Exercise 4.1

Q1. The cost of a notebook is twice the cost of a pen. Write a linear equation in two variables to represent this statement. (Take the cost of a notebook to be ₹ x and that of a pen to be ₹ y).

Answer:

Let the cost of one notebook be ₹ x

Let the cost of one pen be ₹ y

Therefore, we can write the required linear equation in two variables considering the given information as,

Cost of notebook = 2 × Cost of pen

⇒ x = 2y

Since any linear equation in two variables is expressed as: **ax + by + c = 0,**

∴ x - 2y = 0 is the required linear equation in two variables representing the given information.

Q2. Express the following linear equations in the form ax + by + c = 0 and indicate the values of a, b, c in each case:

(i) 2x + 3y = 9.35

(ii) x - y/5 -10 = 0

(iii) -2x + 3y = 6

(iv) x = 3y

(v) 2x = - 5y

(vi) 3x + 2 = 0

(vii) y - 2 = 0

(viii) 5 = 2x

Answer:

(i) Consider 2x + 3y = 9.35

⇒ 2x + 3y - 9.35= 0

Comparing this with the standard form of the linear equation in two variables, ax + by + c = 0 we have,

- a = 2
- b = 3
- c = - 9.35

(ii) Consider x - y/5 - 10 = 0

⇒ 1x - y/5 - 10 = 0

Comparing this with the standard form of the linear equation in two variables, ax + by + c = 0 we have,

- a = 1
- b = -1/5
- c = -10

(iii) Consider -2x + 3y = 6

⇒ -2x + 3y - 6 = 0

Comparing this with the standard form of the linear equation in two variables, ax + by + c = 0 we have,

- a = - 2
- b = 3
- c = - 6

(iv) Consider x = 3y

⇒ 1x - 3y + 0 = 0

Comparing this with the standard form of the linear equation in two variables, ax + by + c = 0, we have

- a = 1
- b = -3
- c = 0

(v) Consider 2x = -5y

⇒ 2x + 5y + 0 = 0

Comparing this with the standard form of the linear equation in two variables, ax + by + c = 0 we have,

- a = 2
- b = 5
- c = 0

(vi) Consider 3x + 2 = 0

⇒ 3x + 0y + 2 = 0

Comparing this with the standard form of the linear equation in two variables, ax + by + c = 0 we have,

- a = 3
- b = 0
- c = 2

(vii) Consider y - 2 = 0

⇒ 0x + 1y - 2 = 0

Comparing this with the standard form of the linear equation in two variables, ax + by + c = 0 we have,

- a = 0
- b = 1
- c = -2

(viii) Consider 5 = 2x

⇒ 2x + 0y - 5 = 0

Comparing this with the standard form of the linear equation in two variables, ax + by + c = 0 we have,

- a = 2
- b = 0
- c = -5

Exercise 4.2

Q1. Which one of the following options is true, and why? y = 3x + 5 has
(i) a unique solution,
(ii) only two solutions,
(iii) infinitely many solutions

Answer:

Given: Linear equation y = 3x + 5

We need to find how many solutions can satisfy the given equation.

We know that,

y = 3x + 5 is a linear equation in two variables in the form of **ax + by + c = 0**

- For x = 0, y = 0 + 5 = 5. Therefore, (0, 5) is one solution.
- For x = 1, y = 3 × 1 + 5 = 8. Therefore, (1, 8) is another solution.
- For y = 0, 3x + 5 = 0, x = -5/3. Therefore, (-5/3, 0) is another solution.

Clearly, for different values of x, we get various values for y. Thus, any value substituted for x in the given equation will constitute another solution for the given equation. So, there is no end to the number of different solutions obtained by substituting real values for x in the given linear equation. Therefore, a linear equation in two variables has infinitely many solutions.

Thus, $y = 3x + 5$ has infinitely many solutions.

Hence (iii) is the correct answer.

Q2. Write four solutions for each of the following equations:

(i) $2x + y = 7$

(ii) $\pi x + y = 9$

(iii) $x = 4y$

Answer:

In the given Linear equations, we can find any number of solutions by putting different values of x and obtain different values of y.

i) $2x + y = 7$

Changing the subject of the equation to y, and solving, we get,

$\therefore y = 7 - 2x$

Let us now take different values of x and substitute them in the given equation.

For $x = 0$, we get $y = 7 - 2(0) \Rightarrow y = 7$. Hence, we get $(x, y) = (0, 7)$

For $x = 1$, we get $y = 7 - 2(1) \Rightarrow y = 5$. Hence, we get $(x, y) = (1, 5)$

For $x = 2$, we get $y = 7 - 2(2) \Rightarrow y = 3$. Hence, we get $(x, y) = (2, 3)$.

For $x = 3$, we get $y = 7 - 2(3) \Rightarrow y = 1$. Hence, we get $(x, y) = (3, 1)$.

Therefore, four solutions of the given equation are (0, 7), (1, 5), (2, 3) and (3, 1).

ii) $\pi x + y = 9$

$\therefore y = 9 - \pi x$

Let us now take different values of x and substitute them in the given equation.

For $x = 0$, $y = 9 - \pi(0) \Rightarrow y = 9$. Hence, we get $(x, y) = (0, 9)$

For $x = 1$, $y = 9 - \pi(1) \Rightarrow 9 - \pi$. Hence, we get $(x, y) = (1, 9 - \pi)$

For $x = 2$, $y = 9 - \pi(2) \Rightarrow 9 - 2\pi$. Hence, we get $(x, y) = (2, 9 - 2\pi)$

For $x = 3$, $y = 9 - \pi(3) \Rightarrow 9 - 3\pi$. Hence, we get $(x, y) = (3, 9 - 3\pi)$

Therefore, four solutions of the given equation are $(0, 9)$, $(1, 9 - \pi)$, $(2, 9 - 2\pi)$ and $(3, 9 - 3\pi)$.

iii) $x = 4y$

∴ $y = x/4$

Let us now take different values of x and substitute them in the given equation.

For x = 0, y = 0/4 = 0. Hence, we get (x, y) = (0, 0)

For x = 1, y = 1/4. Hence, we get (x, y) = (1, 1/4)

For x = 2, y = 2/4 = 1/2. Hence, we get (x, y) = (2, 1/2)

For x = 3, y = 3/4. Hence, we get (x, y) = (3, 3/4)

Therefore, four solutions of the given equation are (0, 0), (1, 1/4), (2, 1/2) and (3, 3/4).

Q3. Check which of the following are solutions of the equation x - 2y = 4 and which are not:

(i) (0, 2)

(ii) (2, 0)

(iii) (4, 0)

(iv) (√2, 4√2)

(v) (1, 1)

Answer:

Given: Linear Equation x - 2y = 4

We can substitute the values in the given equation and can check whether LHS is equal to RHS or not.

if LHS = RHS then it is a solution for the given equation.

x - 2y = 4 --- Equation (1)

i) Consider (0, 2)

By Substituting x = 0 and y = 2 in the given Equation (1)

x - 2y = 4

0 - 2(2) = 4

0 - 4 = 4

- 4 ≠ 4

L.H .S ≠ R.H .S

Therefore, (0, 2) is not a solution to this equation.

ii) Consider (2, 0)

By Substituting, x = 2 and y = 0 in the given Equation (1), x - 2y = 4

2 - 2(0) = 4

2 - 0 = 4

2 ≠ 4

L.H .S ≠ R.H .S

Therefore, (2, 0) is not a solution to this equation.

iii) (4, 0)

By Substituting, x = 4 and y = 0 in the given Equation (1)

x - 2y = 4

4 - 2(0) = 4

4 - 0 = 4

4 = 4

L.H .S = R.H .S

Therefore, (4, 0) is a solution to this equation.

iv) (√2, 4√2)

By Substituting, x = √2 and y = 4√2 in the given Equation (1)

x - 2y = 4

√2 - 8√2 = 4

-7√2 ≠ 4

L.H .S ≠ R.H .S

Therefore, (√2, 4√2) is not a solution to this equation.

v) (1, 1)

By Substituting, x = 1 and y = 1 in the given Equation (1)

x - 2y = 4

1- 2 (1) = 4

1 - 2 = 4

-1 ≠ 4

L.H .S ≠ R.H .S

Therefore, (1, 1) is not a solution to this equation.

