Exercise 10.1

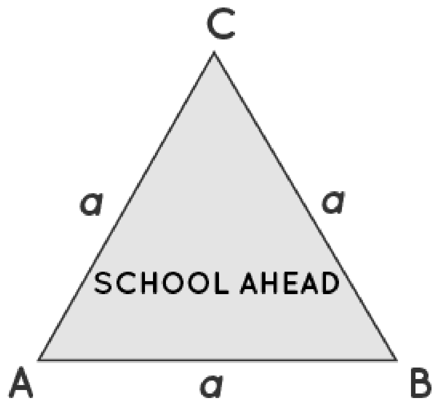
Q1. A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board?

Answer: Given: Dimensions of the traffic signal board (equilateral triangle) and its perimeter.

By using Heron's formula, we can calculate the area of a triangle.

Heron's formula for the area of a triangle is: $\sqrt{s(s - a)(s - b)(s - c)}$

Where a, b and c are the sides of the triangle and s = Semi-perimeter = Half the Perimeter of the triangle



Each side of traffic signal board (equilateral triangle) = 'a' cm

Perimeter of traffic signal board (equilateral triangle) = sum of all the sides = a + a + a = 3a

Semi Perimeter, s = (a + b + c)/2 = (a + a + a)/2 = 3a/2

By using Heron's formula,

Area of a triangle = $\sqrt{s(s - a)(s - b)(s - c)}$

Area of a (triangle) traffic signal board

= √[s(s - a)(s - b)(s - c)]

= √[s(s - a)(s - a)(s - a)] {since all three sides are equal to "a"}

= (s - a) √[s(s - a)].....(1)

We know, s = 3a/2, so substituting this value in equation (1)

Area = (3a/2 - a)√[3a/2(3a/2 - a)]

= (a/2) √[3a/2(a/2)]

= a/2 × a/2 × √3

= (√3/4)a^2

Area of the signal board = (√3/4)a^2 sq. units

Now given perimeter = 180 cm

Each side of triangle = 180/3 cm

a = 60 cm

Substituting the value of 'a'.

Area of the signal board = (√3/4)$(60)^2$

= (√3/4)(3600) = 900√3

Area of the signal board = 900√3 cm^2

2. The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22 m, and 120 m (see Fig. 12.9). The advertisements yield an earning of ₹ 5000 per m^2 per year. A company hired one of its walls for 3 months. How much rent did it pay?

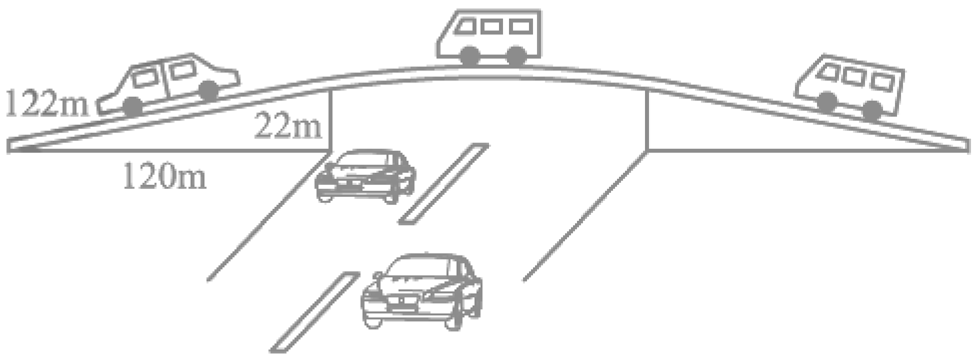


Fig. 12.9

Answer: Given: Dimensions of the triangular sides of walls.

By using Heron's formula, we can calculate the area of triangle.

Heron's formula for the area of a triangle is: Area = $\sqrt{s(s - a)(s - b)(s - c)}$

Where a, b and c are the sides of the triangle, and s = Semi-perimeter = Half the Perimeter of the triangle

Triangular sides of walls are, a = 122 m, b = 22 m, c = 120 m

Semi Perimeter, s = (a + b + c)/2

= (122 + 22 + 120)/2

= 264/2

= 132 m

By using Heron's formula,

Area of a triangle = $\sqrt{s(s - a)(s - b)(s - c)}$

Substuting the values in order to find area of triangular wall,

= √132(132 - 122) (132 - 22) (132 - 120)

= √132 × 10 × 110 × 12

= 1320 m

Rent of 1 m^2 area per year = ₹ 5000

Rent of 1 m^2 area per month = ₹ 5000/12

Rent of 1320 m^2 area for 3 months = ₹ (5000/12) × 3 × 1320

= ₹ 1650000

Therefore, the company paid ₹ 16,50,000 as rent.

Q3. There is a slide in a park. One of its side walls has been painted in some colour with a message "KEEP THE PARK GREEN AND CLEAN" (see Fig. 12.10). If the sides of the wall are 15 m, 11 m and 6 m, find the area painted in colour.

