Exercise 2.1:

Write the correct answer in each of the following:

1. Which one of the following is a polynomial?

(A) $\frac{x^2}{2} - \frac{2}{x^2}$ (B) $\sqrt{2x} - 1$

(C) $x^2 + \frac{3x^{\frac{3}{2}}}{\sqrt{x}}$ (D) $\frac{x-1}{x+1}$

Answer:

(C)

$$x^2 + \frac{3x^{\frac{2}{3}}}{\sqrt{x}} = x^2 + 3x$$

Explanation:

(A)

$$\frac{x^2}{2} - \frac{2}{x^2} = \frac{x^2}{2} - 2x^{-2}$$

The equation contains the terms x^2 and $-2x^{-2}$.

Here, the exponent of x in the second term = – 2, which is not a whole number.

Hence, the given algebraic expression is not a polynomial.

(B)

$$\sqrt{2x} - 1 = \sqrt{2}x^{\frac{1}{2}} - 1$$

The equation contains the term $\sqrt{2}x^{½}$.

Here, the exponent of x in the first term = ½ , which is not a whole number.

Hence, the given algebraic expression is not a polynomial.

(C)

$$x^2 + \frac{3x^{\frac{2}{3}}}{\sqrt{x}} = x^2 + 3x$$

The equation contains the term x^2 and 3x .

Here, the exponent of x in first term and second term= 2 and 1, respectively, which is a whole number.

Hence, the given algebraic expression is a polynomial.

(D)

$$\frac{x-1}{x+1}$$

The equation is a rational function.

Here, the given equation is not in the standard form of a polynomial.

Hence, the given algebraic expression is not a polynomial.

Hence, option C is the correct answer

2. √2 is a polynomial of degree

(A) 2

(B) 0

(C) 1

(D) ½

Answer:

(B) 0

Explanation:

√2 can be written as $\sqrt{2}x^0$

i.e., $\sqrt{2} = \sqrt{2}x^0$

Therefore, the degree of the polynomial = 0

3. Degree of the polynomial $4x^4 + 0x^3 + 0x^5 + 5x + 7$ is
(a) 4
(b) 5
(c) 3
(d) 7
Answer: (a) 4
$4x^4 + 0x^3 + 0x^5 + 5x + 7 = 4x^4 + 5x + 7$
As we know that the degree of a polynomial is equal to the highest power of variable x.
Here, the highest power of x is 4. Therefore, the degree of the given polynomial is 4.

4. Degree of the zero polynomial is
(A) 0
(B) 1
(C) Any natural number
(D) Not defined

Answer: (D)

In zero polynomial, the coefficient of any power of variable is zero i.e., $0x^2$, $0x^5$ etc. Therefore, we can not exactly determine the highest power of variable, hence cannot define the degree of zero polynomial.

5. If $p(x)= x^2 - 2\sqrt{2}x + 1$, then $p(2\sqrt{2})$ is equal to

(A) 0

(B) 1

(C) 4√2

(D) 8√2 +1

Answer: (B) 1

Explanation: According to the question,

$p(x) = x^2 - 2\sqrt{2}x + 1$

To get $p(2\sqrt{2})$,

We substitute $x = 2\sqrt{2}$,

$p(2\sqrt{2}) = (2\sqrt{2})^2 - (2\sqrt{2} \times (2\sqrt{2})) + 1$

$= (4 \times 2) - (4 \times 2) + 1$

$= 8 - 8 + 1$

$= 1$

6. The value of the polynomial $5x - 4x^2 + 3$, when $x = -1$ is

(A) – 6

(B) 6

(C) 2

(D) – 2

Answer: (A) – 6

Explanation: According to the question,

$p(x) = 5x - 4x^2 + 3$

To get $p(-1)$,

We substitute $x = -1$,

$p(-1) = 5(-1) - 4(-1)^2 + 3$

$= 5(-1) - 4(1) + 3$

$= -5 - 4 + 3$

$= -9 + 3$

$= -6$

7. If $p(x) = x + 3$, then $p(x) + p(-x)$ is equal to
(a) 3
(b) 2x
(c) 0

(d) 6
Answer: (d)

