 25 observation is =36.

Therefore, Sum of 25 observation is =25 × 36 = 900.

Now, the mean of the first 13 observation is 32.

So, sum of first 13 observation =13 × 32 = 416

And mean of last 13 observation is 40

So, sum of last 13 observation =13 × 40 = 520

Therefore, 13^{th} observation = (Sum of last 13^{th} observation + sum of first 13^{th} observation) – (sum of 25 observation) = (520 + 416) – 900 = 936 − 900 = 36

Hence, 13^{th} observation is 36.

21. The median of the data 78,56,22,34,45,54,39,68,54,84 is:

(A) 45 **(B) 49.5** **(C) 54** **(D) 56**

Answer: (C) 54

We know that, when number of observations n is even, the median is given by

Mean of the average = $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$

For finding the median of the given data it is first arranged in ascending order as follows:

22,34,39,45,54,54,56,68,78 and 84

Now, number of observations (n) = 10

∴ $\left(\frac{n}{2}\right)^{th}$ term = $\left(\frac{10}{2}\right)^{th}$ = 5th term i.e., 54 from the data arranged in ascending order

Similarly, for $\left(\frac{n}{2}+1\right)^{th} = \left(\frac{10}{2}+1\right)^{th}$ = (5^{th} + 1) term ⇒ 6^{th} term i.e., 54 from the data arranged in ascending order

Now,

Median = $\frac{\left(\left(\frac{n}{2}\right)^{th}+\left(\frac{n}{2}+1\right)^{th}\right)}{2}$

$= \frac{54+24}{2}$

$= \frac{108}{2} = 54$

Hence, the median of given data is 54.

22. For drawing a frequency polygon of a continuous frequency distribution, we plot the points whose ordinates are the frequencies of the respective classes and abscissa are respectively:

(A) upper limits of the classes

(B) lower limits of the classes

(C) class marks of the classes

(D) upper limits of preceding classes

Answer: (C) class marks of the classes

The mid-points of the classes i.e., Class marks are the abscissa of the points, which we plot for frequency polygon.

23. **Median of the following number 4,4,5,7,6,7,7,12,3 is:**

(A) 44 (B) 55 (C) 66 (D) 77

Answer: (C) 66

Median of the given data with number of observations (n)(n) is odd = $\left(\frac{n}{2}+1\right)^{th}$ term

Arranging the given data in ascending order 3,4,4,5,6,7,7,7,12

Median = $\left(\frac{n}{2}+1\right)^{th}$ term

= $\left(\frac{9}{2}+1\right)^{th}$ = $\left(\frac{10}{2}\right)^{th}$ = 5^{th} term i.e 6 from the data arranged in ascending order

= 6

Hence, the median is 6.

24. Mode of the data

15,14,19,20,14,15,16,14,15,18,14,19,15,17,15 **is**

(A) 14 (B) 15 (C) 16 (D) 17

Answer: (B) 15

The observation that has maximum frequency is called Mode. In the given set of data observation 15 has frequency 5 i.e., it occurs more times than other observations. Hence mode of the given data is 15.

25. In a sample study of 642 people, it was found that 514 people have a high school certificate. If a person is selected at random, the probability that the person has a high school certificate is:

(A) 0.5 (B) 0.6 (C) 0.7 (D) 0.8

Answer: (D) 0.8

Probability of an event P(E) = $\frac{Number\ of\ favorable\ outcomes}{Total\ outcomes}$

Number of people with high school certificate = 514

Total number of people = 642

Probability that a person has high school certificate = $\frac{number\ of\ people\ with\ high\ school\ certificate}{total\ number\ of\ people}$

= $\frac{514}{642}$ = 0.8

Hence, probability is 0.8.

26. In a survey of 364 children aged 19 - 36 months, it was found that 91 liked to eat potato chips. If a child is selected at random, the probability that he/she does not like to eat potato chips is:

(A) 0.25 (B) 0.50 (C) 0.75 (D) 0.80

Answer: (C) 0.75

Given that, Total number of children aged from 19- 36 = 364

Number of children that liked to eat potato = 91

Favourable outcomes = n(E)

Total outcome = n(S)

Probability that a child likes potato chips = $\frac{Number\ of\ children\ that\ like\ potatoes}{Total\ number\ of\ children}$

Probability that a child likes potato chips = $\frac{n\ (E)}{n\ (S)}$

= $\frac{91}{364}$ = 0.25

∴ P(E) = 0.25

Now, the probability that he/she does not like to eat potato chips:

=1 − P(E) =1−0.25

= 0.75

Hence, the probability that he/she does not like to eat potato chips is 0.75.

27. In a medical examination of students of a class, the following blood groups are recorded:

Blood Group	**A**	**AB**	**B**	**O**
Number of Students	**10**	**13**	**12**	**5**

A student is selected at random from the class. The probability that he/she has blood group B, is:

(A) $\frac{1}{4}$

(B) $\frac{13}{40}$

(C) $\frac{3}{10}$

(D) $\frac{1}{8}$

Ans: Option (C)

Number of students having blood group B, n(E) = 12

Total number of students, n(S) = (10 + 12 + 13 + 5) = 40

Probability that a student selected at random has blood group B, P(E) = $\frac{n\,(E)}{n\,(S)}$

$= \frac{12}{40} = 0.3$

$= \frac{3}{10}$

Hence, the probability that he/she has blood group B, is $\frac{3}{10}$.

28. Two coins are tossed 1000 times and the outcomes are recorded as below:

Number of Heads	**2**	**1**	**0**
Frequency	**200**	**550**	**250**

Based on this information, the probability for at most one head is

(A) $\frac{1}{5}$ (B) $\frac{1}{4}$ (C) $\frac{4}{5}$ (D) $\frac{3}{4}$

Answer: (C) $\mathbf{\frac{4}{5}}$

Total number of coins tossed, n(S) =1000.

Number of outcomes with utmost one head, n(E) = 550 + 250 = 800

Therefore,

P(E) = $\frac{n\ (E)}{n\ (S)}$

= $\frac{8000}{1000}$ = $\frac{4}{5}$

Hence, the probability of utmost one head is $\frac{4}{5}$

29. 80 **bulbs are selected at random from a lot and their life time (in hrs) is recorded in the form of a frequency table given below:**

Lifetime (in hours)	**300**	**500**	**700**	**900**	**1100**
Frequency	**10**	**12**	**23**	**25**	**10**

One bulb is selected at random from the lot. The probability that its life is 1150 **hours, is**

(A) $\frac{1}{80}$ (B) $\frac{7}{16}$ (C) 0 (D) 1

Answer: (C) 0

Total bulb, n(S) = 80

Number of bulb with lifetime 1150 hrs, n(E) = 0

Probability that lifetime is 1150 hrs, P(E) =$\frac{n\,(E)}{n\,(S)}$

➔ $\frac{0}{80}$ = 0

Hence, the probability of lifetime 1150 hrs is 0.

30. Refer to Q.29 above:

The probability that bulbs selected randomly from the lot has life less than 900 **hours is:**

(A) $\frac{11}{40}$ (B) $\frac{5}{16}$ (C) $\frac{7}{16}$ (D) $\frac{9}{16}$

Answer: (D) $\frac{9}{16}$

Number of bulbs with lifetime less than 900 hours, n(E) = 10 + 12 + 23 = 45

Total number of bulbs in a lot, n(S) = 80

Probability that number of bulbs with lifetime less than 900 hours,

P(E) = $\frac{n\,(E)}{n\,(S)}$ = $\frac{45}{80}$ = $\frac{9}{16}$

Hence, the of probability that number of bulbs with lifetime less than 900 hours, is $\frac{9}{16}$ **.**

Short Answer Questions with Reasoning

1. The mean of the data: 2,8,6,5,4,5,6,3,6,4,9,1,5,6,5 **is given to be 5. Based on this information, is it correct to say that the mean of the data:** 10, 12,10,2,18,8,12,6,12,10,8,10,12,16,4 is 10?

Give a reason.

Answer: It is correct. The mean of the 2nd data will be 2 times the mean of the 1st data because the 2nd data is obtained by multiplying each observation of the 1st data by 2.

2. In a histogram, the areas of the rectangles are proportional to the frequencies. Can we say that the lengths of the rectangles are also proportional to the frequencies?

Ans: Yes. We can say that the lengths of the rectangles are proportional to the frequencies in a histogram because the area of the rectangle is proportional to its frequency.

The shape of rectangles in a histogram shows how different sets of data are represented according to class intervals and frequency. The following list of points will help you understand this.

- The rectangles used to represent any data in a histogram are determined by the class interval and frequency.
- The length of the rectangle determines the area of the rectangle.
- The length of the rectangle and the frequency are proportional because the area of the rectangle is proportional to its frequency.
- As a result, the rectangle's length is proportional to the frequencies.
- As a result, in a histogram, the lengths of the rectangles are proportional to the frequencies.