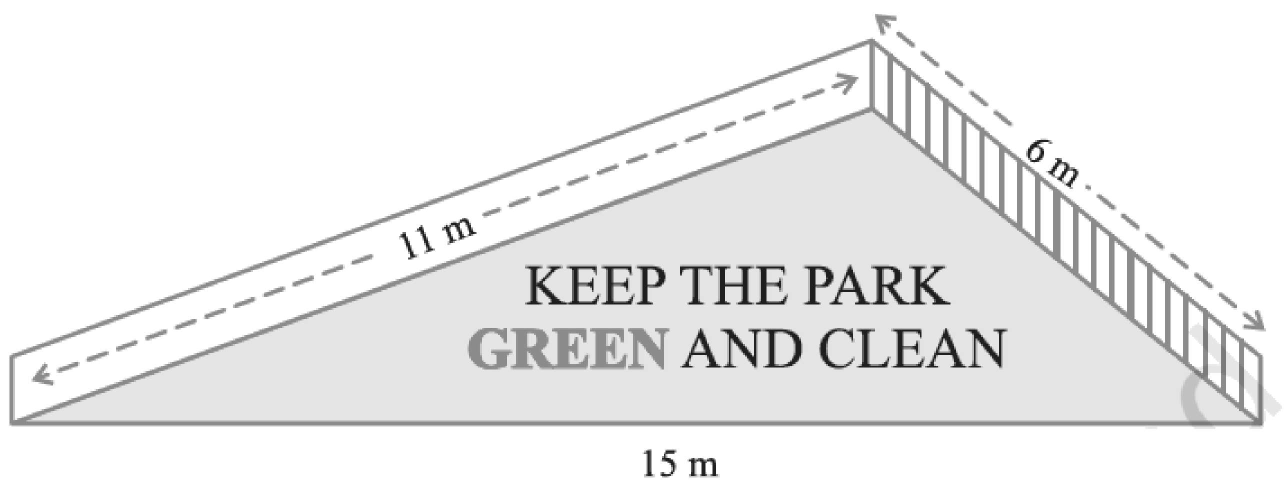


Fig. 12.10

Answer: Given: Dimensions of the triangular wall.

By using Heron's formula, we can calculate the area of a triangle.

Heron's formula for the area of a triangle is: Area = $\sqrt{s(s - a)(s - b)(s - c)}$

Where a, b and c are the sides of the triangle, and s = Semi-perimeter = Half the perimeter of a triangle.

The sides of the triangular walls are a = 11 m, b = 6 m and c = 15 m.

Semi Perimeter:

s = (a + b + c)/2

= (11 + 6 + 15)/2

= 32/2

= 16 m

By using Heron's formula,

Area of triangular wall = $\sqrt{s(s - a)(s - b)(s - c)}$

Substituting the given values in formula,

= $\sqrt{16(16 - 11)(16 - 6)(16 - 15)}$

= $\sqrt{16 \times 5 \times 10 \times 1}$

= $\sqrt{800}$ m^2

= $20\sqrt{2}$ m^2

Area of the wall of the park to be painted in colour = $20\sqrt{2}$ m^2.

4. Find the area of a triangle two sides of which are 18cm and 10cm and the perimeter is 42cm.

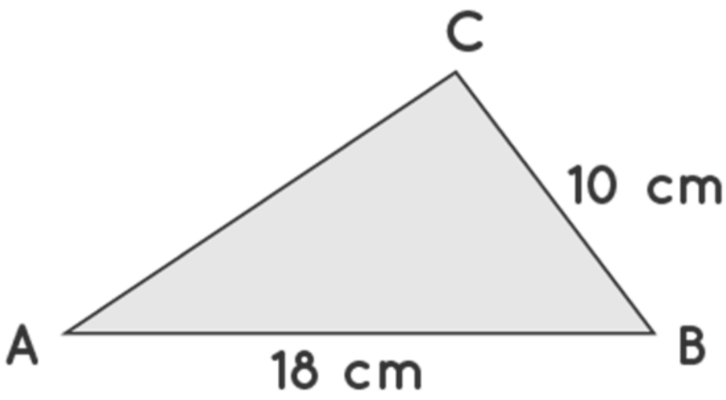
Answer:

Given: Two sides of the triangle and its perimeter.

By using Heron's formula, we can calculate the area of triangle.

Heron's formula for the area of a triangle is: Area = $\sqrt{s(s - a)(s - b)(s - c)}$

Where a, b and c are the sides of the triangle, and s = Semi-perimeter = Half the perimeter of the triangle



The sides of triangle given: a =18 cm, b = 10 cm

Perimeter of the triangle = (a + b + c)

42 = 18 + 10 + c

42 = 28 + c

c = 42 - 28

c = 14 cm

Semi Perimeter

s = (a + b + c) = 42/2 = 21 cm

By using Heron's formula,

Area of a triangle = $\sqrt{s(s - a)(s - b)(s - c)}$

= √21(21 - 18) (21 - 10) (21 - 14)

= √21 × 3 × 11 × 7

= 21√11 cm^2

Area of the triangle = 21√11 cm^2.

5. Sides of a triangle are in the ratio of 12:17:25 and its perimeter is 540cm. Find its area.

Answer : Given: Ratio of sides of the triangle and its perimeter.

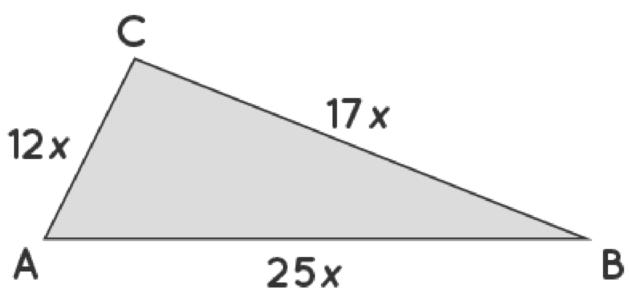
By using Heron's formula, we can calculate the area of a triangle.

