Exercise 2.1 Page: 32

Q1. Which of the following expressions are polynomials in one variable, and which are not? State reasons for your answer.

(i) $4x^2 – 3x + 7$

Answer: The equation $4x^2 – 3x + 7$ can be written as $4x^2 – 3x^1 + 7x^0$

Since *x* is the only variable in the given equation and the powers of x (i.e. 2, 1 and 0) are whole numbers, we can say that the expression $4x^2 – 3x + 7$ is a polynomial in one variable.

(ii) $y^2 + \sqrt{2}$

Answer: The equation $y^2+ \sqrt{2}$ can be written as $y^2 + \sqrt{2}y^0$

Since y is the only variable in the given equation and the powers of y (i.e., 2 and 0) are whole numbers, we can say that the expression y2+√2 is a polynomial in one variable.

(iii) $3\sqrt{t} + t\sqrt{2}$

Answer: The equation $3\sqrt{t} + t\sqrt{2}$ can be written as $3t^{1/2} + \sqrt{2}t$

Though *t* is the only variable in the given equation, the power of *t* (i.e., 1/2) is not a whole number. Hence, we can say that the expression $3\sqrt{t}+t\sqrt{2}$ is **not** a polynomial in one variable.

(iv) y+2/y

Answer: The equation y+2/y can be written as $y + 2y^{-1}$

Though *y* is the only variable in the given equation, the power of *y* (i.e., -1) is not a whole number. Hence, we can say that the expression y+2/y is **not** a polynomial in one variable.

(v) $x^{10}+y^3+t^{50}$

Answer: Here, in the equation x10 + y3 + t50

Though the powers, 10, 3, 50, are whole numbers, there are 3 variables used in the expression

$x^{10}+y^3+t^{50}$. Hence, it is not a polynomial in one variable.

Q2. Write the coefficients of x^2 in each of the following:

(i) $2 + x^2 + x$

Answer: The equation 2+x2+x can be written as $2 + (1) x^2 + x$

We know that the coefficient is the number which multiplies the variable.

Here, the number that multiplies the variable x^2 is 1

Hence, the coefficient of x^2 in $2 + x + x$ is 1.

ii) $2 – x^2 + x^3$

Answer: The equation 2–x2+x3 can be written as $2 + (–1) x^2 + x^3$

We know that the coefficient is the number (along with its sign, i.e. – or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is -1

Hence, the coefficient of x^2 in $2 – x^2 + x^3$ is -1.

(iii) (π/2) x2 + x

Answer: The equation $(\pi/2)x^2 + x$ can be written as $(\pi/2)x^2 + x$

We know that the coefficient is the number (along with its sign, i.e. – or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is π/2.

Hence, the coefficient of x^2 in $(\pi/2) x^2 + x$ is π/2.

iv) √2x - 1

Answer: The equation √2x-1 can be written as $0 x^2 + \sqrt{2}x - 1$ [Since $0x^2$ is 0]

We know that the coefficient is the number (along with its sign, i.e. – or +) which multiplies the variable.

Here, the number that multiplies the variable x2is 0

Hence, the coefficient of x2 in √2x-1 is 0.

Q3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.
(i) Abmomial of degree 35 can be $3x^{35}$ - 4.
(ii) A monomial of degree 100 can be $\sqrt{2}y^{100}$.

Answer: Binomial of degree 35: A polynomial having two terms and the highest degree 35 is called a binomial of degree 35.

For example, $3x^{35} + 5$

Monomial of degree 100: A polynomial having one term and the highest degree 100 is called a monomial of degree 100.

For example, $4x^{100}$

4. Write the degree of each of the following polynomials:

(i) $5x^3+4x^2+7x$

Answer:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, 5x3+4x2+7x = 5x3+4x2+7x1

The powers of the variable x are: 3, 2, 1

The degree of 5x3+4x2+7x is 3, as 3 is the highest power of x in the equation.

(ii) $4–y^2$

Answer:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, in $4–y^2$,

The power of the variable y is 2

The degree of $4–y^2$ is 2, as 2 is the highest power of y in the equation.

(iii) $5t–\sqrt{7}$

Answer:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, in $5t–\sqrt{7}$

The power of the variable t is: 1

The degree of $5t–\sqrt{7}$ is 1, as 1 is the highest power of y in the equation.

(iv) 3

Answer:

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, 3 = 3×1 = 3× x0

The power of the variable here is: 0

Hence, the degree of 3 is 0.

Q5. Classify the following as linear, quadratic and cubic polynomials:

Answer:

We know that,

Linear polynomial: A polynomial of degree one is called a linear polynomial.

Quadratic polynomial: A polynomial of degree two is called a quadratic polynomial.

Cubic polynomial: A polynomial of degree three is called a cubic polynomial.

(i) $x^2 + x$

Answer: The highest power of $x^2 + x$ is 2

The degree is 2

Hence, $x^2 + x$ is a quadratic polynomial

(ii) $x – x^3$

Answer: The highest power of $x – x^3$ is 3

The degree is 3

Hence, $x – x^3$ is a cubic polynomial

iii) $y + y^2 + 4$

Answer: The highest power of $y+y^2+4$ is 2

The degree is 2

Hence, $y+y^2+4$ is a quadratic polynomial

(iv) 1+x

Answer: The highest power of 1+x is 1

The degree is 1

Hence, 1+x is a linear polynomial.

(v) 3t

Answer: The highest power of 3t is 1

The degree is 1

Hence, 3t is a linear polynomial.

(vi) r2

Answer: The highest power of r2 is 2

The degree is 2

Hence, r2is a quadratic polynomial.

(vii) $7x^3$

Answer: The highest power of $7x^3$ is 3

The degree is 3

Hence, $7x^3$ is a cubic polynomial.