Q4. Find the value of k, if x = 2, y = 1 is a solution of the equation 2x + 3y = k.

Answer:

Given: Linear equation 2x + 3y = k.

We can find the value of k by substituting the values of x and y in the given equation.

By substituting the values of x = 2 and y = 1 in the given equation.

$2x + 3y = k$

$\Rightarrow 2(2) + 3(1) = k$

$\Rightarrow 4 + 3 = k$

$\Rightarrow$ Hence, k = 7

Therefore, the value of k is 7.

Exercise 4.3

Q1. Draw the graph of each of the following linear equations in two variables:
(i) x + y = 4
(ii) x - y = 2
(iii) y = 3x
(iv) 3 = 2x + y

Answer:

First of all, we can draw a table for different values of x and y, and then with the help of the values, we can plot a graph for each linear equation.

i) x + y = 4

Re-write the equation as y = 4 - x --- Equation (1)

By substituting the different values of x in Equation (1), we get different values for y

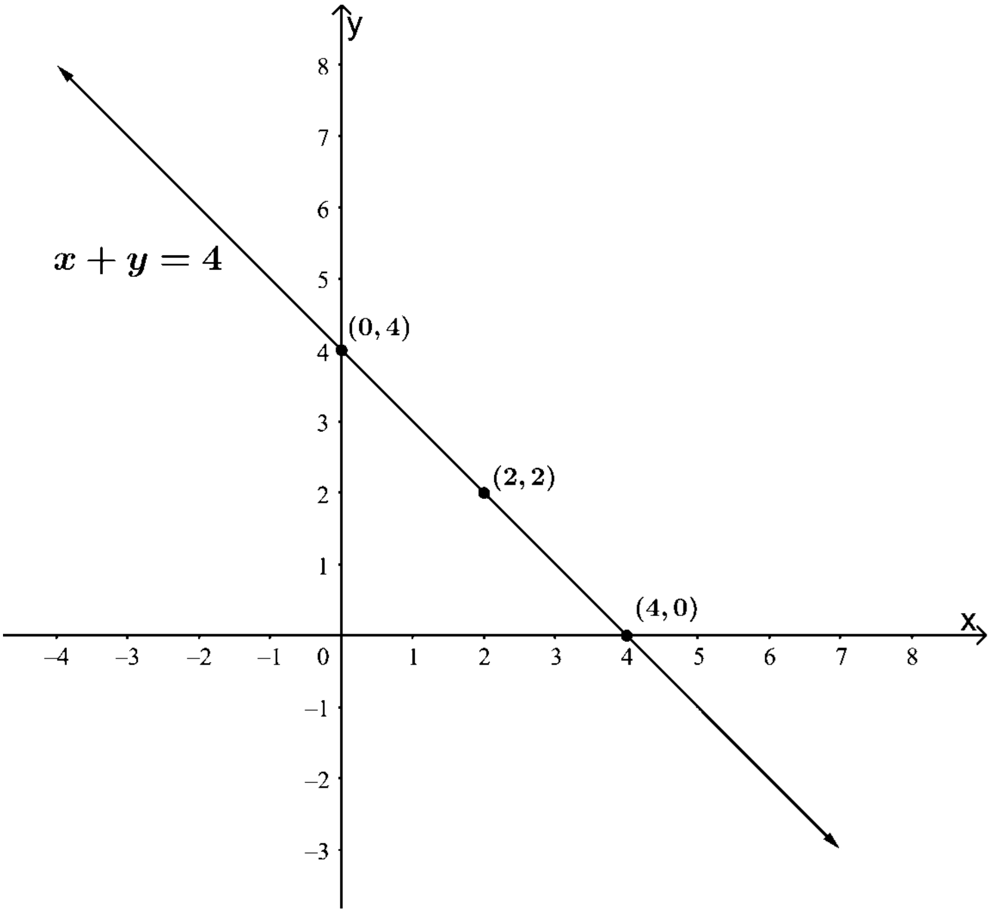
- When x = 0, we have: y = 4 - 0 = 4
- When x = 2, we have: y = 4 - 2 = 2
- When x = 4, we have: y = 4 - 4 = 0

Thus, we have the following table with all the obtained solutions:

x	0	2	4
y	4	2	0

By plotting the points (0, 4) (2, 2) and (4, 0) on the graph paper and drawing a line joining the corresponding points, we obtain the graph.

The graph of the line represented by the given equation is as shown.



ii) x – y = 2

Re-write the equation as y = x – 2 --- Equation (1)

By substituting the different values of x in Equation (1) we get different values for y

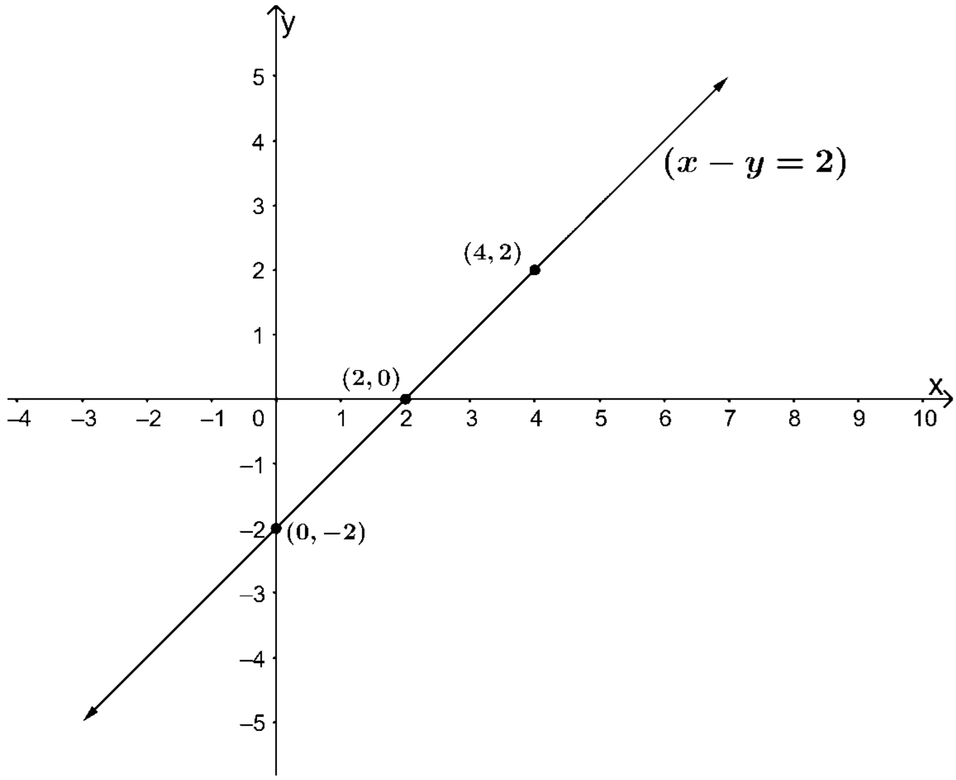
- When x = 0, we have y = 0 - 2 = -2
- When x = 2, we have y = 2 - 2 = 0
- When x = 4, we have y = 4 - 2 = 2

Thus, we have the following table with all the obtained solutions:

x	0	2	4
y	-2	0	2

By Plotting the points (0, -2), (2, 0), and (4, 2) on the graph paper and drawing a line joining the corresponding points, we obtain the graph.