Heron's formula for the area of a triangle is: Area = $\sqrt{s(s - a)(s - b)(s - c)}$

Where a, b, and c are the sides of the triangle, and s = Semi-perimeter = Half the perimeter of the triangle

Since the ratios of the sides of the triangle are given as 12:17:25

So, we can assume the length of the sides of the triangle as 12x cm, 17x cm, and 25x cm.



So the perimeter of the triangle will be

Perimeter = 12x + 17x + 25x

12x + 17x + 25x = 540 (given)

54x = 540

x = 540/54

x = 10 cm

Therefore, the sides of the triangle:

12x = 12 × 10 = 120 cm, 17x = 17 × 10 = 170 cm, 25x = 25 × 10 = 250 cm

a = 120cm, b = 170 cm, c = 250 cm

Semi-perimeter(s) = 540/2 = 270 cm

By using Heron's formula,

Area of a triangle = $\sqrt{s(s - a)(s - b)(s - c)}$

= √270(270 - 120)(270 - 170)(270 - 250)

= √270 × 150 × 100 × 20

= √81000000

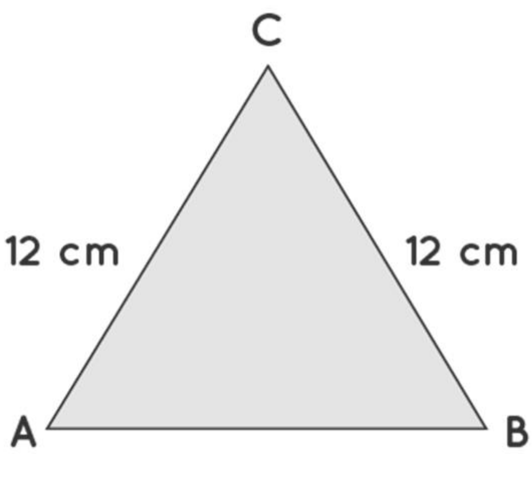
= 9000 cm^2

Area of the triangle = 9000 cm^2.

6. An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle

Answer: Given: Equal sides of the triangle and its perimeter.

Heron's formula for the area of a triangle is: Area = $\sqrt{s(s - a)(s - b)(s - c)}$



Where a, b, and c are the sides of the triangle, and s = Semi-perimeter = Half the perimeter of the triangle

Equal sides: a = b = 12 cm (given)

The formula for the perimeter of a triangle: Perimeter(P) = a + b + c

30 = 12 + 12 + c (Given, perimeter = 30 cm)

c = 30 – 24 = 6 cm

Now, Semi Perimeter (s) = P/2 = (a + b + c)/2

s = 30/2

s = 15 cm

By using Heron's formula,

Area of a triangle = $\sqrt{s(s - a)(s - b)(s - c)}$

= √15 (15 - 12) (15 -12) (15 - 6)

= √15 × 3 × 3 × 9

= √1215

= 9√15 cm^2

Area of the triangle = 9√15 cm^2

Exercise 10.2

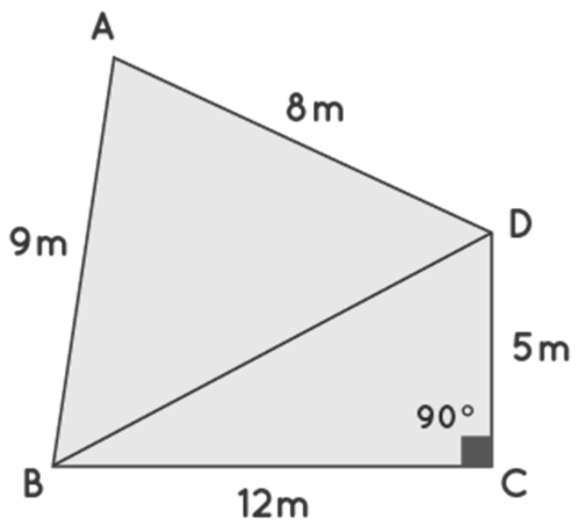
1. A park, in the shape of a quadrilateral ABCD, has ∠C = 90°, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m. How much area does it occupy?

Answer: By dividing the quadrilateral into two triangles and applying Heron's formula, we can calculate the area of triangles.

Heron's formula for the area of a triangle is: Area = $\sqrt{s(s - a)(s - b)(s - c)}$

Where a, b, and c are the sides of the triangle, and s = Semi-perimeter = Half the Perimeter of the triangle

Now, ABCD is the park shown in the figure below



We have ∠C = 90°, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m.

Let's connect B and D, such that BCD is a right-angled triangle.

In ΔBDC, apply Pythagoras theorem in order to find the length of BD

$BD^2 = BC^2 + CD^2$ [Pythagoras theorem]

$BD^2 = 12^2 + 5^2$

$BD^2 = 144 + 25$

$BD = \sqrt{169}$

BD = 13 m

Area of quadrilateral ABCD = area of ΔBCD + area of ΔABD

Now, Area of ΔBCD = 1/2 × base × height

= 1/2 × 12 m × 5 m

= 30 m^2

Now, in ΔABD, AB = a = 9 m, AD = b = 8 m, BD = c = 13 m

Semi Perimeter of ΔABD

s = (a + b + c)/2

= (9 + 8 + 13)/2

= 30/2

= 15 m

By using Heron's formula,

Area of ΔABD = $\sqrt{s(s - a)(s - b)(s - c)}$

= $\sqrt{15(15 - 9)(15 - 8)(15 - 13)}$

= $\sqrt{15 \times 6 \times 7 \times 2}$

= $6\sqrt{35}$

= 35.5 m^2 (approx.)