Exercise 2.2 Page: 34

Q1. Find the value of the polynomial $5x – 4x^2 + 3$ at
(i) x=0
(ii) x=-1
(iii) x=2

Let $p(x) = 5x - 4x^2 + 3$

(i) At $x = 0$; $p(0) = 5(0) - 4(0)^2 + 3$
$= 0 - 0 + 3 = 3$

(ii) At $x = -1$; $p(-1) = 5(-1) - 4(-1)^2 + 3$
$= -5 - 4 + 3 = -6$

(iii) At $x = 2$; $p(2) = 5(2) - 4(2)^2 + 3 = 10 - 16 + 3$
$= 13 - 16 = -3$

Q2. Find p (0), p (1) and p (2) for each of the following polynomials.

(i) $p(y) = y^2 – y + 1$

(ii) $p(t) = 2 + 1 + 2t^2 - t^3$

(iii) $P(x) = x^3$

(iv) p (x) = (x-1) (x+1)

Solution:

(i) Given that $p(y) = y^2 – y + 1$.

$\therefore P(0) = (0)^2 – 0 + 1 = 0 – 0 + 1 = 1$

$p(1) = (1)^2 – 1 + 1 = 1 – 1 + 1 = 1$

$p(2) = (2)^2 – 2 + 1 = 4 – 2 + 1 = 3$

(ii) Given that $p(t) = 2 + t + 2t^2 – t^3$

$\therefore p(0) = 2 + 0 + 2(0)^2 – (0)^3$

$= 2 + 0 + 0 – 0 = 2$

$P(1) = 2 + 1 + 2(1)^2 – (1)^3$

$= 2 + 1 + 2 – 1 = 4$

$p(2) = 2 + 2 + 2(2)^2 – (2)^3$

$= 2 + 2 + 8 – 8 = 4$

(iii) Given that $p(x) = x^3$

$\therefore p(0) = (0)^3 = 0$,

p(1) = (1)3 = 1

$p(2) = (2)^3 = 8$

(iv) Given that $p(x) = (x – 1)(x + 1)$

$\therefore p(0) = (0 – 1)(0 + 1) = (-1)(1) = -1$

$p(1) = (1 – 1)(1 +1) = (0)(2) = 0$

$P(2) = (2 – 1)(2 + 1) = (1)(3) = 3$

Q3. Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x) = 3x + 1, x = -\frac{1}{3}$

(ii) $p (x) = 5x – \pi, x = \frac{4}{5}$

(iii) $p (x) = x^2 – 1, x = x – 1$

(iv) p (x) = (x + 1) (x – 2), x = – 1,2

(v) p (x) = x^2, x = 0

(vi) p (x) = 1x + m, x = $-\frac{m}{1}$

(vii) P (x) = $3x^2 - 1$, x = $-\frac{1}{\sqrt{3}}$V, $\frac{2}{\sqrt{3}}$

(viii) p (x) = 2x + 1, x = $\frac{1}{2}$

Answer:

(i) $p(x) = 3x + 1$

If $x = -\frac{1}{3}$ is zero of $p(x)$, then $p\left(-\frac{1}{3}\right) = 0$

So, $p\left(-\frac{1}{3}\right) = 3 \times \left(-\frac{1}{3}\right) + 1 = -1 + 1 \quad \Rightarrow p\left(-\frac{1}{3}\right) = 0$

Hence, $x = -\frac{1}{3}$ is zero of $p(x)$.

(ii) $p(x) = 5x - \pi$

If $x = \frac{4}{5}$ is zero of $p(x)$, then $p\left(\frac{4}{5}\right) = 0$

So, $p\left(\frac{4}{5}\right) = 5 \times \frac{4}{5} - \pi \quad \Rightarrow p\left(\frac{4}{5}\right) = 4 - \pi$

Hence, $x = \frac{4}{5}$ is not zero of $p(x)$.

(iii) $p(x) = x^2 - 1$

If $x = -1$ and 1 are zeroes of $p(x)$, then $p(-1)$, $p(1) = 0$

So, $p(-1) = (-1)^2 - 1 = 1 - 1 \quad \Rightarrow \quad p(-1) = 0$

and $p(1) = (1)^2 - 1 = 1 - 1 \Rightarrow p(1) = 0$

Hence, $x = -1$ and 1 are zeroes of $p(x)$.

(iv) $p(x) = (x + 1)(x - 2)$

If $x = -1$ and 2 are zeroes of $p(x)$, then $p(-1)$, $p(2) = 0$

So, $p(-1) = (-1 + 1)(-1 - 2) \quad \Rightarrow p(-1) = 0$

and $p(2) = (2 + 1)(2 - 2) = 3(0)$

$\Rightarrow \quad p(2) = 0$

Hence, $x = -1$ and 2 are zeroes of $p(x)$.

(v) $p(x) = x^2$

If $x = 0$ is zero of $p(x)$, then $p(0) = 0$

So, $p(0) = 0^2 \Rightarrow p(0) = 0$

Hence, $x = 0$ is zero of $p(x)$.

(vi) $p(x) = lx + m$

If $x = -\frac{m}{l}$ is zero of $p(x)$, then $p\left(-\frac{m}{l}\right) = 0$

So, $p\left(-\frac{m}{l}\right) = l \times \left(\frac{-m}{l}\right) + m$

$\Rightarrow$ $p\left(-\frac{m}{l}\right) = -m + m \Rightarrow p\left(-\frac{m}{l}\right) = 0$

Hence, $x = -\frac{m}{l}$ is zero of $p(x)$.

(vii) $p(x) = 3x^2 - 1$

If $x = -\frac{1}{\sqrt{3}}$ and $\frac{2}{\sqrt{3}}$ are zeroes of $p(x)$, then

$p\left(-\frac{1}{\sqrt{3}}\right), p\left(\frac{2}{\sqrt{3}}\right) = 0$

So, $p\left(-\frac{1}{\sqrt{3}}\right) = 3\left(-\frac{1}{\sqrt{3}}\right)^2 - 1 \Rightarrow p\left(-\frac{1}{\sqrt{3}}\right) = 1 - 1 = 0$

and $p\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 1 \Rightarrow p\left(\frac{2}{\sqrt{3}}\right) = 4 - 1 = 3$

Hence, $x = -\frac{1}{\sqrt{3}}$ is zero of $p(x)$ and $x = \frac{2}{\sqrt{3}}$ is not zero of $p(x)$.