The graph of the line represented by the given equation is as shown



iii) y = 3x --- Equation (1)

By substituting the different values of x in Equation (1) we get different values for y.

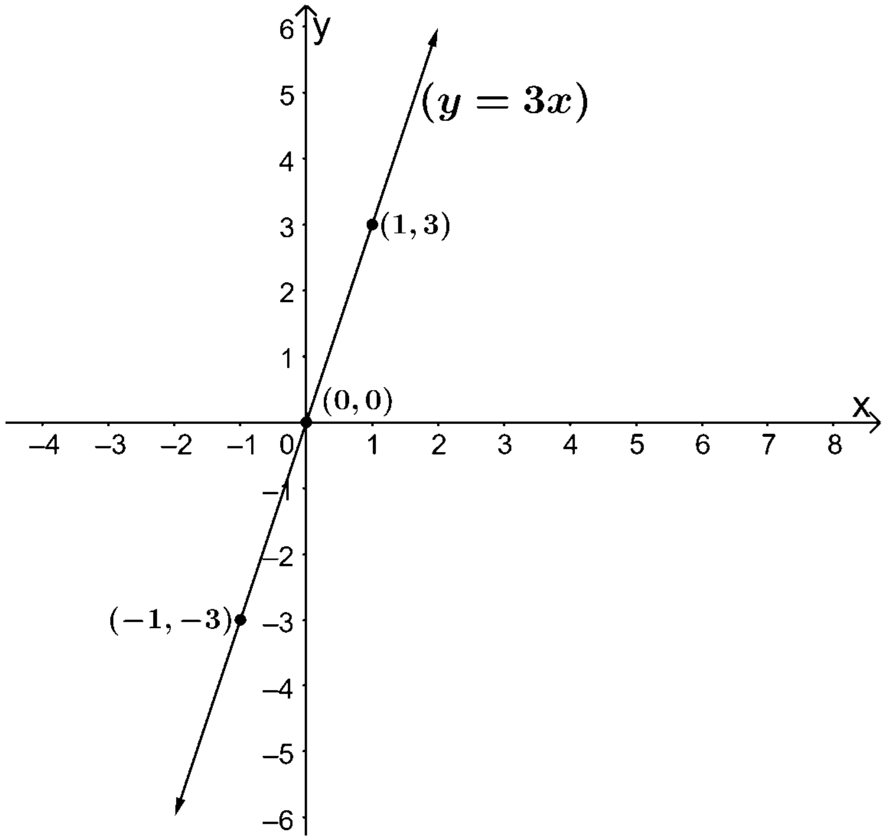
- When x = 0, we have: y = 3(0) = 0
- When x = 1, we have: y = 3 (1) = 3
- When x = -1, we have: y = 3(-1) = -3

Thus, we have the following table with all the obtained solutions:

x	0	1	-1
y	0	3	-3

By Plotting the points (0, 0), (1, 3), and (-1, -3) on the graph paper and drawing a line joining the corresponding points, we obtain the graph.

The graph of the line represented by the given equation is as shown:



iv) 3 = 2x + y

Re-write the equations

y = 3 - 2x --- Equation (1)

By substituting the different values of x in Equation (1) we get different values for y

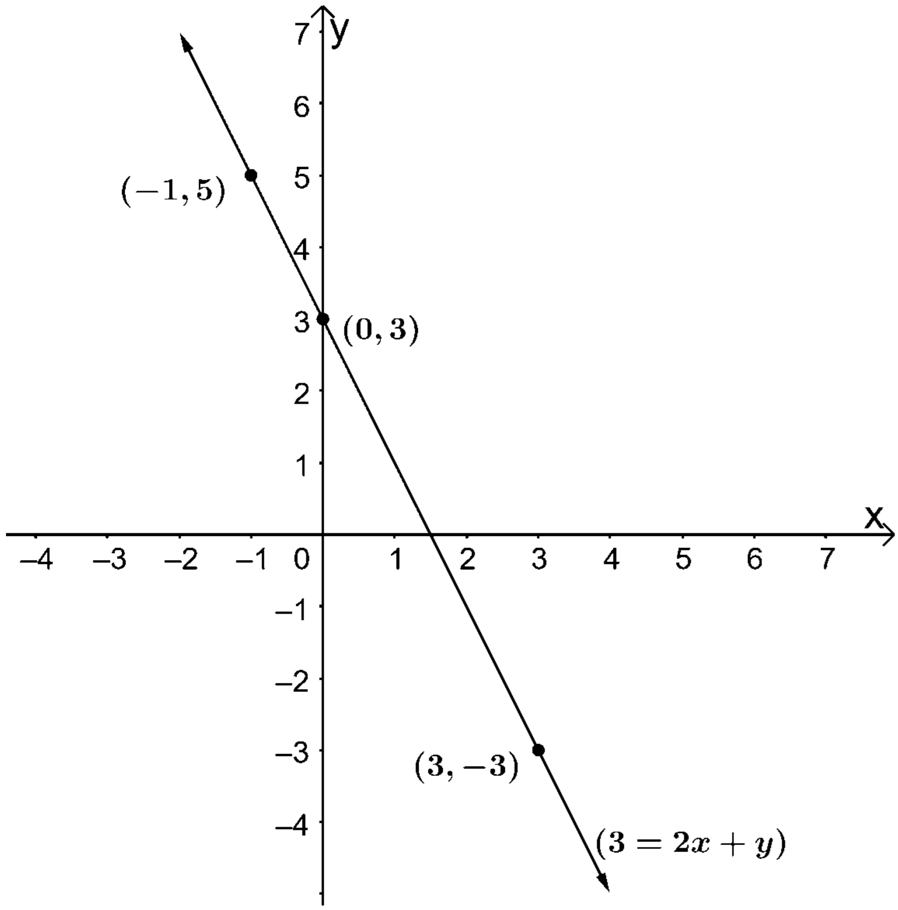
- When x = 0, we have: y = 3 - 2(0) = 3 - 0 = 3
- When x = 3, we have: y = 3 - 2(3) = 3 - 6 = - 3
- When x = -1, we have: y = 3 - 2 (-1) = 3 + 2 = 5

Thus, we have the following table with all the obtained solutions:

x	0	3	-1
y	3	-3	5

By plotting the points (0, 3), (3, -3), and (-1, 5) on the graph paper and drawing a line joining the corresponding points, we obtain the graph.

The graph of the line represented by the given equation is as shown.



Q2. Give the equations of two lines passing through (2, 14). How many more such lines are there, and why?