3. Consider the data: 2,3,9,16,9,3,9.2,3,9,16,9,3,9. **Since 16 is the highest value in the observations, is it correct to say that it is the mode of the data? Give a reason.**

Answer: The data is not in the mode of 16. The observation with the highest frequency, not the observation with the highest value, is the mode of a given set of data.

Exercise 14.2

1. The frequency distribution has been represented graphically as follows:

Marks	**0-20**	**20-40**	**40-60**	**60-100**
Number of students	**10**	**15**	**20**	**25**

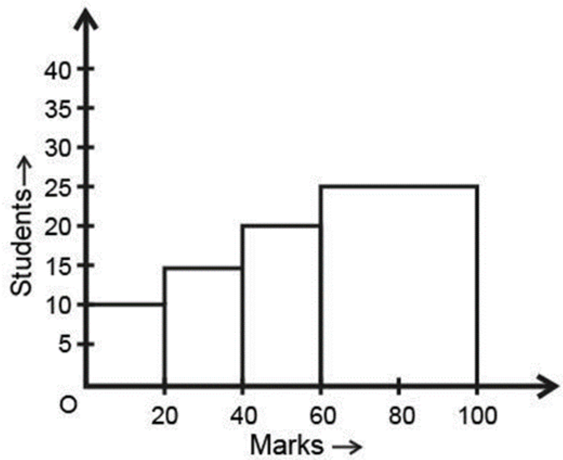


Fig. 14.1

Do you think this representation is correct? Why?

Answer: No, here the widths of the rectangles are varying, so we need to make certain modifications in the length of the rectangles so that the areas are proportional to the frequencies. We proceed as follows:

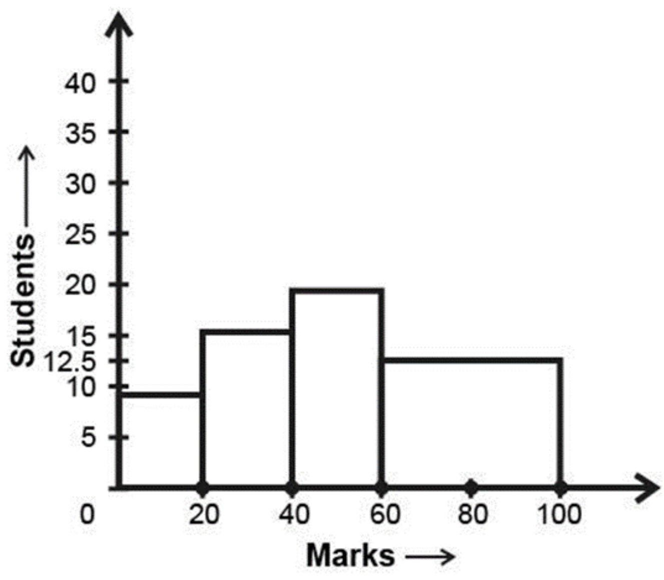
(i) Select a class interval with the minimum class size, here the minimum class size is 20.

(ii) The length of the rectangles is then modified to be proportionate to the class size 20.

Now, we get the following modified table

Marks	Number of students (frequency)	Width of class	Length of Rectangle
0−200−20	10	20	$\frac{10}{20}$ x 20 = 10
20−4020−40	15	20	$\frac{15}{20}$ x 20 = 15
40−6040−60	20	20	$\frac{10}{20}$ x 20 = 10
60−10060−100	25	40	$\frac{25}{40}$ x 20 = 125

So, the correct histogram with varying width is given below



2. In a diagnostic test in mathematics given to students, the following marks (out of 100) are recorded: 46,52,48,11,41,62,54,53,96,40,98,44 Which 'average' will be a good representative of the above data and why?

Answer: We can represent the given data using median as,

(i) each value occurs once.

(ii) The data is influenced by extreme values.

3. A child says that the median of 3,14,18,20,5 is 18. What doesn't the child understand about finding the median?

Answer: Since the child says that the median of

3, 14, 18, 20, 5 is 18, it is clear that the child doesn't understand the fact that the given data should be arranged in ascending or descending order before finding the middle term, i.e., median.

Arranging the given data in ascending order, we get,

3, 5, 14, 18, 20

As the number of terms is odd, median will be the middle term $\left(\frac{n}{2}+1\right)^{th}$ term

Median = $\left(\frac{5}{2}+1\right)^{th}$ = $\left(\frac{5+1}{2}\right)^{th}$ term = $\left(\frac{6}{2}\right)^{th}$ = 3^{rd} term = 14

4. A football player scored the following number of goals in the 10 matches:
1,3,2,5,8,6,1,4,7,91,3,2,5,8,6,1,4,7,9 **Since the number of matches is 10 (an even number), therefore, the**

Median = $\frac{5^{th}\ Observation + 6^{th}\ Observation}{2} = \frac{8+6}{2}$ = 7, is it the correct answer and why?

Answer: No, the obtained data solution in the question is not the correct answer, because the data has to be arranged in either ascending or descending order before finding the median. Now, arranging the data in ascending order, we get, 1,1,2,3,4,5,6,7,8,9

Here, the number of observations is 10, which is even.

So, Median = $\frac{\left(\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}\right)}{2}$

$$= \left(\frac{\left(\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th} \right)}{2} \right)$$

$$= \left(\frac{\left(\left(\frac{10}{2}\right)^{th} + \left(\frac{10}{2}+1\right)^{th} \right)}{2} \right)$$

$$= \left(\frac{\left((5)^{th} + (6)^{th} \right)}{2} \right) = \left(\frac{(4+5)}{2} \right) = \frac{9}{2} = 4.5$$

5. Is it correct to say that in a histogram, the area of each rectangle is proportional to the class size of the corresponding class interval? If not, correct the statement.

Ans: It is not correct, because in a histogram, the area of each rectangle is proportional to the corresponding frequency of its class.

6. The class marks of a continuous distribution are: 1.04, 1.14 ,1.24 ,1.34 ,1.44 ,1.54 and 1.64 Is it correct to say that the last interval will be 1.55−1.73? Justify your answer.

Answer: No, the last interval will not be 1.55 − 1.73. Because the difference between two consecutive class marks should be equal to the class size. Here, the difference between two consecutive marks is 0.1 and class size of 1.55−1.73 is 0.18, which are not equal.

7. 30 children were asked about the number of hours they watched TV programmes last week. The result are recorded as under

Number of Hours	**0-5**	**5-10**	**10-15**	**15-20**
Frequency	**8**	**16**	**4**	**2**

Can we say that the number of children who watched TV for 10 or more hours in a week is 22? Justify your answer.

Answer: No, in fact the number of children who watched TV for 10 or more hours in a week is 4 + 2 i.e. 6.

8. Can the experimental probability of an event be a negative number? If not, why?

Answer: No, since the number of trials in which the event can happen cannot be negative and the total number of trials is always positive.

9. Can the experimental probability of an event be greater than 1? Justify your answer.

Answer: No, since the number of trials in which the event can happen cannot be greater than the total number of trials.

10. As the number of tosses of a coin increases, the ratio of the number of heads to the total number of tosses will be $\frac{1}{2}$. Is this correct? If not, write the correct one.

Answer: No, since the number of coin increases, the ratio of the number of heads to the total number of tosses will be nearer to $\frac{1}{2}$ but not exactly $\frac{1}{2}$.

11. Heights (in cm) of 30 girls of Class IX are given below:

140, 140, 160, 139, 153, 153, 146, 150,

148, 150, 152, 146, 154, 150, 160, 148,

150, 148, 140, 148, 153, 138, 152, 150,

148, 138, 152, 140, 146, 148.

Prepare a frequency distribution table for this data.

Answer: Frequency distribution of heights of 30 girls

Height (in cm)	Tally Marks	Frequency
138	||	2
139	|	1
140	||||	4

146	|||	3
148	~~||||~~ |	6
150	~~||||~~	5
152	|||	3
153	|||	3
154	|	1
160	||	2
		Total 30

2. The following observations are arranged in ascending order:

26, 29 ,42 , 53 , x , x+2 , 70 , 75 , 82 , 93

If the median is 65, find the value of x**.**

Answer: Number of observations (n)=10, which is even. Therefore, median is the mean of

Median = $\frac{\left(\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}\right)}{2}$

$= \frac{\left(\left(\frac{10}{2}\right)^{th} + \left(\frac{10}{2}+1\right)^{th}\right)}{2} = \frac{\left((5)^{th} + (6)^{th}\right)}{2} = \left(\frac{x + (x+2)}{2}\right) = x + 1$

Now, x + 1 = 65x + 1 = 65 (Given)

Therefore, x = 64

Thus, the value of x is 64.