(viii) $p(x) = 2x + 1$

If $x = \frac{1}{2}$ is zero of $p(x)$, then $p\left(\frac{1}{2}\right) = 0$

So, $p\left(\frac{1}{2}\right) = 2 \times \frac{1}{2} + 1 \Rightarrow p\left(\frac{1}{2}\right) = 1 + 1 = 2$

Hence, $x = \frac{1}{2}$ is not zero of $p(x)$.

Q4. Find the zero of the polynomial in each of the following cases :

(i) p(x) = x + 5

Answer:

p(x) = x + 5

$\Rightarrow x + 5 = 0$

$\Rightarrow x = -5$

∴ -5 is a zero polynomial of the polynomial p(x).

(ii) p(x) = x – 5

Answer:

$p(x) = x - 5$

$\Rightarrow x - 5 = 0$

$\Rightarrow x = 5$

∴ 5 is a zero polynomial of the polynomial p(x).

(iii) p(x) = 2x + 5

Answer:

$p(x) = 2x + 5$

$\Rightarrow 2x + 5 = 0$

$\Rightarrow 2x = -5$

$\Rightarrow x = \frac{-5}{2}$

$\therefore x = \frac{-5}{2}$ is a zero polynomial of the polynomial p(x).

(vi) p(x) = 3x-2

Answer:

$p(x) = 3x-2$

$\Rightarrow 3x-2 = 0$

$\Rightarrow 3x = 2$

$\Rightarrow x = \frac{2}{3}$

∴ x = $\frac{2}{3}$ is a zero polynomial of the polynomial p(x).

(v) p(x) = 3x

Answer:

p(x) = 3x

⇒ 3x = 0

⇒ x = 0

∴ 0 is a zero polynomial of the polynomial p(x).

(vi) p(x) = ax, a≠0

Answer:

p(x) = ax

⇒ ax = 0

⇒ x = 0

∴ x = 0 is a zero polynomial of the polynomial p(x).

(vii) p(x) = cx + d, c≠0, c, d are real numbers.

Answer:

p(x) = cx + d

⇒ cx + d =0

⇒ x = $\frac{-d}{c}$

∴ x = $\frac{-d}{c}$ is a zero polynomial of the polynomial p(x).

<u>Exercise 2.3 Page: 40</u>

Q1. Find the remainder when x3 + 3x2 + 3x + 1 is divided by

(i) x + 1

Answer:

x+1= 0

⇒x = −1

∴ Remainder:

$p(-1) = (-1)^3 + 3(-1)^2 + 3(-1) + 1$

$= -1 + 3 - 3 + 1$

$= 0$

(ii) $x - \frac{1}{2}$ (x = ½)

(iii) x (x = 0)

(iv) x + π (x = - π)

(v) 5 + 2x ($x = \frac{-5}{2}$)

(ii) By remainder theorem, the required remainder is equal to $p\left(\frac{1}{2}\right)$.

Now, $p\left(\frac{1}{2}\right) = x^3 + 3x^2 + 3x + 1$

$= \left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) + 1$

$= \frac{1}{8} + \frac{3}{4} + \frac{3}{2} + 1 = \frac{1 + 6 + 12 + 8}{8} = \frac{27}{8}$

Hence, the required remainder $= p\left(\frac{1}{2}\right) = \frac{27}{8}$

(iii) By remainder theorem, the required remainder is equal to $p(0)$.

Now, $p(x) = x^3 + 3x^2 + 3x + 1$

$\therefore$ $p(0) = 0 + 0 + 0 + 1 = 1$

Hence, the required remainder $= p(0) = 1$

(iv) By remainder theorem the required remainder is $p(-\pi)$.

Now, $p(x) = x^3 + 3x^2 + 3x + 1$

$\therefore$ $p(-\pi) = (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1$

$= -\pi^3 + 3\pi^2 - 3\pi + 1$

(v) By remainder theorem, the required remainder is $p\left(-\frac{5}{2}\right)$.

Now, $p(x) = x^3 + 3x^2 + 3x + 1$

$\therefore$ $p\left(-\frac{5}{2}\right) = \left(-\frac{5}{2}\right)^3 + 3\left(\frac{-5}{2}\right)^2 + 3\left(\frac{-5}{2}\right) + 1$

$= -\frac{125}{8} + \frac{75}{4} - \frac{15}{2} + 1$

$= \frac{-125 + 150 - 60 + 8}{8} = \frac{-27}{8}$

Q2. Find the remainder when x3−ax2+6x−a is divided by x-a.

Answer:

Let p(x) = $x^3 - ax^2 + 6x - a$

x−a = 0

∴ x = a

Remainder:

p(a) = $(a)^3 - a(a^2) + 6(a) - a$

= a3 − a^3+ 6a − a = 5a

Thus, the required remainder is 5a.

Q3.Check whether 7 + 3x is a factor of $3x^3+7x$.

Answer:

$7+3x = 0$

$\Rightarrow 3x = -7$

$\Rightarrow x = \frac{-7}{3}$

Thus zero pf g(x) = $\frac{-7}{3}$

On putting $x = -\frac{7}{3}$ in $f(x)$, we get

$$f\left(\frac{-7}{3}\right) = 3\left(\frac{-7}{3}\right)^3 + 7\left(\frac{-7}{3}\right) = 3\left(-\frac{343}{27}\right) - \frac{49}{3}$$

$$= -\frac{343}{9} - \frac{49}{3} = \frac{-343-147}{9} = \frac{-490}{9}$$

Here, $f\left(\frac{-7}{3}\right) \neq 0$ *i.e.*, the remainder obtained on dividing $f(x)$ by $7+3x$ is not zero.