Area of ΔABD = 35.5 m^2

Therefore,

Area of park ABCD = 30 m^2 + 35.5 m^2 = 65.5 m^2

Thus, the park ABCD occupies an area of 65.5 m^2.

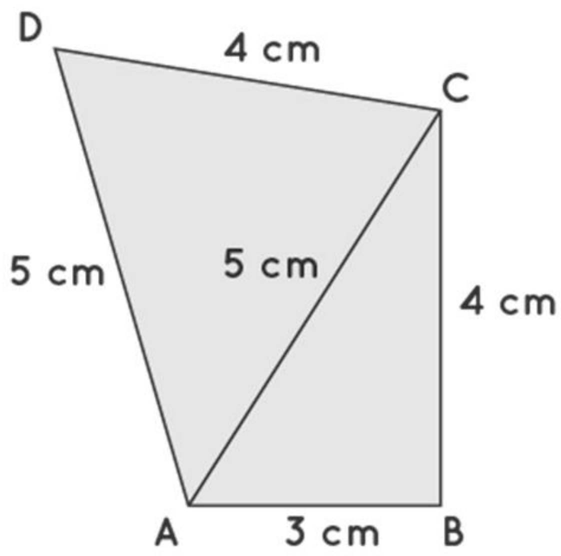
2. Find the area of a quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm

Answer: We divide the quadrilateral into two triangles, and by using Heron's formula, we can calculate the area of triangles.

Heron's formula for the area of a triangle is: Area = $\sqrt{s(s - a)(s - b)(s - c)}$

Where a, b, and c are the sides of the triangle, and s = Semi-perimeter = Half the Perimeter of the triangle

Now, ABCD is quadrilateral shown in the figure



For ΔABC, consider

$AB^2 + BC^2 = 3^2 + 42 = 25 = 5^2$

$\Rightarrow 5^2 = AC^2$

Since ΔABC obeys the Pythagoras theorem, we can say ΔABC is right-angled at B.

Therefore, the area of ΔABC = 1/2 × base × height

= 1/2 × 3 cm × 4 cm = 6 cm^2

Area of ΔABC = 6 cm^2

Now, In ΔADC

we have a = 5 cm, b = 4 cm and c = 5 cm

Semi Perimeter: s = Perimeter/2

s = (a + b + c)/2

s = (5 + 4 + 5)/2

s = 14/2

s = 7 cm

By using Heron's formula,

Area of ΔADC = $\sqrt{s(s - a)(s - b)(s - c)}$

= √[7(7 - 5)(7 - 4)(7 - 5)]

= √[7 × 2 × 3 × 2]

= 2√21 cm^2

Area of ΔADC = 9.2 cm^2 (approx.)

Area of the quadrilateral ABCD = Area of ΔADC + Area of ΔABC

= 9.2 cm^2 + 6 cm^2

= 15.2 cm^2

Thus, the area of the quadrilateral ABCD is 15.2 cm^2.

3. Radha made a picture of an aeroplane with coloured paper as shown in Fig 12.15. Find the total area of the paper used.

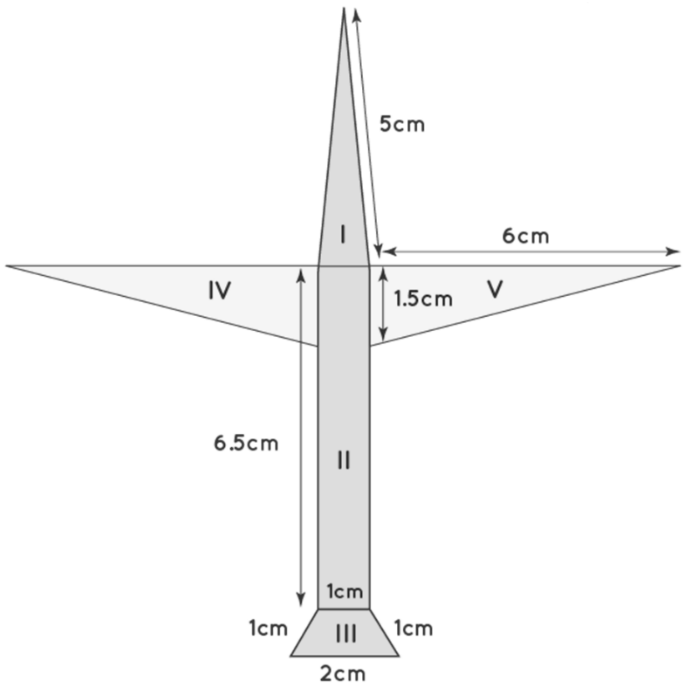


Fig. 12.15

Answer: The picture of the aeroplane with coloured paper is a combination of multiple diagrams: triangle, rectangle, trapezium.

By using Heron's formula, we can calculate the area of a triangle.