3. Here is an extract from a mortality table.

Age (in years)	Number of Persons Surviving out of a Sample of One Million
60	16090
61	11490
62	8012
63	5448
64	3607
65	2320

(i) Based on this information, what is the probability of a person 'aged 60' of dying within a year?

(ii) What is the probability that a person 'aged 61' will live for 4 years?

Answer: (i) We see that 16090 persons aged 60, (16090−11490), i.e., 4600 died before reaching their 61st birthday

Therefore, P(a person aged 60 die within a year) = $\frac{4600}{16090} = \frac{460}{1609}$

(ii) Number of persons aged 61 years = 11490

Number of persons surviving for 4 years = 2320

P (a person aged 61 will live for 4 years) = $\frac{2320}{11490} = \frac{232}{1149}$

Exercise 14.3

1. The blood groups of 30 students are recorded as follows:

A, B, O, A, AB, O, A, O, B, A,

O, B, A, AB, B, A, AB, B, A, A,

O, A, AB, B, A, O, B, A, B, A.

Prepare a frequency distribution table for the data.

Answer: The number of students who have a certain type of blood group is called the frequency of those blood groups. A frequency distribution table for the given data is given below

Blood Groups	**Tally Marks**	**Number of students (frequency)**
A	~~IIII~~ ~~IIII~~ II	12
B	~~IIII~~ III	8
AB	IIII	4
O	~~IIII~~ I	6
Total		30

2. The value of π up to 35 decimal places is given below:

3.14159265358979323846264338327950288

Make a frequency distribution of the digits 0 to 9 after the decimal point.

Answer: The frequency of those digits is defined as the number of times they are repeated. The following is a frequency distribution table for the given data.

Digit	**Tally Marks**	**Frequency**
0	I	1
1	II	2
2	~~IIII~~	5

3	~~||||~~ |	6
4	|||	3
5	||||	4
6	|||	3
7	||	2
8	~~||||~~	5
9	||||	4
Total		35

3. The scores (out of 100) obtained by 33 students in a mathematics test are as follows:

69, 48, 84, 58, 48, 73, 83, 48, 66, 58, 84,

66, 64, 71, 64, 66, 69, 66, 83, 66, 69, 71,

81, 71, 73, 69, 66, 66, 64, 58, 64, 69, 69.

Scores	**Tally Marks**	**Frequency**
48	|||	3
58	|||	3
64	||||	4
66	~~||||~~ ||	7
69	~~||||~~ |	6

71	|||	3
73	||	2
81	|	1
83	||	2
84	||	2
Total		**33**

Represent this data in the form of a frequency distribution.

Answer: The Number of students who have the same marks in mathematics is called the frequency of that mark. A frequency distribution table for the given data is given below

4. Prepare a continuous grouped frequency distribution from the following data:

Mid-point	Frequency
5	**4**
15	**8**
25	**13**
35	**12**
45	**6**

Also find the size of class intervals.

Answer: Here, we see that the difference between two midpoints is 15−5 i.e., 10. It means the width of the class interval is 10. Let the lower limit of the first-class interval be a. Then, its upper limit = a+10

Now, mid value of the first-class interval = 5

➔ Mid value = $\frac{Lower\ limit\ +\ Upper\ limit}{2}$

➔ 5 = $\frac{a\ +\ a+10}{2}$

➔ 2a + 10 = 10

➔2a = 0

➔a = 0

So, the first class interval is 0−100−10. Now, we prepare a continuous grouped frequency distribution table is given below

Mid-point	**Class Interval**	**Frequency**
5	0-10	4
15	10-20	8
25	20-30	13
35	30-40	12
45	40-50	6

Hence, the size of the class interval is 10 i.e., 0−10 .

5. Convert the given frequency distribution into a continuous grouped frequency distribution:

Class Interval	Frequency
150-153	7
154-157	7
158-161	15
162-165	10
166-169	5
170-173	6

In which intervals would 153.5 and 157.5 be included?

Answer: It is clear that the given table is in inclusive (discontinuous) form. So, we first convert it into an exclusive form.

Now, consider the classes 150−153, 154−157 Lower limit of 154−157 = 154 and upper limit of 150 −153 = 153

Required difference = 154 −153 = 1

So, half the difference = $\frac{1}{2}$ = 0.5

So, we subtract 0.5 from each lower limit and add 0.5 to each upper limit.
The table for continuous grouped frequency distribution is given below

Class Interval	Frequency
149.5 - 153.5	7
153.5 - 157.5	7
157.5 - 161.5	15
161.5 - 165.5	10
165.5 - 169.5	5
169.5 - 173.5	6

Thus, 153.5 and 157.5 will lie in the class interval 153.5 - 157.5 and 157.5 - 161.5, respectively.

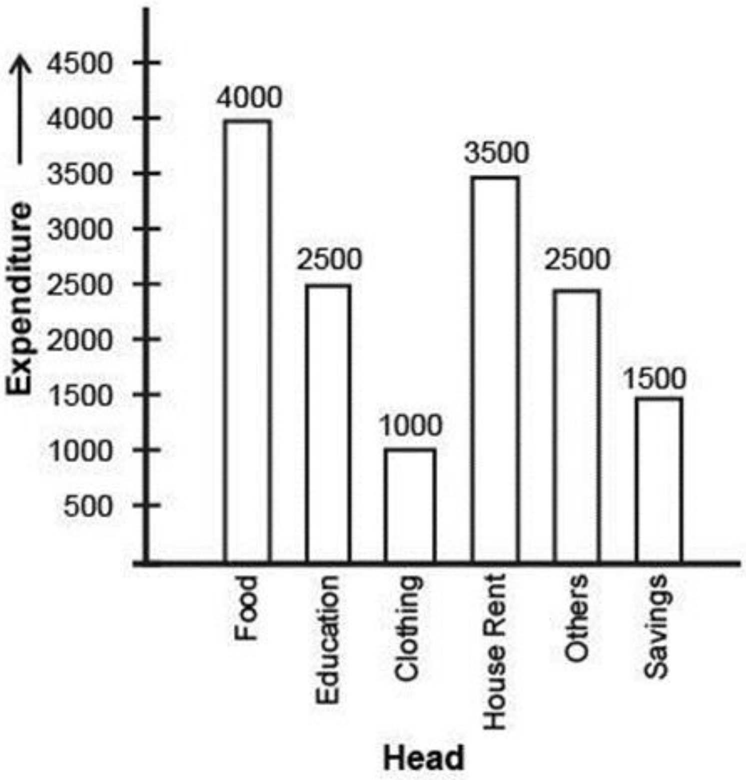
6. The expenditure of a family on different heads in a month is given below:

Head	Food	Education	Clothing	House rent	Others	Savings
Expenditure (in Rs)	4000	2500	1000	3500	2500	1500

Draw a bar graph to represent the data above.

Ans: The bar graph for the following data is represented by

Scale : 1 unit = Rs. 5001 unit = Rs. 500



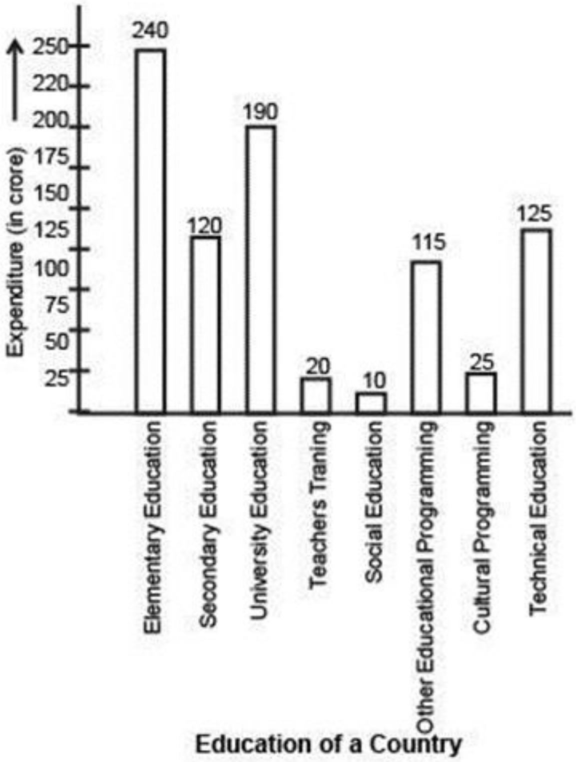
7. Expenditure on education of a country during a five years period (2002-2006), in crore of rupees, is given below

Elementary Education	**240**
Secondary Education	**120**
University Education	**190**
Teacher's Training	**20**
Social Education	**10**
Other Educational Programmes	**115**
Cultural Programmes	**25**
Technical Education	**125**

Represent the information above by a bar graph.