Hence, $g(x) = 7 + 3x$ is not a factor of $f(x) = 3x^3 - 7x$.

Since,($-\frac{-490}{9}$) ≠ 0

i.e. the remainder is not 0.

∴ $3x^3 + 7x$ is not divisible by 7 + 3x.

Thus, 7 + 3x is not a factor of $3x^3 + 7x$.

Exercise 2.4 Page: 43

Q1. Determine which of the following polynomials has (x + 1) a factor:

(i) x^3+x^2+x+1

Solution:

Let p(x) = x^3+x^2+x+1

The zero of x+1 is -1. [x+1 = 0 means x = -1]

$p(-1) = (-1)^3+(-1)^2+(-1)+1$

$= -1+1-1+1$

$= 0$

∴ By factor theorem, x+1 is a factor of x3+x2+x+1

(ii) $x^4+x^3+x2+x+1$

Solution:

Let p(x)= $x^4+x^3+x2+x+1$

The zero of x+1 is -1. [x+1= 0 means x = -1]

$p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$

$= 1-1+1-1+1$

$= 1 \neq 0$

∴ By factor theorem, x+1 is not a factor of $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4+3x^3+3x^2+x+1$

Solution:

Let p(x)= $x^4 + 3x^3 + 3x^2 + x + 1$

The zero of x+1 is -1.

$p(-1)=(-1)^4 + 3(-1)^3 +3 (-1)^2 +(-1) + 1$

$=1-3 +3 -1+1$

$=1 \neq 0$

∴ By factor theorem, x+1 is not a factor of x4+3x3+3x2+x+1

(iv) Let p(x) = $x^3 - x^2 - (2 + 2\sqrt{2}) x + \sqrt{2}$

Then, p(-1) = (-1)3 – (-1)2 – (2 + √2)(-1) + √2

= —1—1 + 2 + √2 + √2

= 2√2

So, by the Factor theorem (x + 1) is not a factor of x^3 -x^2 -(2 + √2)x + √2

Q2. Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases:

(i) p(x) = $2x^3 + x^2 – 2x – 1$, g(x) = x + 1

Answer:

p(x) = $2x^3+x^2–2x–1$, g(x) = x+1

g(x) = 0

⇒ x + 1 = 0

⇒ x = −1

∴ Zero of g(x) is -1.

Now,

p(−1) = $2(−1)^3+ (−1)^2 −2 (−1) – 1$

= −2+1+2−1

= 0

∴ By factor theorem, g(x) is a factor of p(x).

(ii) p(x)=x3+3x2+3x+1, g(x) = x+2

Solution:

p(x) = x3+3x2+3x+1, g(x) = x+2

g(x) = 0

⇒ x+2 = 0

⇒ x = −2

∴ Zero of g(x) is -2.

Now,

p(−2) = (−2)3+3(−2)2+3(−2)+1

= −8+12−6+1

= −1 ≠ 0

∴ By factor theorem, g(x) is not a factor of p(x).

(iii) p(x) = x^3– 4x^2 + x + 6, g(x) = x – 3

Answer:

p(x) = x^3 – 4x^2 + x + 6, g(x) = x -3

g(x) = 0

⇒ x−3 = 0

⇒ x = 3

∴ Zero of g(x) is 3.

Now,

p(3) = $(3)^3$−4$(3)^2$+(3)+6

= 27−36 + 3 + 6

= 0

∴ By factor theorem, g(x) is a factor of p(x).

3. Find the value of k, if x–1 is a factor of p(x) in each of the following cases:

(i) p(x) = x^2 + x + k

Answer:

If x-1 is a factor of p(x), then p(1) = 0

By Factor Theorem

⇒ $(1)^2$+(1) + k = 0

⇒ 1+1 + k = 0

⇒ 2 + k = 0

⇒ k = −2

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

Answer:

If x-1 is a factor of p(x), then p(1) = 0

$\Rightarrow 2(1)^2 + k(1) + \sqrt{2} = 0$

$\Rightarrow 2 + k + \sqrt{2} = 0$

$\Rightarrow k = -(2 + \sqrt{2})$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

Answer:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow k(1)^2 - \sqrt{2}(1) + 1 = 0$

$\Rightarrow k = \sqrt{2} - 1$

(iv) $p(x) = kx^2 - 3x + k$

Solution:

If x-1 is a factor of p(x), then p(1) = 0

By Factor Theorem

$\Rightarrow k(1)^2 - 3(1) + k = 0$

$\Rightarrow k - 3 + k = 0$

$\Rightarrow 2k - 3 = 0$

$\Rightarrow k = \frac{3}{2}$

Q4. Factorise:

(i) 12x2–7x+1

Answer:

Using the splitting the middle term method, We have to find a number whose sum = -7 and product = 1 × 12 = 12

We get -3 and -4 as the numbers [-3 + -4 = -7 and -3 × -4 = 12]

$12x^2 – 7x + 1 = 12x^2 - 4x -3x + 1$

= 4x (3x-1) -1 (3x-1)

= (4x-1)(3x-1)

(ii) $2x^2+7x+3$

Answer:

Using the splitting the middle term method,

We have to find a number whose sum = 7 and product = 2 × 3 = 6

We get 6 and 1 as the numbers [6+1 = 7 and 6 × 1 = 6]

$2x^2 + 7x + 3 = 2x^2 + 6x + 1x + 3$

= 2x (x+3) +1 (x+3)

= (2x+1) (x+3)

(iii) $6x^2+5x-6$

Answer:

Using the splitting the middle term method,

We have to find a number whose sum = 5 and product = 6 × -6 = -36

We get -4 and 9 as the numbers [-4 + 9 = 5 and -4 × 9 = -36]

$6x^2 +5x -6 = 6x^2 + 9x – 4x –6$

= 3x (2x+3) –2 (2x+3)

= (2x+3) (3x–2)