Given p(x) = x + 3, put x = -x in the given equation, we get p(-x) = -x + 3
Now, p(x) + p(-x) = x + 3 + (-x) + 3 = 6

8. Zero of the zero polynomial is

(A) 0

(B) 1

(C) Any real number

(D) Not defined

Answer:

(C) Any real number

Explanation:

A zero polynomial is a constant polynomial whose coefficients are all equal to 0.

Zero of a polynomial is the value of the variable that makes the polynomial equal to zero.

Therefore, zero of the zero polynomial is any real number.

9. Zero of the polynomial p(x) = 2x + 5 is
(a) -2/5
(b) -5/2
(c) 2/5
(d) 5/2
Answer:(b)

Given, p(x) = 2x + 5
For zero of the polynomial, put p(x) = 0
∴ 2x + 5 = 0
⇒ -5/2
Hence, zero of the polynomial p(x) is -5/2.

Q10. One of the zeroes of the polynomial $2x^2 + 7x - 4$ is

(a) 2

(b) ½

(c) -1

(d) -2

Answer:(b)

Let p (x) = $2x^2 + 7x - 4$
= $2x^2 + 8x - x - 4$ [by splitting middle term]
= 2x(x+ 4)-1(x + 4)
= (2x-1)(x+ 4)
For zeroes of p(x), put p(x) = 0
⇒ (2x -1) (x + 4) = 0
⇒ 2x – 1 = 0 and x + 4 = 0
⇒ x = ½ and x = -4
Hence, one of the zeroes of the polynomial p(x) is ½.

11. If $x^{51} + 51$ is divided by x + 1, then the remainder is

(a) 0

(b) 1

(c) 49

(d) 50

Answer:(d)

Let p(x) = $x^{51} + 51$ . ...(i)
When we divide p(x) by x+1, we get the remainder p(-1)
On putting x = -1 in Eq. (i), we get p(-1) = $(-1)^{51} + 51$
= -1 + 51 = 50
Hence, the remainder is 50.

12. If x + 1 is a factor of the polynomial $2x^2 + kx$, then the value of k is

(a) -3

(b) 4

(c) 2

(d)-2

Answer: (c)

Let p(x) = $2x^2 + kx$
Since, (x + 1) is a factor of p(x), then
p(-1)=0

2(-1)2 + k(-1) = 0

⇒ 2 – k = 0

⇒ k = 2

Hence, the value of k is 2.

13. x + 1 is a factor of the polynomial

(a) $x^3 + x^2 – x + 1$

(b) $x^3 + x^2 + x + 1$

(c) $x^4 + x^3 + x^2 + 1$

(d) $x^4 + 3x^3 + 3x^2 + x + 1$

Answer: (b)

Let assume (x + 1) is a factor of $x^3 + x^2 + x + 1$.

So, x = -1 is zero of $x^3 + x^2 + x + 1$

$(-1)^3 + (-1)^2 + (-1) + 1 = 0$

⇒ -1 + 1 – 1 + 1 = 0

⇒ 0 = 0 Hence, our assumption is true.

14. One of the factors of $(25x^2 – 1) + (1 + 5x)^2$ is

(a) 5 + x

(b) 5 – x

(c) 5x -1

(d) 10x

Answer: (d)

Now, $(25x^2 -1) + (1 + 5x)^2$

$= 25x^2 -1 + 1 + 25x^2 + 10x$ [using identity, $(a + b)^2 = a^2 + b^2 + 2ab$]

$= 50x^2 + 10x = 10x (5x + 1)$

Hence, one of the factor of given polynomial is 10x.