Answer:

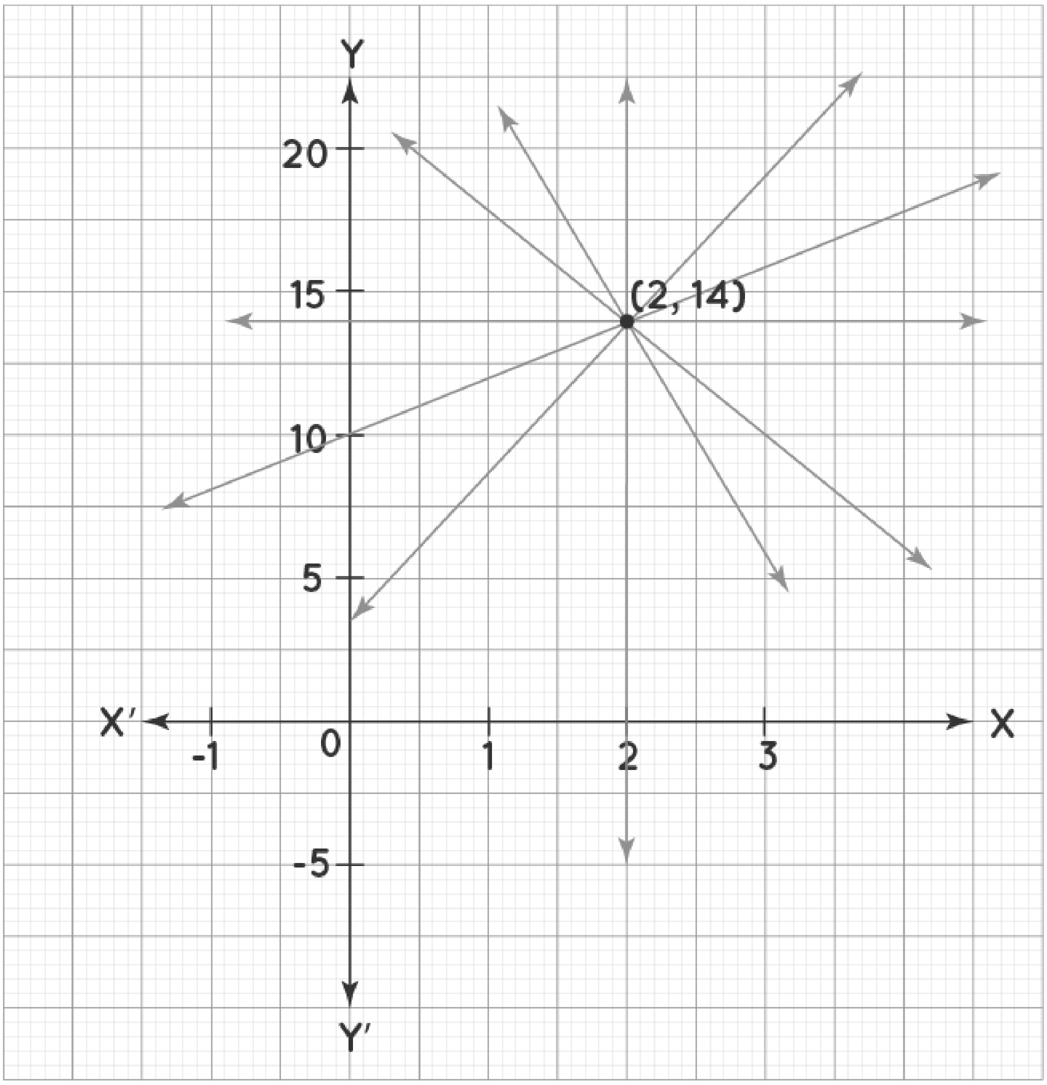
Given: Point (2, 14) at which lines passing through.

We can think of framing equations that could satisfy the given point (2, 14) on the line.

It can be observed that point (2, 14) satisfies the equation $7x - y = 0$ and $x - y + 12 = 0$.

Therefore, $7x - y = 0$ and $x - y + 12 = 0$ are two lines passing through point (2, 14).

Since we know infinite lines can pass through a single point, therefore there are infinite more lines passing through the given point



Q3. If the point (3, 4) lies on the graph of the equation 3y = ax + 7, find the value of a.

Answer:

Given: Linear equation 3y = ax + 7.

We can find the value of 'a' by substituting the values of x and y in the given equation

By substituting the value of x = 3 and y = 4 in the the equation 3y = ax + 7, we get

3(4) = a(3) + 7

12 = 3a + 7

3a = 5

a = 5/3

Hence, the value a = 5/3

Q4. The taxi fare in a city is as follows: For the first kilometer, the fare is ₹ 8, and for the subsequent distance, it is ₹ 5 per km. Taking the distance covered as x km and total fare as ₹ y, write a linear equation for this information, and draw its graph.

Answer:

Given: The distance covered is x km and the total fare is ₹ y. Also, the fare of the first kilometre is ₹ 8 and for the subsequent distance, it is ₹5 per km.

- Let the total distance covered = x km
- Therefore, subsequent distance = (x - 1) km
- The total fare covered = ₹ y
- Fare for the 1^{st} kilometre = ₹ 8
- Fare per km = ₹ 5
- Fare rate for the subsequent distance = 5(x - 1)

The linear equation for the above information is given by,

Total fare = Fare for the first kilometre + Subsequent distance × Fare per km

$y = 8 + (x - 1) 5$

$y = 8 + 5x - 5$

$y = 5x + 3$

$5x - y + 3 = 0$

This can be written as,

$y = 5x + 3$ --- Equation (1)

By substituting different values of x in Equation (1) we get different values for y.