(iv) $3x^2 –x – 4$

Answer:

Using the splitting the middle term method,

We have to find a number whose sum = -1 and product = 3 × -4 = -12

We get -4 and 3 as the numbers [-4 + 3 = -1 and -4 × 3 = -12]

$3x^2 - x - 4 = 3x^2 - 4x + 3x - 4$

$= x(3x-4) + 1(3x-4)$

$= (3x-4)(x+1)$

5. Factorise:

(i) x^3-2x^2-x+2

Answer:

Let $p(x) = x^3-2x^2-x+2$

Factors of 2 are ±1 and ± 2

Now,

$p(x) = x^3 - 2x^2 - x + 2$

$p(-1) = (-1)^3 - 2(-1)^2 - (-1) + 2$

$= -1 - 2 + 1 + 2$

$= 0$

Therefore, (x+1) is the factor of p(x)

Now, Dividend = Divisor × Quotient + Remainder

$(x+1)(x^2 - 3x + 2) = (x+1)(x^2 - x - 2x + 2)$

$= (x+1)(x(x-1) - 2(x-1))$

$= (x+1)(x-1)(x-2)$

(ii) x^3-3x^2-9x-5

Solution:

Let $p(x) = x^3 - 3x^2 - 9x - 5$

Factors of 5 are ±1 and ±5

By the trial method, we find that

p(5) = 0

So, (x – 5) is factor of p(x)

Now,

$p(x) = x^3 - 3x^2 - 9x - 5$

$p(5) = (5)^3 - 3(5)^2 - 9(5) - 5$

= 125 – 75 – 45 – 5

= 0

Therefore, (x-5) is the factor of p(x)

Now, Dividend = Divisor × Quotient + Remainder

$(x-5)(x^2 + 2x + 1) = (x - 5)(x^2 + x + x + 1)$

$= (x - 5)(x(x+1) + 1(x + 1))$

$= (x-5)(x+1)(x+1)$

$= (x - 5)(x + 1)^2$

(iii) $x^3+13x^2+32x+20$

Solution:

Let $p(x) = x^3 + 13x^2 + 32x + 20$

Factors of 20 are ±1, ±2, ±4, ±5, ±10 and ±20

By the trial method, we find that

p(-1) = 0

So, (x+1) is factor of p(x)

Now,

$p(x) = x^3 + 13x^2 + 32x + 20$

$p(-1) = (-1)^3 + 13(-1)^2 + 32(-1) + 20$

$= -1 + 13 - 32 + 20$

= 0

Therefore, (x+1) is the factor of p(x)

Now, Dividend = Divisor × Quotient +Remainder

$(x + 1)(x^2 + 12x + 20) = (x + 1)(x^2 + 2x + 10x + 20)$

$= (x + 1)\ x(x + 2) + 10(x + 2)$

$= (x + 1)(x + 2)(x + 10)$

(iv) $2y^3 + y^2 - 2y - 1$

Answer:

Let $p(y) = 2y^3 + y^2 - 2y - 1$

Factors $= 2 \times (-1) = -2$ are ± 1 and ± 2

By the trial method, we find that

$p(1) = 0$

So, (y-1) is factor of p(y)

Now,

$p(y) = 2y^3 + y^2 - 2y - 1$

$p(1) = 2(1)^3 + (1)^2 - 2(1) - 1$

$= 2 + 1 - 2 - 1$

$= 0$

Therefore, (y-1) is the factor of p(y)

Now, Dividend = Divisor × Quotient + Remainder

$(y - 1)(2y^2 + 3y + 1) = (y - 1)(2y^2 + 2y + y + 1)$

$= (y - 1)(2y(y + 1) + 1(y + 1))$

$= (y - 1)(2y + 1)(y + 1)$

Exercise 2.5 Page: 48

Q1. Use suitable identities to find the following products:

(i) (x + 4)(x + 10)

Answer:

Using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

[Here, a = 4 and b = 10]

We get,

$(x+4)(x+10) = x^2 + (4 + 10)x + (4 \times 10)$

$= x^2 + 14x + 40$

(ii) (x+8) (x –10)

Answer:

Using the identity, $(x+a)(x+b) = x^2 + (a + b)x + ab$

[Here, a = 8 and b = −10]

We get,

$(x+8)(x-10) = x^2 + (8 + (-10))x + (8 \times (-10))$

$= x^2 + (8-10)x - 80$

$= x^2 - 2x - 80$

(iii) (3x + 4)(3x – 5)

Answer:

Using the identity, $(x+a)(x+b) = x^2 + (a + b)x + ab$

[Here, x = 3x, a = 4 and b = −5]

We get,

$(3x+4)(3x-5) = (3x)^2 + [4+(-5)]3x + 4\times(-5)$

$= 9x^2 + 3x(4-5) - 20$

$= 9x^2 - 3x - 20$

(iv) $(y^2+3/2)(y^2-3/2)$

Answer:

Using the identity, $(x + y)(x - y) = x^2 - y^2$

[Here, $x = y^2$ and $y = 3/2$]

We get,

$(y^2 + 3/2)(y^2 - 3/2) = (y^2)2 - (3/2)^2$

$= y^4 - 9/4$

Q2. Evaluate the following products without multiplying directly:

(i) 103×107

Answer:

$103 \times 107 = (100 + 3) \times (100 + 7)$

Using identity, $[(x + a)(x + b) = x^2 + (a + b)x + ab$

Here, $x = 100$

$a = 3$

$b = 7$

We get, $103 \times 107 = (100 + 3) \times (100 + 7)$

$= (100)^2 + (3+7)\ 100 + (3 \times 7)$

$= 10000+1000+21$

$= 11021$

(ii) 95 ×96

Answer:

$95 \times 96 = (100 - 5) \times (100 - 4)$

Using identity, $[(x - a)(x - b) = x^2 - (a + b)x + ab$

Here, $x = 100$

$a = -5$

b = -4

We get, 95 × 96 = (100 - 5) × (100 - 4)