15. The value of $249^2 – 248^2$ is

(a) 1^2

(b) 477

(c) 487

(d) 497

Answer:(d)

Now, $249^2 – 248^2 = (249 + 248) (249 – 248)$ [using identity, $a^2 – b^2 = (a – b)(a + b)$]

= 497 × 1 = 497.

16. The factorization of $4x^2 + 8x + 3$ is

(a) $(x + 1)(x + 3)$

(b) $(2x + 1)(2x + 3)$

(c) $(2x + 2)(2x + 5)$

(d) $(2x – 1)(2x – 3)$

Answer: (b)

Now, $4x^2 + 8x + 3 = 4x^2 + 6x + 2x + 3$ [by splitting middle term]

$= 2x(2x + 3) + 1(2x + 3)$

$= (2x + 3)(2x + 1)$

17. Which of the following is a factor of $(x + y)^3 – (x^3 + y^3)$?

(a) $x^2 + y^2 + 2xy$

(b) $x^2 + y^2 – xy$

(c) xy^2

(d) $3xy$

Answer:(d)

Now, $(x+ y)3 – (x^3 + y^3) = (x + y) – (x + y)(x^2 – xy + y^2)$

[using identity, $a^3 + b^3 = (a + b)(a^2 – ab + b^2)$] $= (x + y)[(x + y)^2 - (x^2 – xy + y^2)]$

$= (x+ y)(x^2 + y^2 + 2xy – x^2 + xy – y^2)$

[using identity, $(a + b)^2 = a^2 + b^2 + 2ab$)]

$= (x + y)(3xy)$

Hence, one of the factor of given polynomial is 3xy.

18.The coefficient of x in the expansion of $(x + 3)^3$ is

(a) 1

(b) 9

(c) 18

(d) 27

Answer: (d)

Now, $(x + 3)^3 = x^3 + 3^3 + 3x(3)(x + 3)$

[using identity, $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$]

$= x^3 + 27 + 9x(x + 3)$

$= x^3 + 27 + 9x^2 + 27x$ Hence, the coefficient of x in $(x + 3)^3$ is 27.

19.

If $\frac{x}{y}+\frac{y}{x}=-1$ $(x, y \neq 0)$, the value of x^3-y^3 is

(a) 1

(b) -1

(c) 0

(d) 1/2

Answer:

(C) : We have, $\frac{x}{y}+\frac{y}{x}=-1$

$\Rightarrow \frac{x^2+y^2}{xy}=-1$

$\Rightarrow x^2+y^2+xy=0$ ... (i)

Now, $x^3-y^3=(x-y)(x^2+xy+y^2)$

$=(x-y)\times 0$ [Using (i)]

$=0$

20.

If $49x^2-b=\left(7x+\frac{1}{2}\right)\left(7x-\frac{1}{2}\right)$, then the value of b is

(A) 0

(B) $\frac{1}{\sqrt{2}}$

(C) $\frac{1}{4}$

(D) $\frac{1}{2}$

Answer:

(C) : We have,

$49x^2-b=\left(7x+\frac{1}{2}\right)\left(7x-\frac{1}{2}\right)$

$\Rightarrow 49x^2-(\sqrt{b})^2=\left[(7x)^2-\left(\frac{1}{2}\right)^2\right]$

[$\because (a+b)(a-b)=a^2-b^2$]

$\Rightarrow 49x^2-(\sqrt{b})^2=49x^2-\left(\frac{1}{2}\right)^2$

Comparing both sides, we get $(\sqrt{b})^2=\left(\frac{1}{2}\right)^2$

$\Rightarrow b=\frac{1}{4}$

Exercise 2.2

1. Which of the following expressions are polynomials? Justify your answer:

(i) 8 (ii) $\sqrt{3}x^2 - 2x$ (iii) $1-\sqrt{5x}$

(iv) $\frac{1}{5x^{-2}} + 5x + 7$ (v) $\frac{(x-2)(x-4)}{x}$ (vi) $\frac{1}{x+1}$

(vii) $\frac{1}{7}a^3 - \frac{2}{\sqrt{3}}a^2 + 4a - 7$ (viii) $\frac{1}{2x}$

Answer:

(i) 8

8 can be written as $8x^0$.

i.e., $8 = 8x^0$,

Here, the power of x = 0, which is a whole number.