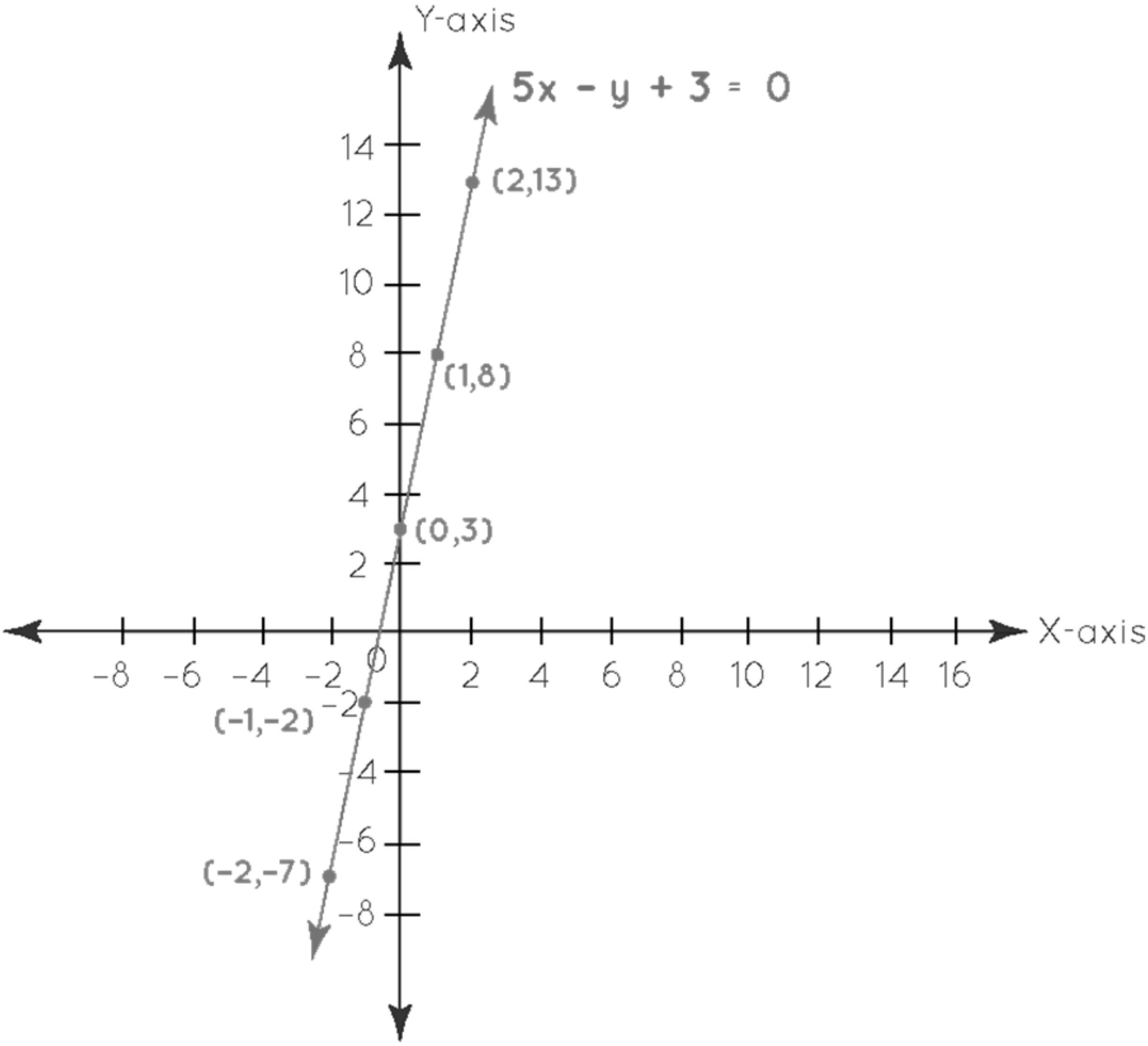
- When x = 0, y = 5 × 0 + 3 = 0 + 3 = 3
- When x = 1, y = 5 × (1) + 3 = 5 + 3 = 8
- When x = 2, y = 5 × (2) + 3 = 10 + 3 = 13
- When x = -1, y = 5 × (-1) + 3 = -5 + 3 = -2
- When x = -2, y = 5 × (-2) + 3 = -10 + 3 = -7

Thus, we have the following table with all the obtained solutions:

x	**0**	**1**	**2**	**-1**	**-2**
y	**3**	**8**	**13**	**-2**	**-7**

By Plotting the points (0, 3), (1, 8), (2, 13), (-1, -2) and (-2, -7) on the graph paper and drawing a line joining them, we obtain the required graph.

The graph of the line represented by the given equation 5x - y + 3 = 0 is shown below:



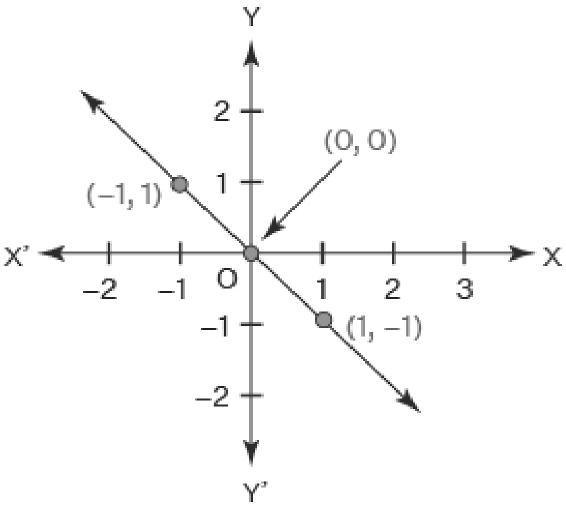
Here, the variables x and y represent the distance covered and the fare paid for that distance respectively, and these quantities cannot be negative.

Hence, only those values of x and y which are lying in the 1st quadrant will be considered as they are positive.

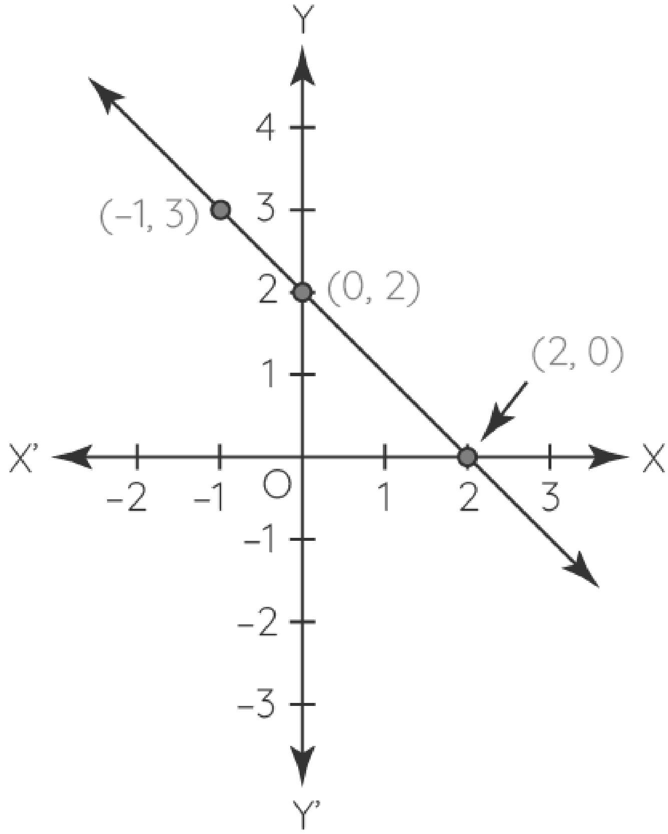
Q5. **From the choices given below, choose the equation whose graphs are given in Fig. 4.6 and Fig. 4.7.**

Answer:

For Figure 4.6	For Figure 4.7
i. $y = x$	i. $y = x + 2$
ii. $x + y = 0$	ii. $y = x - 2$
iii. $y = 2x$	iii. $y = -x + 2$
iv. $2 + 3y = 7x$	iv. $x + 2y = 6$

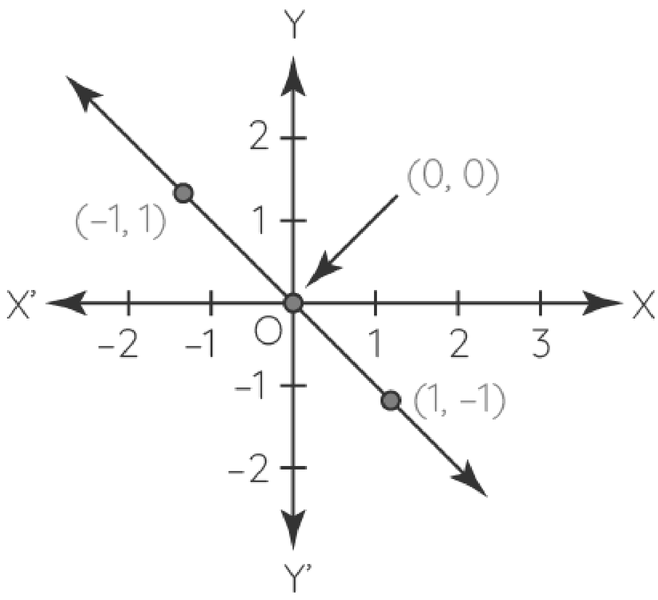


For Figure 4.6



For Figure 4.7

Answer:



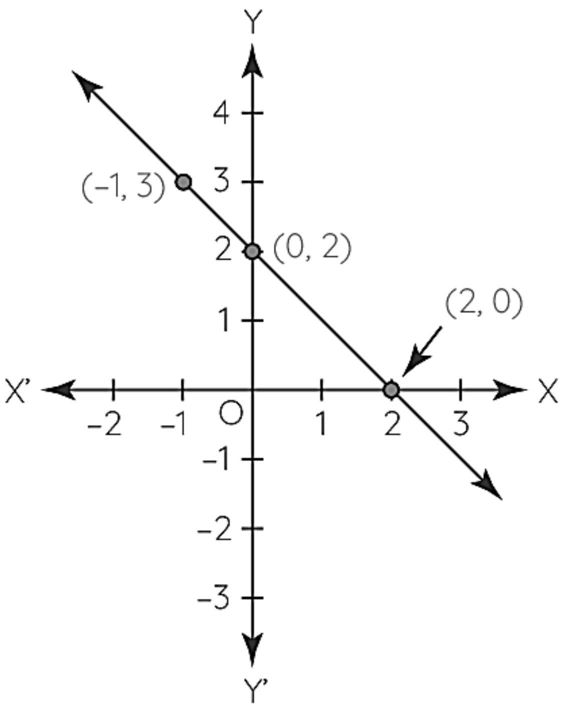
For Figure 4.6

Points on the given line are (-1, 1), (0, 0), and (1, -1).

It can be observed that the coordinates of the points of the graph satisfy the equation x + y = 0.

Therefore, x + y = 0 is the equation corresponding to the graph as shown in the first figure.

Hence, (ii) is the correct answer.



For Figure 4.7

Points on the given line are (-1, 3), (0, 2), and (2, 0).

It can be observed that the coordinates of the points of the graph satisfy the equation y = -x + 2.

Therefore, y = - x + 2 is the equation corresponding to the graph shown in the second figure.

Hence, (iii) is the correct answer.

Q6. If the work done by a body on application of a constant force is directly proportional to the distance travelled by the body, express this in the form of an equation in two variables and draw the graph of the same by taking the constant force as 5 units. Also, read from the graph the work done when the distance travelled by the body is

(i) 2 units (ii) 0 units

Answer:

We can consider the distance travelled by the body and the work done as 'x' and 'y' respectively and then apply the direct proportion property.

- Let the distance travelled and the work done by the body be x and y respectively.
- The constant force applied to the body is 5 units.
- Work done ∝ distance travelled.

Hence, y ∝ x

y = kx --- Equation (1)

Where k is the constant force applied on the body, given as 5. Thus, y = 5x.

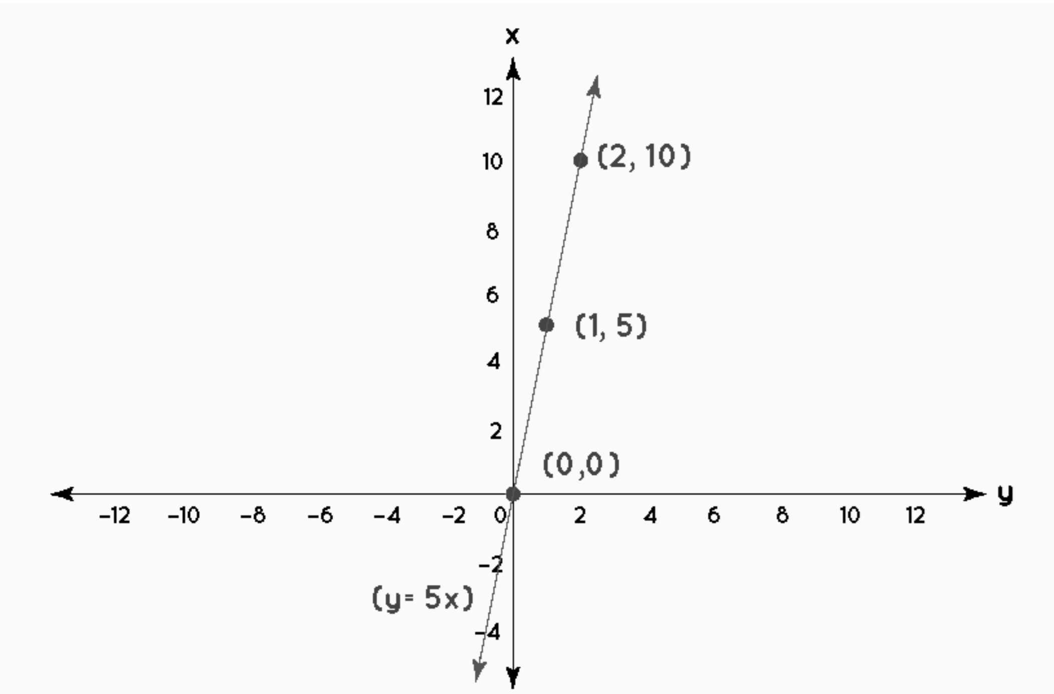
By substituting different values of x in equation (1) we get different values for y

- When x = 0, y = 0
- When x = 1, y = 5
- When x = 2, y = 10

Thus, we have the following table with all the obtained solutions:

x	0	1	2
y	0	5	10

The graph of the line represented by the given equation is shown below.



(i) From the graphs, it can be observed that the value of y corresponding to x = 2 is 10 units. This implies that the work done by the body is 10 units when the distance travelled by it is 2 units.

(ii) From the graphs, it can be observed that the value of y corresponding to x = 0 is 0. This implies that the work done by the body is 0 units when the distance travelled by it is 0 units.

Q7. Yamini and Fatima, two students of Class IX of a school, together contributed ₹ 100 towards the Prime Minister's Relief Fund to help the earthquake victims. Write a linear equation that satisfies this data. (You may take their contributions as ₹ x and ₹ y.) Draw the graph of the same.

Answer:

We can assume Yamini and Fatima's contributions as ₹ x and ₹ y respectively, and form a linear equation.