Answer: The bar graph for the above given data is represented by

Scale : 1 unit = 25 crores1 unit = 25 crores



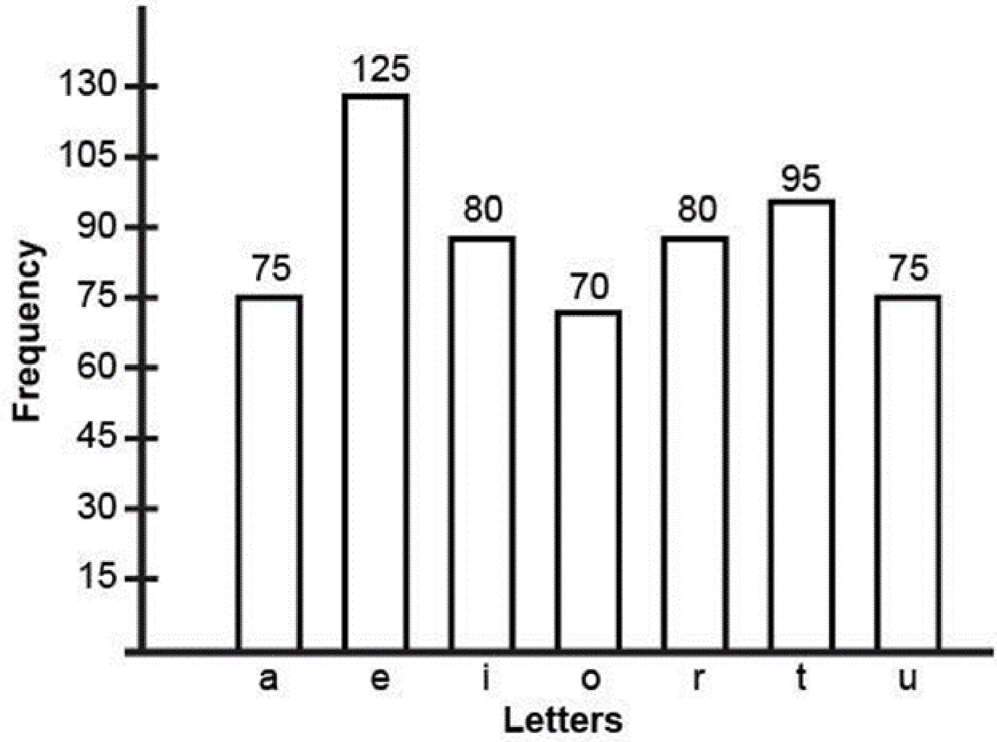
8. The following table gives the frequencies of most commonly used letters o, e, i, o, r, t, u, from a page of a book

Letters	a	E	i	o	r	t	u
Frequency	**75**	**125**	**80**	**70**	**80**	**95**	**75**

Represent the information above by a bar graph.

Answer: Representing the information above by a bar graph.

Scale : 1 unit =15 frequency



9. If the mean of the following data is 20.2, find the value of p:

x	**10**	**15**	**20**	**25**	**30**
f	**6**	**8**	**p**	**10**	**6**

Answer: We know that, Mean of the data $(\bar{x})$ = $\frac{\sum_{i=1}^{5} f_i\, x_i}{\sum_{i=1}^{5} f_i}$ = 20.2 given

➔ $\frac{f_1x_1 + f_2x_2 + f_3x_3 + f_4x_4 + f_5x_5}{t_1 + t_2 + t_3 + f_4 + f_5}$ = 20.2

➔ $\frac{(6)(10) + (8)(15) + (p)(20) + (10)(25) + (6)(30)}{6 + 8 + p + 10 + 6}$ = 20.2

➔ $\frac{60 + 120 + 20\,p + 250 + 180}{30 + p}$ = 20.2

➔ 610 – 606 = 02p

➔ $\frac{2p}{10}$ = 4

➔ ∴p = 10 × 2 = 20

Hence, the value of p is 20.

10. Obtain the mean of the following distribution:

Frequency	Variable
4	4
8	6
14	8
11	10
3	12

Answer: We know that,

Mean of the data $(\bar{x})$ = $\frac{\sum_{i=1}^{5} f_i x_i}{\sum_{i=1}^{5} f_i}$ = $\frac{f_1x_1 + f_2x_2 + f_3x_3 + f_4x_4 + f_5x_5}{t_1 + t_2 + t_3 + f_4 + f_5}$

= $\frac{4 X 4+8 x 6+14 x 8+11 x 10+3 x 12}{4+8+14+11+ 3}$ = $\frac{16+48+112+110+36}{40}$

= $\frac{322}{40}$ = 8.05

Hence, the mean of the given data is 8.05.

11. A class consists of 50 students out of which 30 are girls. The mean of marks scored by girls in a test is 73 (out of 100) and that of boys is 71. Determine the mean score of the whole class.

Answer: A class consists of 50 students out of which 30 are girls.

The mean marks of 30 girls =73

Total score of 30 girls

$=73 \times 30 = 2190$

Now, number of boys are

$50 - 30 = 20$

The mean marks of 20 boys =71

Total score of boys = $71 \times 20 = 1420$

Here, the total score of whole class = Total score of all girls + Total score of all boys

$= 2190 + 1420 = 3610$

Mean of all students = $\frac{Total\ marks\ of\ all\ students}{Total\ number\ of\ students}$

$= \frac{3610}{50} = 72.2$

Hence, the mean of whole class is 72.2

12. Mean of 50 observations was found to be 80.4. But later on, it was discovered that 96 was misread as 69 at one place. Find the correct mean.

Answer: Mean of 50 observations was found to be 80.4 (Incorrect mean) The incorrect sum of all the numbers.

The incorrect sum of all the numbers = Incorrect mean x Total numbers

= 80.4 x 50 = 4020

It was discovered that 96 was misread as 69 at one place

Hence the correct sum of all the numbers = 4020 – 69 + 96 = 4047

So the correct mean $\frac{4047}{50} = 80.94$

Hence, the correct mean is 80.94 .

13. Ten observations 6, 14, 15, 17, x + 1, 2x−13, 30, 32, 34, 436, 14, 15, 17, x + 1, 2x−13, 30, 32, 34, 43 are written in an ascending Order. The median of the data is 24. Find the value of x.

Answer: Here, the observations are: 6, 14, 15, 17, x + 1, 2x−13, 30, 32, 34, 43

To calculate the median, arrange the given data in ascending order and then find the middle term. This middle term is called the median. The terms are already given in ascending order so we have to find the middle term.

Number of terms, n = 10 (even)

Median = $\frac{\left(\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}\right)}{2}$

24 = $\frac{\left(\left(\frac{10}{2}\right)^{th} + \left(\frac{10}{2}+1\right)^{th}\right)}{2}$

24 = $\frac{\left(\left(\frac{10}{2}\right)^{th} + \left(\frac{10}{2}+1\right)^{th}\right)}{2}$

24 = $\frac{\left((5)^{th} + (6)^{th}\right)}{2}$

24 = $\frac{x+1+2x-13}{2}$

➔ 2 x 24 = $x + 1 + 2x - 13$

➔ 48 = 3x – 12

➔ 60 = 3x

➔ x = 20

Hence, the correct answer is 20.

14. The points scored by a basketball team in a series of matches are as follows:

17,2,7,27,25,5,14,18,10,24,48,10,8,7,10,28. Find **the median and mode for the data.**

Answer: Median = 12 and mode = 10

To calculate the median, arrange the given data in ascending order and then find the middle term. This middle term is called the median.

Here total elements, n =16 (even)

the terms as arranged in ascending order: 17,2,7,27,25,5,14,18,10,24,48,10,8,7,10,28

Number of observations =16 (even number)

now using the formula of the median in case number of terms is even

Median = $\frac{\left(\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}\right)}{2}$

Median = $\frac{\left(\left(\frac{16}{2}\right)^{th} + \left(\frac{16}{2}+1\right)^{th}\right)}{2}$

Median = $\frac{\left((8)^{th} + (9)^{th}\right)}{2}$

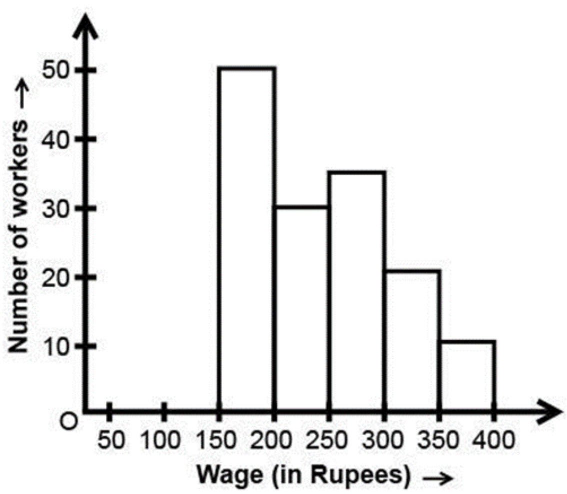
➔ $\frac{10+14}{2} = \frac{24}{2} = 12$

Median = 12

The mode is the value that occurs the maximum number of times in a given set of values. Now mode is 10 because it is the most repeating number.