Heron's formula for the area of a triangle is:
Area = $\sqrt{s(s - a)(s - b)(s - c)}$

Where a, b and c are the sides of the triangle, and s = Semi-perimeter = Half the Perimeter of the triangle

i) For the triangle marked as I:

It is an isosceles triangle, therefore a = 5 cm, b = 5 cm, c = 1 cm

Semi Perimeter: (s) = (Perimeter/2)

s = (a + b + c)/2

= (5 + 5 + 1)/2

= 11/2

= 5.5 cm

By using Heron's formula,

Area of triangle marked I = $\sqrt{s(s - a)(s - b)(s - c)}$

= √5.5 (5.5 - 5) (5.5 - 5) (5.5 - 1)

= √5.5 × 0.5 × 0.5 × 4.5

= √6.1875 cm

= 2.5 cm^2 (approx.)

Area of triangle (I) = 2.5 cm^2

(ii) For the rectangle marked as II:

The measures of the sides are 6.5 cm, 1 cm, 6.5 cm, and 1 cm.

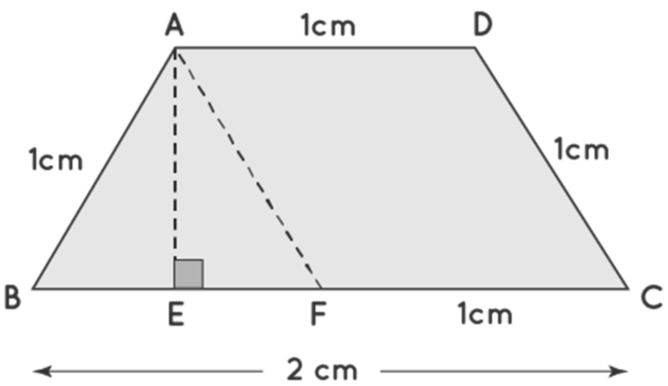
Area of rectangle = length × breadth

= 6.5 cm × 1 cm

= 6.5 cm^2

Area of a rectangle (II) = 6.5 cm^2

(iii) For the trapezium marked as III



Name it as ABCD.

Draw AE ⊥ BC from A on BC and AF parallel to DC

AD = FC = 1 cm (opposite sides of a parallelogram)

AB = DC = 1 cm

BC = 2 cm

AF = DC = 1 cm (opposite sides of parallelogram)

BF = BC - FC = 2 cm - 1 cm = 1 cm

Here ΔABF is an equilateral triangle, Hence E will be the mid-point of BF.

So, BE = EF = BF/2

EF = 1/2 = 0.5 cm

In ΔAEF,

$AF^2 = AE^2 + EF^2$ [Pythagoras theorem]

$1^2 = AE^2 + 0.5^2$

$AE^2 = \sqrt{1^2 - 0.5^2}$

$AE^2 = \sqrt{0.75}$

AE = 0.9 cm (approx.)

Area of trapezium = 1/2 × sum of parallel sides × distance between them

= 1/2 × (BC + AD) × AE

= 1/2 × (2 + 1) × 0.9

= 1/2 × 3 × 0.9

= 1.4 cm^2 (approx.)

Areaof trapezium (III) = 1.4 cm^2

(iv) For the triangle marked as IV and V

Triangles IV and V are congruent right-angled triangles with base 6 cm and height 1.5 cm.

Area of the triangle = 1/2 × base × height

= 1/2 × 6 × 1.5

= 4.5 cm^2

Area of two triangles (IV and V) = 4.5 cm^2 + 4.5 cm^2 = 9 cm^2

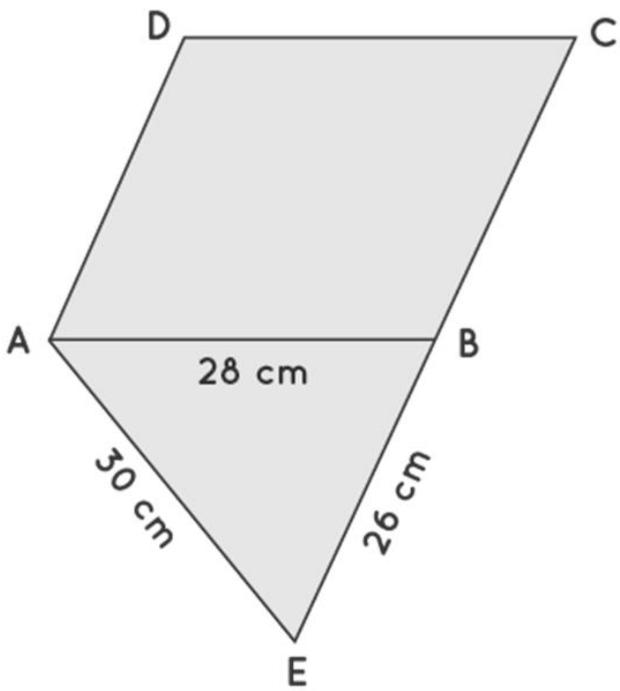
Total area of the paper used = Area I + Area II + Area III + Area IV + Area V

= 2.5 cm^2 + 6.5 cm^2 + 1.4 cm^2 + 9 cm^2 = 19.4 cm^2

Thus, the total area of the paper used is 19.4 cm^2 (approx.).