= $(100)^2$ + 100 (-5+(-4)) + (-5 × -4)

= 10000 – 900 + 20

= 9120

(iii) 104 × 96

Answer:

104 × 96 = (100 + 4) × (100 –4)

Using identity, [(a + b)(a - b)= a^2- b^2]

Here, a = 100

b = 4

We get, 104 × 96 = (100 + 4) × (100 –4)

= $(100)^2$–$(4)^2$

= 10000 – 16

= 9984

Q3. Factorize the following using appropriate identities:

(i) $9x^2 + 6xy + y^2$

Answer:

$9x^2 + 6xy + y^2 = (3x)^2 + (2 \times 3x \times y) + y^2$

Using identity, $x^2 + 2xy + y^2 = (x + y)^2$

Here, x = 3x

y = y

$9x^2 + 6xy + y^2 = (3x)^2 + (2 \times 3x \times y) + y^2$

$= (3x + y)^2$

= (3x + y) (3x +y)

(ii) $4y^2-4y+1$

Answer:

$4y^2-4y+1 = (2y)^2-(2\times2y\times1)+1$

Using identity, $x^2 - 2xy + y^2 = (x - y)^2$

Here, $x = 2y$

$y = 1$

$4y^2 -4y +1 = (2y)^2-(2 \times 2y \times 1) +12$

$= (2y-1)^2$

$= (2y - 1) (2y -1)$

(iii) $x^2 - y^2/100$

Solution:

$x^2-y^2 / 100 = x^2- (y / 10)^2$

Using identity, $x^2-y^2 = (x - y) (x +y)$

Here, $x = x$

$y = y /10$

$x^2 -y^2/100 = x^2 - (y / 10)^2$

$= (x -y / 10)(x + y /10)$

Q4.Expand each of the following, using suitable identities:

(i) $(x + 2y + 4z)^2$

Answer:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz +2zx$

Here, $x = x$

$y = 2y$

$z = 4z$

$(x + 2y + 4z)^2 = x^2 + (2y)^2 + (4z)^2 + (2 \times x \times 2y) + (2 \times 2y \times 4z)+(2 \times 4z \times x)$

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8xz$

(ii) (2x–y+z)2

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x = 2x$

$y = -y$

$z = z$

$(2x - y + z)^2 = (2x)^2 + (-y)^2 + z^2 + (2 \times 2x \times -y) + (2 \times -y \times z) + (2 \times z \times 2x)$

$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$

(iii) $(-2x + 3y + 2z)^2$

Answer:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x = -2x$

$y = 3y$

$z = 2z$

$(-2x + 3y + 2z)^2 = (-2x)^2 + (3y)^2 + (2z)^2 + (2 \times -2x \times 3y) + (2 \times 3y \times 2z) + (2 \times 2z \times -2x)$

$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8xz$

(iv) (3a –7b–c)2

Answer:

Using identity $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x = 3a$

$y = -7b$

$z = -c$

$(3a - 7b - c)^2 = (3a)^2 + (-7b)^2 + (-c)^2 + (2 \times 3a \times -7b) + (2 \times -7b \times -c) + (2 \times -c \times 3a)$

$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ca$

(v) $(-2x + 5y - 3z)^2$

Answer:

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here, $x = -2x$

$y = 5y$

$z = -3z$

$(-2x+5y-3z)^2 = (-2x)^2+(5y)^2+(-3z)^2+(2 \times -2x \times 5y)+(2 \times 5y \times -3z)+(2 \times -3z \times -2x)$

$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx$

(vi) $((1/4)a - (1/2) b + 1)^2$

Answer:

(vi) $$\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2 = \left[\frac{1}{4}a + \left(-\frac{1}{2}b\right) + 1\right]^2$$

$$= \left(\frac{1}{4}a\right)^2 + \left(-\frac{1}{2}b\right)^2 + (1)^2 + 2\left(\frac{1}{4}a\right)\left(-\frac{1}{2}b\right) + 2\left(-\frac{1}{2}b\right)(1) + 2(1)\left(\frac{1}{4}a\right)$$

$$= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{1}{2}a$$

5. Factorize:

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

Answer:

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

We can say that, $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$

$4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz = (2x)^2 + (3y)^2 + (-4z)^2 + (2 \times 2x \times 3y) + (2 \times 3y \times -4z) + (2 \times -4z \times 2x)$

$= (2x + 3y – 4z)^2$

$= (2x + 3y – 4z) (2x + 3y – 4z)$

(ii) 2 x 2 + y^2+ $8z^2$ – 2√2xy + 4√2yz – 8xz

Using identity, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

We can say that, $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$

$2x^2 + y^2 + 8z^2 – 2\sqrt{2}xy + 4\sqrt{2}yz – 8xz$

$= (-\sqrt{2}x)^2+(y)^2+(2\sqrt{2}z)^2+(2 \times -\sqrt{2}x \times y) + (2\times y \times 2\sqrt{2}z) + (2 \times 2\sqrt{2}\times - \sqrt{2}x)$

$= (-\sqrt{2}x + y + 2\sqrt{2}z)^2$

$= (-\sqrt{2}x + y + 2\sqrt{2}z) (-\sqrt{2}x + y + 2\sqrt{2}z)$

6. Write the following cubes in expanded form:

(i) $(2x + 1)^3$

(ii) $(2a - 3b)^3$

(iii) $((3/2) x +1)^3$

(iv) $(x - (2/3)y)^3$

(i) $(2x+1)^3 = (2x)^3 + 1^3 + 3(2x)(1)(2x+1)$

[Using identity $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$]

$= 8x^3 + 1 + 6x(2x+1)$

$= 8x^3 + 1 + 12x^2 + 6x = 8x^3 + 12x^2 + 6x + 1$

(ii) $(2a-3b)^3 = (2a)^3 - (3b)^3 - 3(2a)(3b)(2a-3b)$

[Using identity $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$]