- Let the amount that Yamini and Fatima have contributed individually be ₹ x and ₹ y respectively towards the Prime Minister's Relief Fund.
- The total amount contributed by Yamini and Fatima together is = ₹ 100

Therefore, x + y = 100

On transposing, y = 100 - x

This is a linear equation in two variables of the form ax + by + c = 0

- Let us consider this y = 100 - x --- Equation (1)

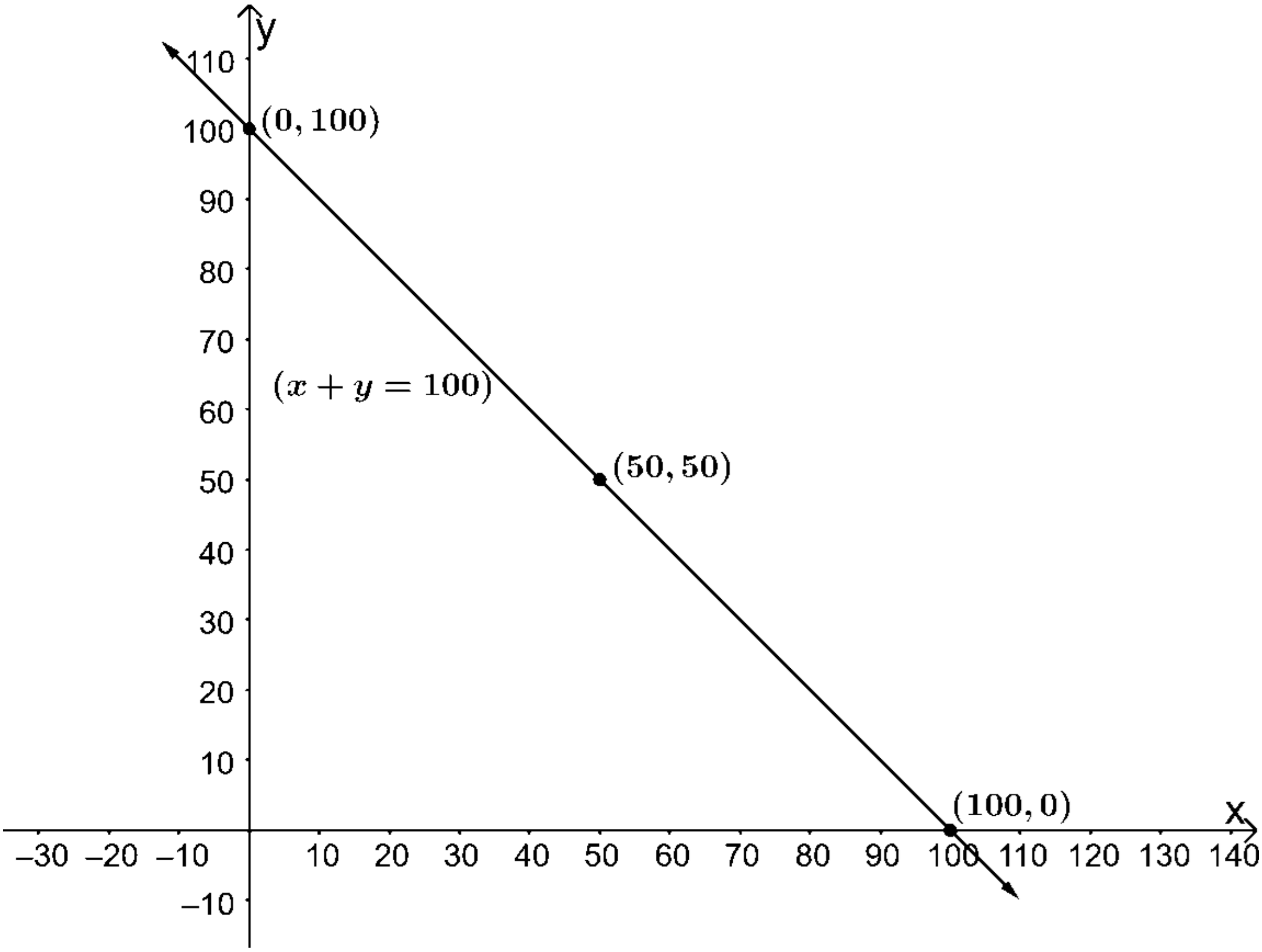
By substituting different values of x in the equation (1), we get different values for y

- When x = 0, y = 100
- When x = 50, y = 50
- When x = 100, y = 0

Thus, we have the following table with all the obtained values of x and y:

x	0	50	100
y	100	50	0

The graph of the line represented by the given equation is as shown.



Here, variables x and y represent the amount contributed by Yamini and Fatima respectively, and these quantities cannot be negative. Hence, only those values of x and y which are lying in the 1st quadrant are considered.

Q8. In countries like the USA and Canada, the temperature is measured in Fahrenheit, whereas in countries like India, it is measured in Celsius. Here is a linear equation that converts Fahrenheit to Celsius:

$$F = \left(\frac{9}{5}\right)C + 32$$

(i) Draw the graph of the linear equation above using Celsius for the x-axis and Fahrenheit for the y-axis.

(ii) If the temperature is 30°C, what is the temperature in Fahrenheit?

(iii) If the temperature is 95°F, what is the temperature in Celsius?

(iv) If the temperature is 0°C, what is the temperature in Fahrenheit, and if the temperature is 0°F, what is the temperature in Celsius?

(v) Is there a temperature which is numerically the same in both Fahrenheit and Celsius? If yes, find it.

Answer:

i) F = (9/5) C + 32 --- Equation (1)

Equation (1) represents a linear equation of the form ax + by + c = 0, where C and F are the two variables.

By substituting different values of C in equation (1), we obtain different values for F.

When C = 0, F = (9/5) C + 32 = (9/5)0 + 32 = 32

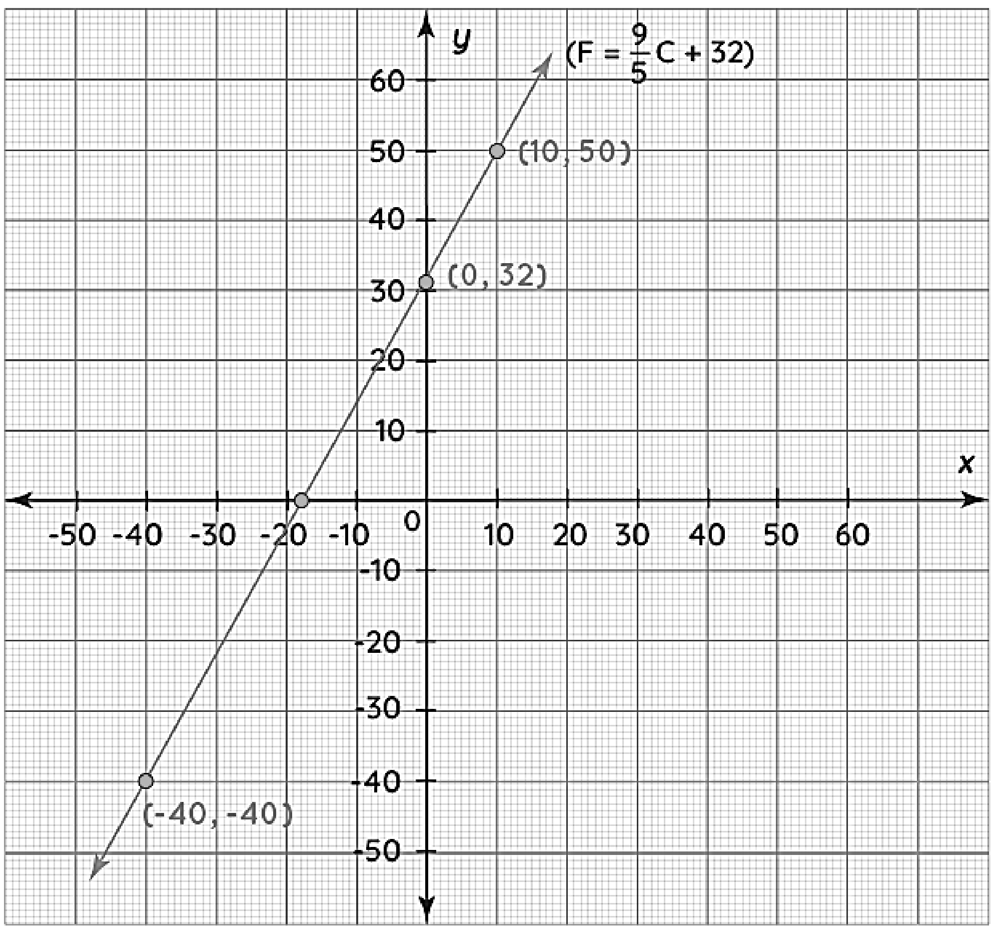
When C = - 40, F = (9/5) C + 32 = (9/5) × (- 40) + 32 = - 72 + 32 = - 40

When C = 10, F = (9/5) C + 32 = (9/5) × (10) + 32 = 18 + 32 = 50

Thus, we have the following table with all the obtained solutions:

C	0	- 40	10
F	32	- 40	50

The graph of the line represented by the given equation is shown below.



ii) Given: Temperature = 30°C

To find: F = ?