4. A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 26 cm, 28 cm, and 30 cm, and the parallelogram stands on the base 28 cm, find the height of the parallelogram

Answer: Given: Area of the parallelogram = Area of the triangle



By using the area of the parallelogram formula, we can calculate the height of the parallelogram

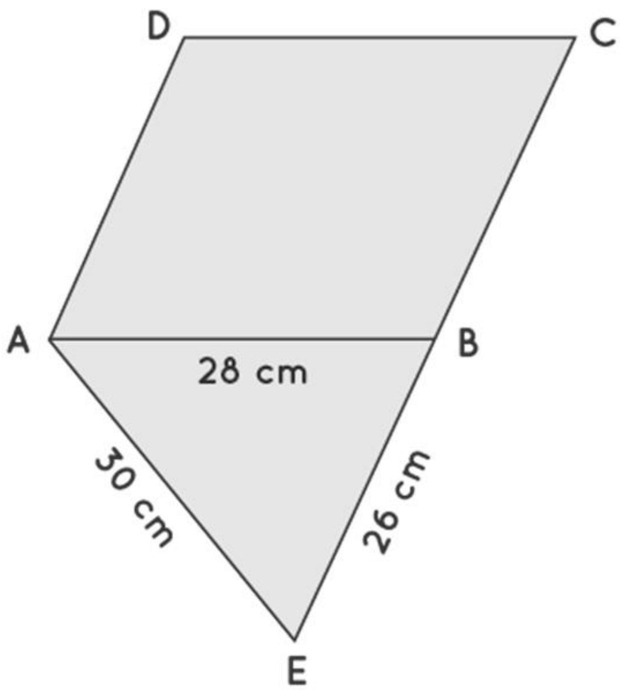
By using Heron's formula, we can calculate the area of a triangle.

Heron's formula for the area of a triangle is:
Area = $\sqrt{s(s - a)(s - b)(s - c)}$

Where a, b, and c are the sides of the triangle, and s = Semi-perimeter = Half the Perimeter of the triangle

Let ABCD is a parallelogram and ABE is a triangle having a common base with parallelogram ABCD.

For ΔABE, a = 30 cm, b = 26 cm, c = 28 cm



Semi Perimeter: (s) = Perimeter/2

s = (a + b + c)/2

= (30 + 26 + 28)/2

= 84/2

= 42 cm

By using Heron's formula,

Area of a ΔABE = $\sqrt{s(s - a)(s - b)(s - c)}$

= √42(42 - 30) (42 - 28) (42 - 26)

= √42 × 12 × 14 × 16

= 336 cm^2

Area of parallelogram ABCD = Area of ΔABE (given)

Base × Height = 336 cm^2

28 cm × Height = 336 cm^2

On rearranging, we get

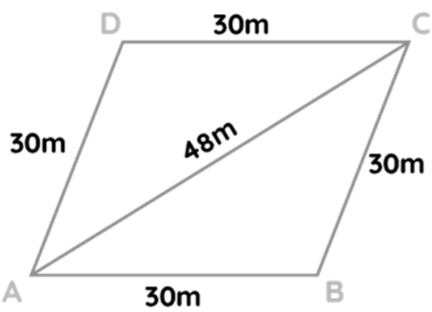
Height = 336/28 cm = 12 cm

Thus, height of the parallelogram is 12 cm.

5. A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of grass field will each cow be getting?

Answer: We will divide the given rhombus into two triangles, and by using Heron's formula, we can calculate the area of triangles.

Heron formula for the area of a triangle = $\sqrt{s(s - a)(s - b)(s - c)}$

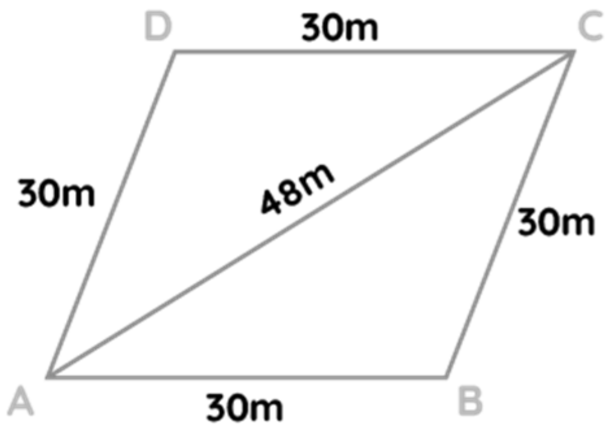


Where a, b and c are the sides of the triangle, and s = Semi-perimeter = Half the Perimeter of the triangle = (a + b + c)/2

Let the longer diagonal AC divides the rhombus ABCD into two congruent triangles.

For ΔABC, a = b = 30 m, c = 48 m

And Semi Perimeter(s) = (a + b + c)/2



s = (30 + 30 + 48)/2

s = 108/2

s = 54 m

By using Heron's formula,

Area of ΔABC = $\sqrt{s(s - a)(s - b)(s - c)}$

= √54(54 - 30)(54 - 30)(54 - 48)

= √54 × 24 × 24 × 6

Area of ΔABC = 432 m^2

Area of rhombus = 2 × Area of a ΔABC

= 2 × 432 m^2

= 864 m^2

Since, number of cows = 18

The area of grass field will each cow get = (Total area of the rhombus) / 18

= 864 m^2/18

= 48 m^2

Thus, each cow will be getting a 48 m^2 area of the grass field.

6. An umbrella is made by stitching 10 triangular pieces of cloth of two different colours (See Fig. 12.16), each piece measuring 20 cm, 50 cm, and 50 cm. How much cloth of each colour is required for the umbrella?