$= 8a^3 - 27b^3 - 18ab(2a-3b)$

$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$

$= 8a^3 - 36a^2b + 54ab^2 - 27b^3$

(iii) $\left(\frac{3}{2}x+1\right)^3 = \left(\frac{3}{2}x\right)^3 + 1^3 + 3\left(\frac{3}{2}x\right)(1)\left(\frac{3}{2}x+1\right)$

[Using identity $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$]

$$= \frac{27}{8}x^3 + 1 + \frac{9}{2}x \times \left(\frac{3}{2}x + 1\right)$$

$$= \frac{27}{8}x^3 + 1 + \frac{27}{4}x^2 + \frac{9}{2}x = \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$$

(iv) $\left(x - \frac{2}{3}y\right)^3 = x^3 - \left(\frac{2}{3}y\right)^3 - 3x\left(\frac{2}{3}y\right)\left(x - \frac{2}{3}y\right)$

[Using identity $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$]

$$= x^3 - \frac{8}{27}y^3 - 2xy\left(x - \frac{2}{3}y\right) = x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$$

$$= x^3 - 2x^2y + \frac{4}{3}xy^2 - \frac{8}{27}y^3$$

Q7.Evaluate the following using suitable identities :

(i) $(99)^3$

(ii) $(102)^3$

(iii) $(998)^3$

Answer:

(i) $(99)^3 = (100-1)^3 = (100)^3 - (1)^3 - 3 \times 100 \times 1(100-1)$

[$\because (a-b)^3 = a^3 - b^3 - 3ab(a-b)$]

$= 1000000 - 1 - 300(100-1)$

$= 1000000 - 1 - 300 \times 99$

$= 1000000 - 29701 = 970299$

(ii) $(102)^3 = (100+2)^3$

$= (100)^3 + (2)^3 + 3 \times 100 \times 2(100+2)$

[$\because (a+b)^3 = a^3 + b^3 + 3ab(a+b)$]

$= 1000000 + 8 + 600(102)$

$= 1000000 + 8 + 61200 = 1061208$

(iii) $(998)^3 = (1000-2)^3 = 1000^3 - 2^3 - 3 \times 1000 \times 2\ (1000-2)$

[$\because (x-y)^3 = x^3 - y^3 - 3xy(x-y)$]

$= 1000000000 - 8 - 6000\ (1000-2)$

$= 1000000000 - 8 - 6000000 + 12000 = 994011992$

8. Factorise each of the following:

(i) $8a^3+b^3+12a2b+6ab2$ (Here, the identity, $(x+y)^3 = x^3+y^3+ 3xy (x+y)$ is used.)

(ii) $8a^3–b^3–12a2b+6ab2$ (Here, the identity,$(x–y)^3 = x^3–y^3–3xy(x–y)$ is used.)

(iii) $27–125a^3–135a +225a^2$ (Here, the identity, $(x–y)^3 = x^3–y^3-3xy(x–y)$ is used.)

(iv) $64a^3–27b^3–144a^2b+108ab^2$ (Here, the identity, $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$ is used.)

(v) $27p^3–(1/216)–(9/2) p^2+(1/4)p$ ($(x – y)^3 = x^3 – y^3 – 3xy (x – y)$)

Answer:

$$\text{(i) } 8a^3 + b^3 + 12a^2b + 6ab^2 = (2a)^3 + (b)^3 + 3(2a)(b)(2a+b)$$
$$= (2a+b)^3$$
$$= (2a+b)(2a+b)(2a+b)$$

$$\text{(ii) } 8a^3 - b^3 - 12a^2b + 6ab^2 = (2a)^3 - b^3 - 3(2a)(b)(2a-b)$$
$$= (2a-b)^3$$
$$= (2a-b)(2a-b)(2a-b)$$

$$\text{(iii) } 27 - 125a^3 - 135a + 225a^2 = (3)^3 - (5a)^3 - 3(3)(5a)(3-5a)$$
$$= (3-5a)^3 = (3-5a)(3-5a)(3-5a)$$

$$\text{(iv) } 64a^3 - 27b^3 - 144a^2b + 108ab^2 = (4a)^3 - (3b)^3 - 3(4a)(3b)(4a-3b)$$
$$= (4a-3b)^3$$
$$= (4a-3b)(4a-3b)(4a-3b)$$

$$\text{(v) } 27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p = (3p)^3 - \left(\frac{1}{6}\right)^3 - 3(3p)\left(\frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$$
$$= \left(3p - \frac{1}{6}\right)^3 = \left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$$

9. Verify:

(i) $x^3+y^3 = (x + y)(x^2– xy + y^2)$

Answer:

We know that, $(x + y)^3 = x^3 + y^3+ 3xy (x + y)$

$\Rightarrow x^3 + y^3 = (x + y)^3 –3xy(x + y)$

$\Rightarrow x^3 + y^3 = (x + y) [(x + y)^2 – 3xy]$

Taking (x + y) common $\Rightarrow x^3 + y^3 = (x + y) [(x^2 + y^2 + 2xy) –3xy]$

$\Rightarrow x^3+ y^3 = (x + y)(x^2 + y^2 – xy)$

= RHS **Hence proved.**

(ii) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

We know that, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$\Rightarrow x^3 - y^3 = (x - y)^3 + 3xy(x - y)$

$\Rightarrow x^3 - y^3 = (x - y)[(x - y)^2 + 3xy]$

Taking (x+y) common $\Rightarrow x^3 - y^3 = (x - y)[(x^2 + y^2 - 2xy) + 3xy]$

$\Rightarrow x^3 + y^3 = (x - y)(x^2 + y^2 + xy)$

= RHS **Hence proved.**

10. Factorize each of the following:

(i) $27y^3 + 125z^3$

(ii) $64m^3 - 343n^3$

Answers:

(i) $27y^3 + 125z^3$

The expression, $27y^3 + 125z^3$ can be written as $(3y)^3 + (5z)^3$

$27y^3 + 125z^3 = (3y)^3 + (5z)^3$

We know that, $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$27y^3 + 125z^3 = (3y)^3 + (5z)^3$

$= (3y + 5z)[(3y)^2 - (3y)(5z) + (5z)^2]$

$= (3y + 5z)(9y^2 - 15yz + 25z^2)$

(ii) $64m^3 - 343n^3$

We know that

$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

We have, $64m^3 - 343n^3 = (4m)^3 - (7n)^3$

$= (4m - 7n)[(4m)^2 + (4m)(7n) + (7n)^2]$

$= (4m - 7n)(16m^2 + 28mn + 49n^2)$

Q11. Factorise $27x^3 + y^3 + z^3 - 9xyz$.