We know that, F = (9/5)C + 32

By Substituting the value of C = 30° C in the equation above,

F = (9/5)C + 32

= (9/5)30 + 32

= 54 + 32

= 86

Therefore, the temperature in Fahrenheit is 86° F .

iii) Given, Temperature = 95° F

To find, C = ?

We know that, F = (9/5)C + 32

By Substituting the value of temperature in the above equation,

95 = (9/5)C + 32

95 - 32 = (9/5)C

63 = (9/5)C

C = (63 × 5)/9

C = 35

Therefore, the temperature in Celsius is 35° C.

iv) We know that, F = (9/5)C + 32

If C = 0°, then by substituting this value in the above equation,

F = (9/5)0 + 32

F = 0 + 32

F = 32

Therefore, if C = 0°, then F = 32°

If F = 0° F, then by substituting this value in the above equation,

0 = (9/5)C + 32

(9/5)C = - 32

C = (- 32 × 5)/9

C = - 17.77

Therefore, if F = 0° F, then C = -17.8° C

v) We know that, F = (9/5)C + 32

Let us consider, F = C

By Substituting this value in the equation above,

F = (9/5)C + 32

(9/5 - 1)F + 32 = 0

(4/5)F = - 32

F = (- 32 × 5)/4

Hence, F = - 40

Yes, there is a temperature, of -40°, which is numerically the same for both Fahrenheit and Celsius.

Exercise 4.4

1. Give the geometric representations of y = 3 as an equation

(i) in one variable

(ii) in two variables

Answer:

(i) In one variable, y = 3 represents a point as shown in the following figure.

(ii) We know that y = 3 can be written as, 0x + y = 3

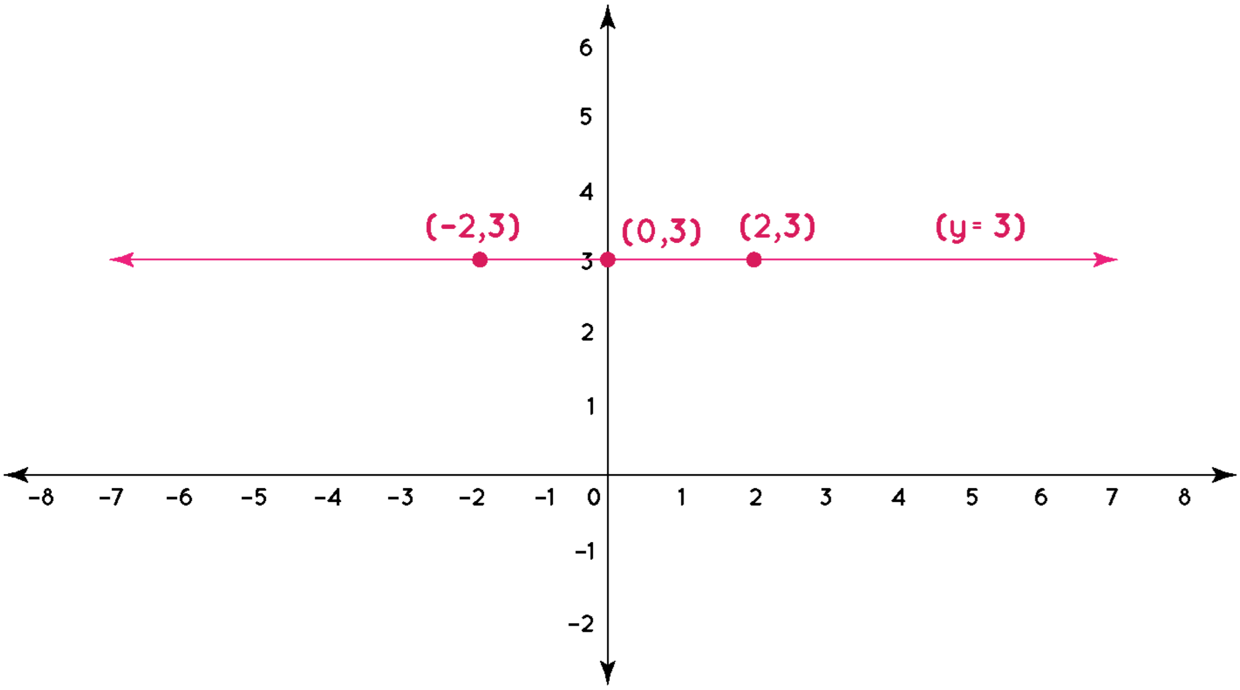
In two variables, y = 3 represents a straight line passing through the point (0, 3) and parallel to the x-axis. It is a collection of all the points on the plane, having their y-coordinate as 3.

Hence,

- When, x = 0, we get y = 3
- When x = 2, we get y = 3
- When x = -2, we get y = 3

Plotting the points (0, 3) (2, 3) and (-2, 3) and on joining them we get the graph AB as a line parallel to the x-axis at a distance of 3 units above it.

The graphical representation is shown below:



2. Give the geometric representations of 2x + 9 = 0 as an equation

(i) in one variable

(ii) in two variables

Answer:

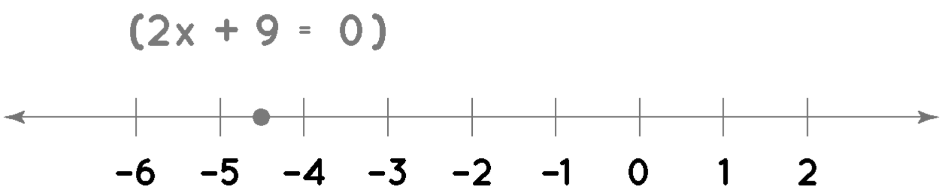
(i) 2x + 9 = 0

2x = - 9

x = - 9/2

x = - 4.5

Hence, in one variable 2x + 9 = 0 represents a point as shown in the following figure.



(ii) We know that 2x + 9 = 0 can be written as 2x + 0y + 9 = 0 as a linear equation in two variables x and y.

Value of y is always 0. However, x must satisfy the relation 2x + 9 = 0 i.e. x = - 9/2 = - 4.5

Hence,

- When, y = 0, we get x = - 4.5
- When y = 2, we get x = - 4.5
- When y = -2, we get x = - 4.5

Therefore, plotting the points and on joining them we get the graph AB as a line parallel to the y-axis at a distance of 4.5 units on the left of the y-axis. It is a collection of all points of the plane, having their x-coordinate as 4.5.