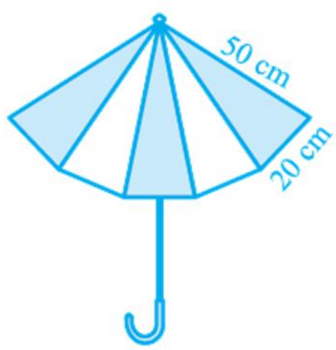


Fig. 12.16

Answer:

Given: Dimensions of each triangular piece used in umbrella.

Reasoning: By using Heron's formula, we can calculate the area of a triangle.

Heron's formula for the area of a triangle, Area = $\sqrt{s(s - a)(s - b)(s - c)}$

Where a, b, and c are the sides of the triangle, and

s = Semi-perimeter = Half the Perimeter of the triangle = (a + b + c)/2

We know that umbrella is made of 10 triangular pieces of cloth of two different colours.

Let us calculate the area of one triangle.

For each triangle, a = b = 50 cm, c = 20 cm

Fig. 12.16

Semi Perimeter

s = (a + b + c)/2

= (50 + 50 + 20)/2

= 120/2

= 60 cm

By using Heron's formula,

Area of triangle = $\sqrt{s(s - a)(s - b)(s - c)}$

= √60(60 - 50)(60 - 50)(60 - 20)

= √60 × 10 × 10 × 40

= 200√6 cm^2

Therefore,

Area of 10 triangular pieces = 10 × 200√6 cm^2 = 2000√6 cm^2

Hence, cloth required for each colour = (Total area of the cloth)/2

= (2000√6)/2

= 1000√6 cm^2

Thus, 1000√6 cm^2 cloth of each colour is required for the umbrella.

7. A kite in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and sides 6 cm each is to be made of three different shades as shown in Fig. 12.17. How much paper of each shade has been used in it?

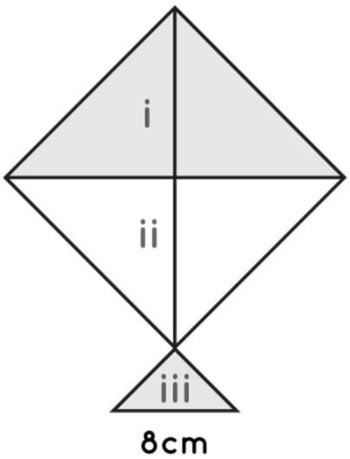


Fig. 12.17

Answer: We divide the kite into three different triangles, and by using Heron's formula, we can calculate the area of triangle.

Heron's formula for the area of a triangle = $\sqrt{s(s - a)(s - b)(s - c)}$

Where a, b, and c are the sides of the triangle, and

s = Semi-perimeter = Half the Perimeter of the triangle = (a + b + c)/2

We know that the diagonals of a square are perpendicular bisectors of each other.

Given diagonal BD = AC = 32 cm, then OA = 1/2 AC = 16 cm.

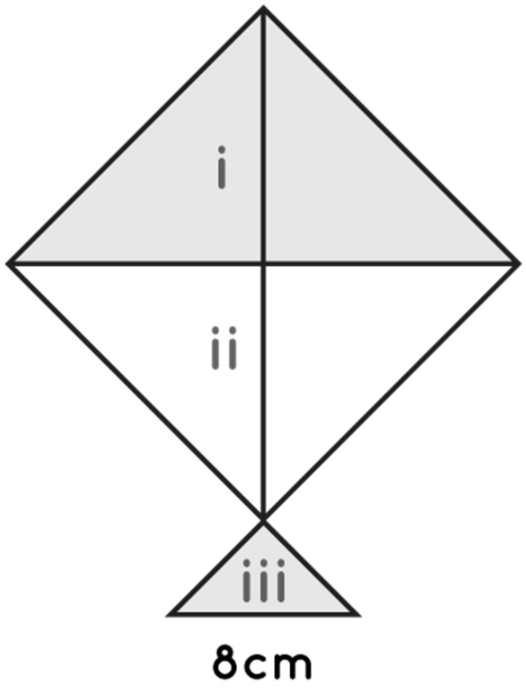


Fig. 12.17

So square ABCD is divided into two isosceles triangles ABD and CBD of base 32 cm and height 16 cm.

Area of ΔABD = 1/2 × base × height

= (32 × 16)/2

= 256 cm^2

Since the diagonal divides the square into two equal triangles. Therefore, Area of ΔABD = Area of ΔCBD = 256 cm^2

Now, for ΔCEF

Semi Perimeter(s) = (a + b + c)/2

s = (6 + 6 + 8)/2

s = 20/2

s = 10 cm

By using Heron's formula,

Area of ΔCEF = $\sqrt{s(s - a)(s - b)(s - c)}$

= √10(10 - 6)(10 - 6)(10 - 8)

= √10 × 4 × 4 × 2

= 8√5

= 8 × 2.24

= 17.92 cm^2

Thus, the area of the paper used to make region I = 256 cm^2, region II = 256 cm^2, and region III = 17.92 cm^2.

8. A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm, and 35 cm (see Fig. 12.18). Find the cost of polishing the tiles at the rate of 50p per cm^2.