Hence, [median = 12 and mode = 10]

15. In the given histogram depicting the daily wages of workers in a factory. Construct the frequency distribution table.



Answer: Based on the given bar graph:

Class-Interval	Frequency
150-200	50
200-250	30
250-300	35
300-350	20
350-400	10
Total workers	145

16. A company selected 4000 households at random and surveyed them to find out a relationship between income level and the number of television sets in a home. The information so obtained is listed in the following table:

Monthly income (in Rs)	**Number of Televisions / household**			
	0	**1**	**2**	**Above 2**
<10000	**20**	**80**	**10**	**0**
10000-14999	**10**	**240**	**60**	**0**
15000-19999	**0**	**380**	**120**	**30**
20000-24999	**0**	**520**	**370**	**80**
25000 and above	**0**	**1100**	**760**	**220**

Find the probability:

(i) of a household earning Rs 10000 – Rs 14999

per year and having exactly one television.

(ii) of a household earning Rs 25000 and more per year and owning 2 televisions.

(iii) of a household not having any television.

Ans: Here, total events = 4000

(i) A household earning Rs 10000 - 14999 per year and having exactly one television.

Favourable outcomes = 240

Probability = $\frac{Favourable\ Outcomes}{Total\ number\ of\ events}$

Required Probability = $\frac{240}{4000} = \frac{3}{50} = 0.06$

(ii) A household earning Rs 25000 and more per year and owning 2 televisions.

Favourable outcomes =760

Probability = $\frac{Favourable\ Outcomes}{Total\ number\ of\ events}$

Required Probability = $\frac{760}{4000} = 0.19$

(iii) A household not having any television.

Favourable outcomes = 20 + 10

Probability = $\frac{Favourable\ Outcomes}{Total\ number\ of\ events}$

Required Probability = $\frac{30}{4000} = \frac{3}{400} = 0.0075$

17. Two dice are thrown simultaneously 500 times. Each time the sum of two numbers appearing on their tops is noted and recorded as given in the following table:

Sum	Frequency
2	14
3	30
Sum	Frequency

4	**42**
5	**55**
6	**72**
7	**75**
8	**70**
9	**53**
10	**46**
11	**28**
12	**15**

If the dice are thrown once more, what is the probability of getting a sum
(i) Equal to 3?

(ii) More than 10 ?

(iii) less than or equal to 5?

(iv) between 8 and 12

Answer: Here, total events = 14 + 30 + 42 + 55 + 72 + 75 + 70 + 53 + 46 + 28 + 15 = 500

(i) Probability of getting a sum = 3

Favourable events = 30

Probability of getting sum 3 = $\frac{30}{500}$ = 0.06

(ii) Probability of getting a sum more than 10

Favourable events = 28 + 15 = 43

Probability of getting sum 10 = $\frac{43}{500}$ = 0.086

(iii) Probability of getting a sum less than or equal to 5

Favourable events = 14 + 30 + 42 + 55

Probability of getting sum less than or equal to 5 = $\frac{141}{500} = 0.282$

(iv) Probability of getting a sum between 8 and 12

Favourable events = 53 + 46 + 28 = 127

Probability of Sum between 8 and 12 = $\frac{127}{500} = 0.254$

18. Bulbs are packed in cartons each containing 40 bulbs. Seven hundred cartons were examined for defective bulbs and the results are given in the following table:

Number of defective bulbs	0	1	2	3	4	5	6	more than 6
Frequency	400	180	48	41	18	8	3	2

One carton was selected at random. What is the probability that it has

(i) No defective bulb?

(ii) Defective bulbs from 2 to 6?

(iii) defective bulbs less than 4?

Answer: Probability = $\frac{Favourable\ Outcomes}{Total\ number\ of\ events}$

Here, total events = total cartons =700

(i) No defective bulb

Favourable outcomes = 400

Probability of Cartoon has no defective bulb = $\frac{400}{700} = \frac{4}{7}$

(ii) Defective bulbs from 2 to 6 = 2 or 3 or 4 or 5 or 6 defective bulbs

Favourable outcomes = 48 + 41 + 18 + 8 + 3 = 118

Probability of Defective bulb from 2 to 6 = $\frac{118}{700} = \frac{59}{350}$

(iii) Defective bulbs less than 4 = defective bulbs equal to 0 or 1 or 2 or 3

Favourable outcomes = 400 + 180 + 48 + 41 = 669

Probability of defective bulbs less than 4 = $\frac{669}{700}$

19. Over the past 200 working days, the number of defective parts produced by a machine is given in the following table:

Number of defective parts	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Days	50	32	22	18	12	12	10	10	10	8	6	6	2	2

Determine the probability that tomorrow's output will have

Answer:
(i) no defective part

Probability = $\frac{Favourable\ Outcomes}{Total\ number\ of\ events}$

Here, total events = total number of working days = 200

Favourable outcomes 50 days = 50

Probability of no defective part = $\frac{50}{200}$ = 0.25

(ii) at least one defective part

Probability that at least one defective part = 1−the probability that no defective part

Probability of no defective part = $\frac{50}{200}$ = 0.25

Probability of at least one defective part = $1 - \frac{50}{200}$ = 0.75

(iii) not more than 5 defective parts

Not more than 5 defective parts =0 or 1 or 2 or 3 or 4 or 51 or 2 or 3 or 4 or 5 defective parts

PP (Not more than 5 defective parts) = P(No defective part) + P (1 defective part) + P (2 defective part) + P (3 Defective part) +P (4 defective part) + P (5 defective part)

$= \frac{50}{200} + \frac{32}{200} + \frac{22}{200} + \frac{18}{200} + \frac{12}{200} + \frac{12}{200} = \frac{50+32+22+18+12+12}{200} = \frac{146}{200}$

$= \frac{73}{200}$ = 0.73

(iv) more than 13 defective parts

More than 13 defective parts = not possible

Favourable outcomes = 0

Probability of more than 13 defective parts = 0

20. A recent survey found that the ages of workers in a factory is distributed as follows:

Age (in years)	20-29	30-39	40-49	50-59	60 and above
Number of workers	38	27	86	46	3

If a person is selected at random, find the probability that the person is:
(i) 40 **years or more**
Answer: Here, total events = total number of worker S= 38 + 27 + 86 + 46 + 3 =200

(i) P (Person is 40 years or more) = P (Person having age 40 to 49 years) + P (Person having age 50 to 59 years) + P (Having age 60 and above) = $\frac{86}{200} + \frac{46}{200} + \frac{3}{200} = \frac{135}{200}$ = 0.675

(ii) under 40 years

Answer: P (Person is under 40 years) = P (Person having age 20 to 29 years) + P (Person having age 30 to 39 years) = $\frac{38}{200} + \frac{27}{200} = \frac{65}{200}$ = 0.135

(iii) having age from 30 to 39 years

Ans: P (Under 60 but over 39 years) = P (person having age 40 to 49 years) + P (Person having age 50 to 59 years) = $\frac{86}{200} + \frac{46}{200} = \frac{132}{200}$ = 0.66

21. Following is the frequency distribution of total marks obtained by the students of different sections of class VIIII**.**

Marks	100–150	150–200	200–300	300–500	500–800
Number of students	60	100	100	80	180

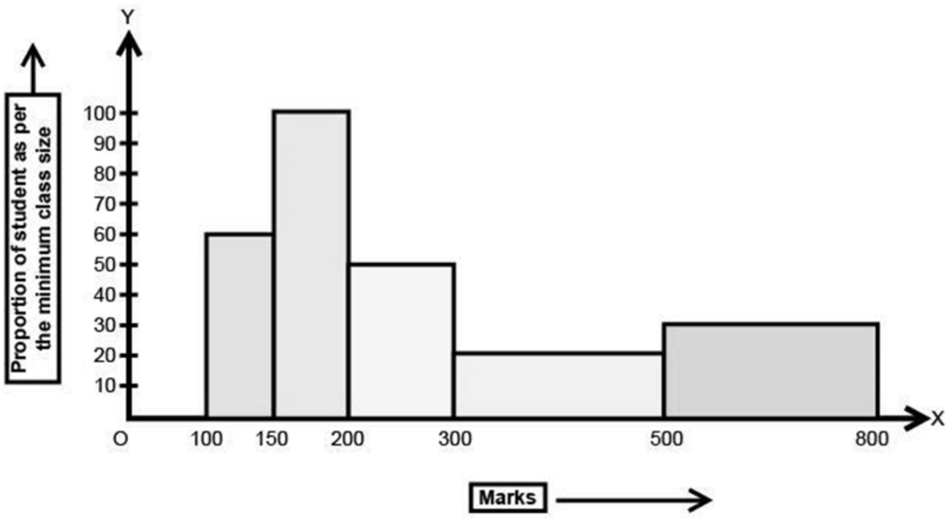
Draw a histogram for the distribution above.