Answer:

We have,

$27x^3 + y^3 + z^3 - 9xyz = (3x)^3 + (y)^3 + (z)^3 - 3\,(3x)\,(y)\,(z)$

Using the identity,

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)\,(x^2 + y^2 + z^2 - xy - yz - zx)$

We have, $(3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z)$

$= (3x + y + z)\,[(3x)^3 + y^3 + z^3 - (3x \times y) - (y \times 2) - (z \times 3x)]$

$= (3x + y + z)\,(9x^2 + y^2 + z^2 - 3xy - yz - 3zx)$

Q12.Verify that $x^3 + y^3 + z^3 - 3xyz = 12\,(x + y + z)\,[(x-y)^2 + (y - z)^2 + (z - x)^2]$

Answer:

R.H.S

$= 12(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

$= 12\,(x + y + 2)[(x^2 + y^2 - 2xy) + (y^2 + z^2 - 2yz) + (z^2 + x^2 - 2zx)]$

$= 12\,(x + y + 2)(x^2 + y^2 + y^2 + z^2 + z^2 + x^2 - 2xy - 2yz - 2zx)$

$= 12\,(x + y + z)[2(x^2 + y^2 + z^2 - xy - yz - zx)]$

$= 2 \text{ x } 12 \text{ x } (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= x^3 + y^3 + z^3 - 3xyz =$ L.H.S.

Hence, verified.

Q13. If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3\,xyz$.

Answer:

Since, $x + y + z = 0$

$\Rightarrow x + y = -z\ (x + y)^3 = (-z)^3$

$\Rightarrow x^3 + y^3 + 3xy(x + y) = -z^3$

$\Rightarrow x^3 + y^3 + 3xy(-z) = -z^3\ [\because x + y = -z]$

$\Rightarrow x^3 + y^3 - 3xyz = -z^3$

$\Rightarrow x^3 + y^3 + z^3 = 3xyz$

Hence, if $x + y + z = 0$, then

$x^3 + y^3 + z^3 = 3xyz$

Hence proved.

Q14.Without actually calculating the cubes, find the value of each of the following

(i) $(-12)^3 + (7)^3 + (5)^3$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Answer:

(i) We have, $(-12)^3 + (7)^3 + (5)^3$

Let $x = -12$, $y = 7$ and $z = 5$.

Then, $x + y + z = -12 + 7 + 5 = 0$

We know that if $x + y + z = 0$, then, $x^3 + y^3 + z^3 = 3xyz$

$\therefore (-12)^3 + (7)^3 + (5)^3 = 3[(-12)(7)(5)]$

$= 3[-420] = -1260$

(ii) We have, $(28)^3 + (-15)^3 + (-13)^3$

Let $x = 28$, $y = -15$ and $z = -13$.

Then, $x + y + z = 28 – 15 – 13 = 0$

We know that if $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$

$\therefore (28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13)$

$= 3(5460) = 16380$

Q15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i) Area : $25a^2 – 35a + 12$

(ii) Area : $35y^2 + 13y – 12$

Answer:

(i) Area : $25a^2 – 35a + 12$

(i) We have,

Area of rectangle = $25a^2 - 35a + 12$ [by splitting the middle term]

Using the splitting the middle term method,

We have to find a number whose sum = -35 and product =25×12=300

We get -15 and -20 as the numbers [-15 + -20 = -35 and -15 × -20=300]

= $25a^2 - 20a - 15a + 12$

= $5a(5a - 4) - 3(5a - 4)$

= $(5a - 4)(5a - 3)$

Possible expression for length = $(5a - 3)$ and

Possible expression for breadth = $(5a - 4)$

(ii) Area : $35y^2+13y-12$

Using the splitting the middle term method,

We have to find a number whose sum = 13 and product = 35 × -12 = 420

We get -15 and 28 as the numbers [-15 + 28 = 13 and -15 × 28 =420]

$35y^2 + 13y - 12 = 35y^2 - 15y + 28y - 12$

= $5y(7y - 3) + 4(7y - 3)$

= $(5y + 4)(7y - 3)$

Possible expression for length = $(5y + 4)$

Possible expression for breadth = $(7y - 3)$

16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i) Volume : $3x^2 – 12x$

(ii) Volume : $12ky^2 + 8ky – 20k$

Answer:

(i) Volume : $3x^2 – 12x$

$3x^2 – 12x$ can be written as $3x(x – 4)$ by taking 3x out of both the terms.

Possible expression for length = 3

Possible expression for breadth = x

Possible expression for height = (x–4)

(ii) Volume:

$12ky^2 + 8ky – 20k$

$12ky^2 + 8ky –20k$ can be written as $4k (3y^2 + 2y –5)$ by taking 4k out of both the terms.

$12ky^2+ 8ky – 20k = 4k (3y^2+2y–5)$

[Here, $3y^2 + 2y –5$ can be written as $3y^2 + 5y – 3y –5$ using splitting the middle term method.]

$= 4k (3y^2 +5y – 3y –5)$

$= 4k [y (3y + 5)–1(3y + 5)]$

$= 4k (3y + 5)(y – 1)$

Possible expression for length = 4k

Possible expression for breadth = (3y +5)

Possible expression for height = (y -1)