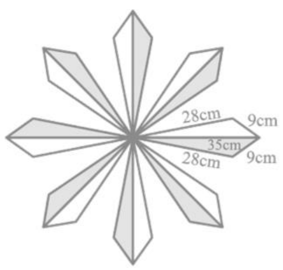


Fig. 12.18

Answer:

Given: Dimensions of the triangular tiles.

By using Heron's formula, we can calculate the area of triangle.

Heron's formula for the area of a triangle, Area = √s(s - a)(s - b)(s - c)

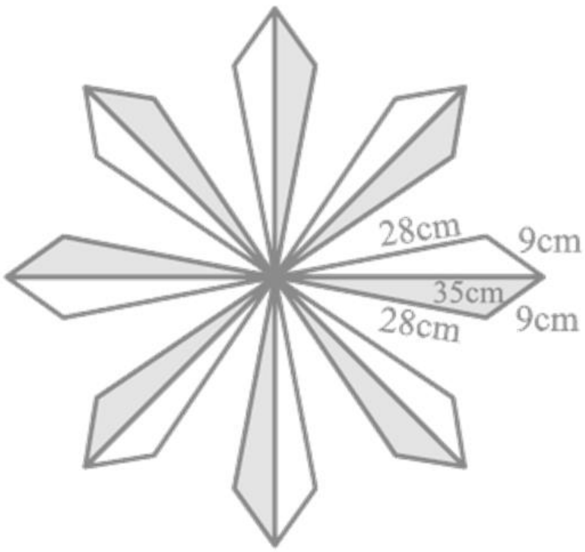


Fig. 12.18

Where a, b and c are the sides of the triangle, and s = Semi-perimeter = Half the Perimeter of the triangle = (a + b + c)/2

We have the dimensions of sides of each triangular tile.

a = 35 cm, b = 28 cm, c = 9 cm

Semi Perimeter(s) = (a + b + c)/2

s = (35 + 28 + 9)/2

s = 72/2

s = 36 cm

By using Heron's formula,

Area of each triangular tile = $\sqrt{s(s-a)(s-b)(s-c)}$

= √36(36 - 35)(36 - 28)(36 - 9)

= √36 × 1 × 8 × 27

= 36√6 cm^2

Area of a 1 triangular tile = 36√6 cm^2

Area of 16 triangular tile = 16 × 36√6 cm^2

= 16 × 36 × 2.45 cm^2

= 1411.2 cm^2

Since, the cost of polishing 1cm^2 of tiles is 50p or ₹ 0.5

Therefore, the cost of polishing 1411.2 cm^2 area of tiles = 1411.2 cm^2 × 0.5 = ₹ 705.60

9. A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.

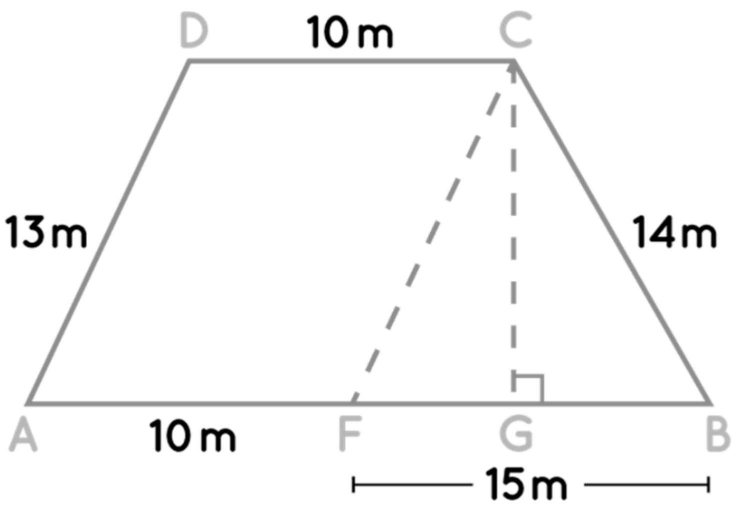
Answer: We draw a triangle inside the trapezium and by using Heron's formula, we can calculate the area of a triangle, and then find the height of the triangle which will also be the height of the trapezium and hence, we will calculate the area of the trapezium.

Heron's formula for the area of a triangle, Area = $\sqrt{s(s-a)(s-b)(s-c)}$

Where a, b and c are the sides of the triangle, and

s = Semi-perimeter = Half the Perimeter of the triangle = (a + b + c)/2

Given ABCD is a field. Draw CG ⊥ AB from C on AB, and CF parallel to DA.



DC = AF = 10 m, DA = CF = 13 m (opposite sides of parallelogram)

So, FB = 25 - 10 = 15 m

In ΔCFB, a = 15 m, b = 14 m, c = 13 m.

Semi Perimeter(s) = (a + b + c)/2

s = (15 + 14 + 13)/2

s = 42/2

s = 21 m

By using Heron's formula,

Area of ΔCFB = $\sqrt{s(s - a)(s - b)(s - c)}$

= √21(21 - 15)(21 - 14)(21 - 13)

= √21 × 6 × 7 × 8

= 84 m^2

Also,

Area of ΔCFB = 1/2 × base × height

84 = 1/2 × BF × CG

84 = 1/2 × 15 × CG

CG = (84 × 2)/15

CG = 11.2 m

Area of trapezium ABCD = 1/2 × sum of parallel sides × distance between them

= 1/2 × (AB + DC) × CG

= 1/2 × (25 + 10) × 11.2

= 196 m^2

Hence the area of the field is 196 m^2.