Answer: The widths of the class intervals vary for each data in the table given. These widths serve as the width of the rectangles in the histograms. So before drawing histograms the length of the rectangles is to be found in each case. This is due to the property of histogram that the area of the rectangles should be proportional to the frequencies.

Length of each rectangle is given as $\frac{c}{C}\ x\ f$, where c is the minimum class width, C is the class width of the particular class and f is the frequency. In the above case c=50. Consider the following table.

Marks	Frequency	Width of the class	Length of the rectangle
100–150	60	50	$\frac{50}{50}\ x\ 60$ = 60
150–200	100	50	$\frac{50}{50}\ x\ 100$ = 100
200–300	100	100	$\frac{50}{100}\ x\ 100$ = 50
300–500	80	200	$\frac{50}{200}\ x\ 80$ = 20
500–800	180	300	$\frac{50}{300}\ x\ 180$ = 30

So the histogram can be shown by taking 1 cm=10 units on the x axis and 1 cm=50 units on the y axis as below : -



2. Two sections of class IX having 30 students each appeared for the mathematics Olympiad. The marks obtained by them are shown below:

46 31 74 68 42 54 14 61 83 48 37 26 8 64 57

93 72 53 59 38 16 88 75 56 46 66 45 61 54 27

27 44 63 58 43 81 64 67 36 48 50 76 38 47 55

77 62 53 40 71 60 58 45 42 34 46 40 59 42 29

Construct a grouped frequency distribution of the data above using the classes 0−9, 10−19 etc., and hence find the number of students who secured more than 49 marks.

Ans: A group frequency distribution of the data is to be made and the classes of those are given like 0−90−9, 10−1910−19 etc. There are three columns in the table. In the first column there will be classes, in the second column the tally marks and in the third column the frequency represented by the tally marks as per the data given. The boundary values will be included.

The lowest mark is 88 and the highest mark is 9393 so the starting class will be 0−90−9 and the ending class will be 90−9990−99.

Class	Tally marks	Frequency
0−9	|	1
10−19	||	2
20−29	||||	4
30−39	~~||||~~ |	6
40−49	~~||||~~ ~~||||~~ ~~||||~~	15
50−59	~~||||~~ ~~||||~~ ||	12
60−69	~~||||~~ ~~||||~~	10
70−79	~~||||~~ |	6

80−89	|||	3
90−99	|	1
		Total = 60

Clearly the number of students who secured more than 49 marks = (12 + 10 + 6 + 3 + 1) = 32

Exercise 14.4

1. The following are the marks (out of 100) of 60 students in mathematics.
16,13,5,80,86,7,51,48,24,56,70,19,61,17,16,36,34,42,34,35,72

55,75,31,52,28,72,97,74,45,62,68,86,35,85,36,81,75,55,26,95,31,7,78

92,62,52,56,15,63,25,36,54,44,47,27,72,17,4,30

Construct a grouped frequency distribution table with width 10 of each class starting from 0−9.

Answer:

Step 1: arrange these numbers in ascending order.

Step 2: Make the frequency distribution table as follows:

The class intervals of (0−9), (10−19)) etc. are given. So we can write the class interval and the corresponding frequency.

Class interval	Frequency
0-10	4
10-20	7
20-30	5
30-40	10
40-50	5
50-60	8

60-70	5
70-80	8
80-90	5
90-100	3
Total	60

2. Construct a grouped frequency distribution table with width 10 of each class, in such a way that one of the classes is 10–20 (20 not included).

Answer:

Step 1: arrange these numbers in ascending order.
4,5,9,7,13,15,16,17,17,19,24,25,26,27,28,30,31,34,34,34,35,35,36,36,36,42,44,45,47,48,51 ,

52,52,54,55,55,56,56,61,62,62,63,68,70,72,72,72,74,75,75,78,80,81,85,86,92,95

Step 2: Make the frequency distribution table as follows:

The class intervals of (0–10), (10–20) etc. are given. So, we can write the class interval and the corresponding frequency. A value like 10 will always be counted in the interval where it is the lower limit, i.e., 10-20 and not in 0- 10 (where is the higher limit)

limit)

Class interval	**Frequency**
0-10	4
10-20	7
20-30	5
30-40	10

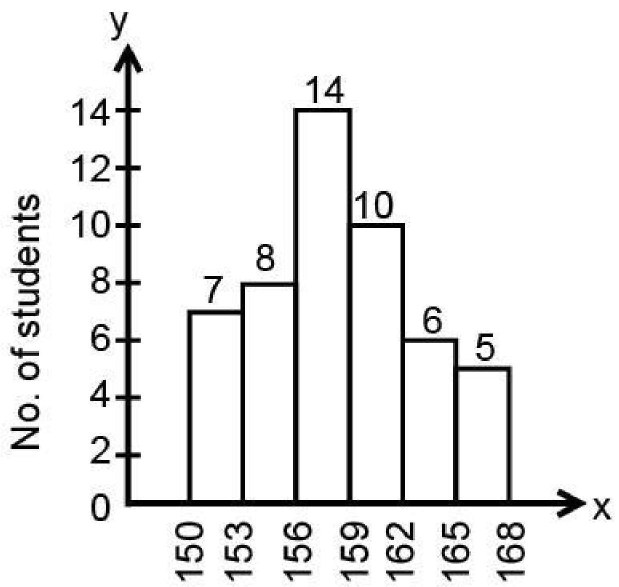
40-50	5
50-60	8
60-70	5
70-80	8
80-90	5
90-100	3
Total	60

3. Draw a histogram of the following distribution:

Heights (in cm)	**Number of students**
150-153	**7**
153-156	**8**
156-159	**14**
159-162	**10**
162-165	**6**
165-168	**5**

Answer: To draw the histogram, we have to plot the frequency, i.e., number of students on the y-axis and class Interval, i.e., heights on the x-axis. We can construct the histogram as follows:

Taking 1 cm on x – axis = 3 units and 1 cm on y – axis =2 units



4. Draw a histogram to represent the following grouped frequency distribution:

Ages (in years)	Number of teachers
20-24	10
25-29	28
30-34	32
35-39	48
40-44	50
45-49	35
50-54	12

Answer: We can see that the intervals are 20 - 24,25 - 29,30 - 34…20 - 24,25 - 29,30 - 34….etc. First we need to adjust the class interval to make it continuous.

For the interval 20 – 24: Upper limit is 24

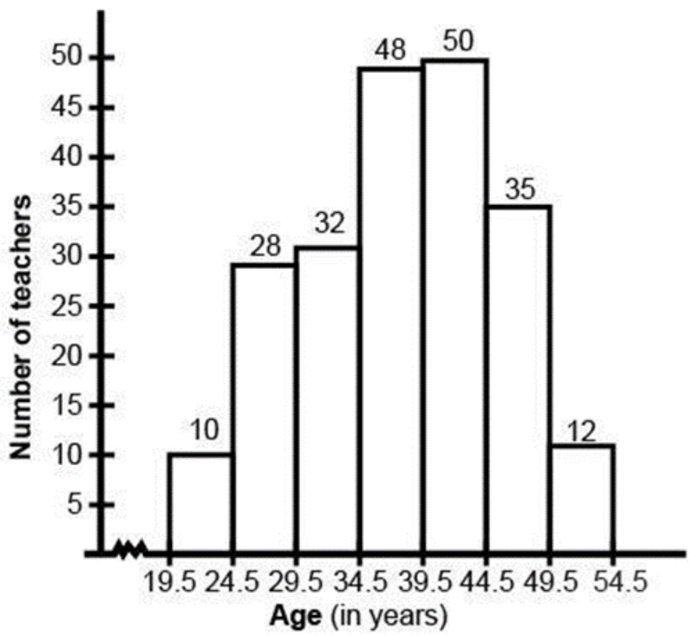
For the interval 25 - 29: Lower limit is 25

Class adjustment value = $\frac{Upper\ class\ limit - lower\ class\ limit}{2}$

$= \frac{25-24}{2} = \frac{1}{2} = 0.5$

Now, we will subtract 0.5 from all lower limits and add 0.5 to all upper limits

Ages	No. of teachers
19.5 - 24.5	10
24.5 - 29.5	28
29.5 - 34.5	32
34.5 - 39.5	48
39.5 - 44.5	50
44.5 - 49.5	35
49.5 - 54.5	12



Now, we draw the diagram.

Taking 1 cm on x – axis = 5 units and 1 cm on y - axis=5 units1 cm on x – axis =5 units and 1 cm on

y – axis = 5 units

5. The lengths of 62 leaves of a plant are measured in millimetres and the data is represented in the following table:

Length (in mm)	Number of leaves
118 - 126	8
127 - 135	10
136 - 144	12
145 - 153	17
154 - 162	7
163 - 171	5
172 - 180	3

Draw a histogram to represent the data above.

Answer: We can see that the intervals are 118 - 126, 127 - 135, 136 - 144…etc. First, we need to adjust the class interval to make it continuous.

For the interval 118 – 126 : Upper limit is 126

For the interval 127 − 135 : Lower limit is 127

Class adjustment value = $\frac{Upper\ class\ limit - lower\ class\ limit}{2}$

$= \frac{127-126}{2} = \frac{1}{2} = 0.5$

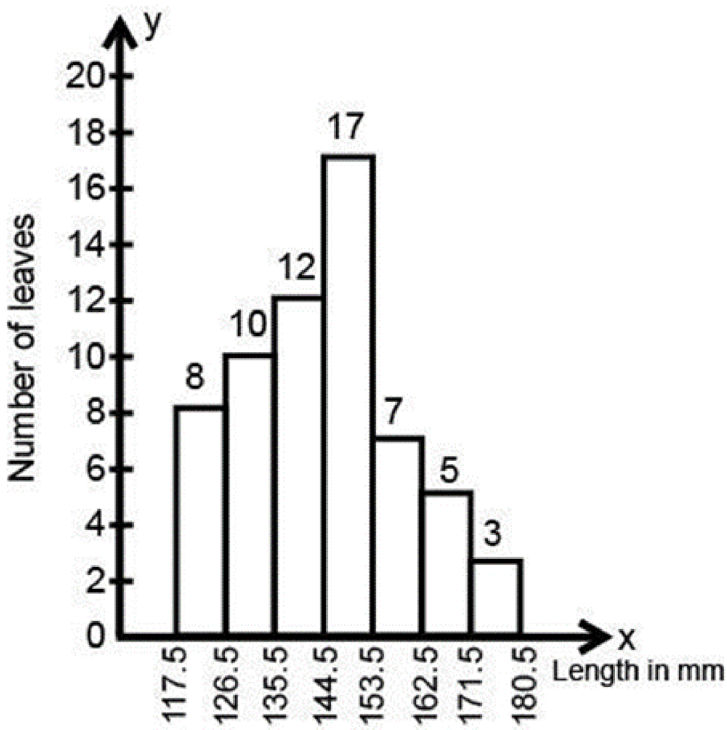
Now, we will subtract 0.5 from all lower limits and add 0.5 to all upper limits

Length (in mm)	Number of leaves
117.5 - 126.5	8

126.5 - 135.5	10
135.5 - 144.5	12
144.5 - 153.5	17
153.5 - 162.5	7
162.5 - 171.5	5
171.5 - 180.5	3

Hence the histogram is:

Taking 1 cm on x – axis = 9 units and 1 cm on y – axis = 2 units



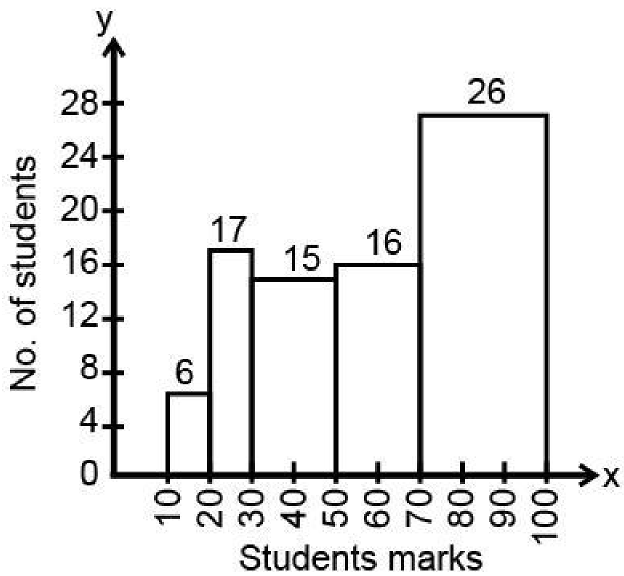
6. The marks obtained (out of 100) by a class of 80 students are given below:

Marks	Number of students
10 - 20	6
20 - 30	17
30 - 50	15
50 - 70	16
70 - 100	26

Construct a histogram to represent the data above.

Answer: To draw the histogram, we have to plot the frequency, i.e., number of students on the y-axis and class interval, i.e., marks on the x-axis. We can construct the histogram as follows:

Taking 1 cm on x – axis = 10 units and 1 cm on y – axis = 4 units

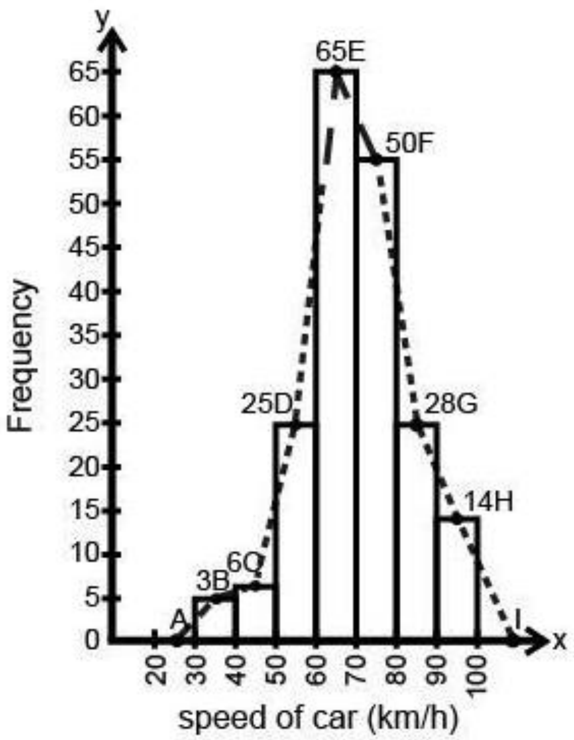


7. Following table shows a frequency distribution for the speed of cars passing through at a particular spot on a highway:

Class interval (km/h)	Frequency
30 - 40	3
40 - 50	6
50 - 60	25
60 - 70	65
70 - 80	50
80 - 90	28
90 - 100	14

Draw a histogram and frequency polygon representing the data above.

Answer: To draw the histogram, we have to plot the frequency on the y-axis and class interval on the x-axis. We can construct the histogram as follows:



Clearly, the given frequency distribution is in exclusive form. Along the horizontal axis, we represent the class intervals on some suitable scale. The corresponding frequencies are represented along the vertical axis on a suitable scale. We construct rectangles with class intervals as the bases and the respective frequencies as the heights. Let us draw a histogram for this data and mark the mid-points of the top of the rectangles as B, C, D, E, F, G and H, respectively. Here, the first class is 30-40 and the last class is 90-100.

Also, consider the imagined classes 20-30 and 100-110 each with frequency O. The class marks of these classes are 25 and 105 at the points A and I, respectively.

Taking 1 cm on x – axis = 10 units and 1 cm on y – axis = 5 units

A frequency polygon is a graph constructed by using lines to join the midpoints of each interval. Accordingly, the curve ABCDEFGHI is the frequency polygon.

8. Following table shows a frequency distribution for the speed of cars passing through at a particular spot on a highway:

Class interval (km/h)	Frequency
30 - 40	3
40 - 50	6
50 - 60	25
60 - 70	65
70 - 80	50
80 - 90	28
90 - 100	14

Draw the frequency polygon representing the above data without drawing the histogram.

Ans: A frequency polygon is a graph constructed by using lines to join the midpoints of each interval. Here the midpoints as follows:

We have to draw a frequency polygon without a histogram.

Firstly, we find the class marks of the classes given that is 30−40, 40−50, 50−60, 60−70…

Class Mark = $\frac{actual\ upper\ limit\ +\ actual\ lower\ limit}{2}$

Class Mark = $\left(\frac{30+40}{2}\right) = \frac{70}{2} = 35$

Similarly, we can determine the class marks of the other classes.
So, table for class marks is shown below

Class Interval (Km/h)	Class Marks	Frequency

30-40	35	3
40-50	45	6
50-60	55	25
60-70	65	65
70-80	75	50
80-90	85	28
90-100	95	14

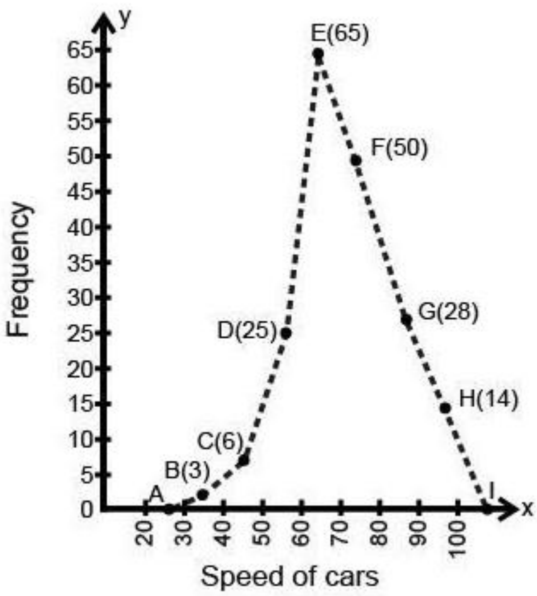
We can draw a frequency polygon by plotting the class marks along the horizontal axis and the frequency along the vertical axis. Now, plotting all the points

B (35, 3), C (45, 6), D (55, 25), E (65, 65), F (75, 50), G (85, 28), H (95, 14) also plot the point corresponding to the considering classes 20-30 and 100-110 each with frequency 0. Join all these point line segments.

Hence, we can construct the points using mid-point on x−axis and corresponding frequency on

y−axis. So, the frequency polygon will be as follows:

Taking 1 cm on x – axis = 10 units and 1 cm on y – axis = 5 units



9. Following table gives the distribution of students of sections AA **and** BB **of a class according to the marks obtained by them.**

Section A		Section B	
Marks	**Frequency (A)**	**Marks**	**Frequency (B)**
0−15	5	0−15	3
15−30	12	15−30	16
30−45	28	30−45	25
45−60	30	45−60	27
60−75	35	60−75	40
75−90	13	75−90	40

Represent the marks of the students of both the sections on the same graph by two frequency polygons. What do you observe?

Answer: To form the frequency polygon we need to find the class marks for each class interval provided. Class mark of a particular class is the average of its upper and lower limit. so we consider the following table.

Marks	Class marks	Frequency (A)	Frequency (B)
0−15	7.5	5	3
15−30	22.5	12	16
30−45	37.5	28	25
45−60	52.5	30	27.
60−75	67.5	35	40

75–90	82.5	13	10

We have to plot the points of class marks versus frequency. But first we have to find the class marks where 12 frequencies will be 0. The difference between any two consecutive class marks is 15. Therefore, the two such class marks can be given as,

Starting point = class mark of first-class interval −15

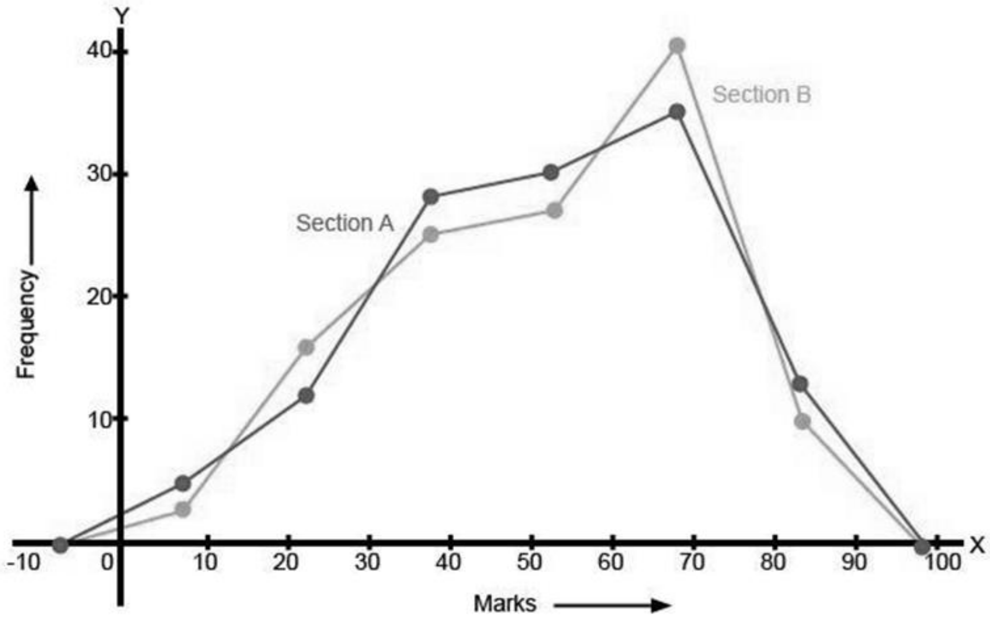
Starting point =7.5 − 15 = 7.5

Ending point = class mark of last class interval +15

Ending point = 82.5 + 15 = 97.5

The frequency polygons can be shown below:

Taking 1 cm on x – axis = 10 units and 1 cm on y – axis = 15 units



From the graph we can see that the maximum marks 82.5 is scored by 13 students of section A and minimum marks 7.5 is scored by 3 students of section B.

10. The mean of the following is 5050**.**

x	*f*
10	17
30	5a + 3
50	32
70	7a – 11
90	19

Find the value of a and hence the frequencies of 30 and 70.

Answer: For the given table of data the mean is given as $(\bar{x}) = \frac{\sum_{i=1}^{n} f_i x_1}{\sum_{i=1}^{n} f_i}$, where n = 5 (the values of x). The provided mean value is 5050, so we have,

➔ $50 = \frac{(10\ x\ 17) + (30\ x\ (5a+3)) + (50\ x\ 32) + (70\ x\ (7a-11)) + (90\ x\ 19))}{17+5a+3+32+7a-11+19}$

➔ $50 = \frac{640a+2800}{12a+60}$

➔ 600a + 3000 = 640a + 2800

➔ a = 5

Substituting the value of aa in the required frequencies we get,

The frequency of 30 = 5a + 3 = 28

The frequency of 70 = 7a – 11 = 24

11. The mean marks (out of 100) of boys and girls in an examination are 70 and 73, respectively. If the mean marks of all the students in that examination is 71, find the ratio of the number of boys to the number of girls.

Ans: Let us assume the number of boys in the class is xx and the number of girls is y.

Now, since the mean marks of all the boys is 70 that means the ratio of sum of marks obtained by all the boys and the total number of boys is 70.

➔ Sum of marks obtained by all the boys = 70 × x………………(i)

Similarly, the mean marks of all the girls is 73 that means the ratio of sum of marks obtained by all the girls and the total number of girls is 73.

➔ Sum of marks obtained by all the girls =73 × y…………………(ii)

Now, the mean marks of all the students is 71 that means the ratio of sum of marks obtained by both girls and boys and the total number of boys and girls is 71.

➔ Sum of marks obtained by all the students (both boys and girls) =71 × (x + y)………(iii)

Clearly, we can see that (i) + (ii) = (iii) so we have,

➔ 70x + 73y = 71 (x + y)

➔ 70x + 73y = 71x + 71y

➔ x = 2y

➔ $\frac{x}{y} = \frac{2}{1}$

Therefore, the ratio of the number of boys to the number of girls is x : y = 2: 1

12. A total of 25 patients admitted to a hospital are tested for levels of blood sugar, (mg/dl) and the results obtained were as follows:

87 71 83 67 85

77 69 76 65 85

85 54 70 68 80

73 78 68 85 73

81 78 81 77 75

Find the mean, median and mode (mg/dl) of the above data.

Answer:

(i) First let us find the meaning.

We know that the mean is the ratio of the sum of observations to the number of observations. For the above data the sum of all the blood sugar levels is 1891 and the total number of patients is 25.

➔ Mean = $\frac{1892}{25}$

➔ Mean = 75.64 (mg/dl)

(ii) Now, let us find the median. Arranging the data in ascending order we have,

54, 65, 67, 68, 68, 69, 70, 71, 73, 73, 75, 76

77, 77, 78, 78, 80, 81, 81, 83, 85, 85, 85, 85, 85

We can see that the total number of patients is 25 which is odd, so the median will be

➔ Median = $\left(\frac{n}{2}+1\right)^{th}$

➔ Median = $\left(\frac{25}{2}+1\right)^{th}$

➔ Median = $\left(\frac{25+1}{2}\right)^{th}$ = 13^{th} term = 77 mg/dl

(iii) Finally, let us find the mode.

The mode is the observation which appears the most in the data. We can see that 85 appears the most number of times.

➔ Mode = 85 (mg/dl)