Hence, 8 is a polynomial.

(ii) $\sqrt{3}x^2 - 2x$

$\sqrt{3}x^2 - 2x$

Here, the power of x are 2 and 1, respectively

2 and 1 both are whole numbers.

Hence, $\sqrt{3}x^2 - 2x$ is a polynomial.

(iii) $1 - \sqrt{(5x)}$

$1 - \sqrt{5}\sqrt{x} = 1 - \sqrt{5}\, x^{½}$

Here, the power of x = ½, which is not a whole number.

Hence, $1 - \sqrt{5x}$ is not a polynomial

(iv)

$$\frac{1}{5x^{-2}} + 5x + 7$$

$1/5x^{-2} + 5x + 7 = 5x^2 + 5x + 7$

Here, the power of x are 2 and 1 respectively

2 and 1 both are whole numbers.

Hence, $1/5x^{-2} + 5x + 7$ is a polynomial.

(v)

$$\frac{(x-2)(x-4)}{x}$$

$((x – 2)(x – 4))/x = (x^2 – 4x – 2x + 8)/x$

$= (x^2 – 6x + 8)/x$

$= x – 6 + (8/x)$

$= x – 6 + 8x^{–1}$

Here, the power of x = – 1, which is not a whole number, but a negative number.

Hence, ((x – 2)(x – 4))/x is not a polynomial

(vi)

$$\frac{1}{x+1}$$

$1/(x+1) = (x+1)^{– 1}$

Here, the power of x is not a whole number.

Hence, 1/(x+1) is not a polynomial

(vii)

$$\frac{1}{7}a^3 - \frac{2}{\sqrt{3}}a^2 + 4a - 7$$

$(1/7)a^3 – (2/\sqrt{3})a^2 + 4a – 7$

Here, the power of a are 3, 2 and 1, respectively

3, 2 and 1 are all whole numbers.

Hence, $(1/7)a^3 – (2/\sqrt{3})a^2 + 4a – 7$ is a polynomial.

(viii)

$$\frac{1}{2x}$$

$1/2x = (x^{-1}/2)$

Here, the power of x = – 1, which is not a whole number, but a negative number.

Hence, 1/2x is not a polynomial

2. Write whether the following statements are True or False. Justify your answer.
(i) A binomial can have atmost two terms
(ii) Every polynomial is a binomial
(iii) A binomial may have degree 5
(iv) Zero of a polynomial is always 0
(v) A polynomial cannot have more than one zero
(vi) The degree of the sum of two polynomials each of degree 5 is always 5.

Answer:
(i) False
Because a binomial has exactly two terms.

(ii) False
Because every polynomial is not a binomial.
e.g., (a)$x^2 + 4x + 3$ [polynomial but not a binomial]
(b) $x^2 + 5$ [polynomial and also a binomial]

(iii) True
Because a binomial is a polynomial whose degree is a whole number which is greater than or equal to one. Therefore, a binomial may have degree 5.

(iv) False
Because zero of a polynomial can be any real number e.g., for p(x) = x – 1, zero of p(x) is 1, which is a real number.

(v) False

Because a polynomial can have any number of zeroes. It depends upon the degree of the polynomial. e.g. for $p(x) = x^2 – 4$, degree is 2, so it has two zeroes i.e., 2 and -2.

(vi) False

Because the sum of any two polynomials of same degree has not always same degree.
e.g., Let $f(x) = x^5 + 2$ and $g(x) = -x^5 + 2x^2$
∴ Sum of two polynomials, $f(x) + g(x) = x^5 + 2 + (-x^5 + 2x^2) = 2x^2 + 2$, which is not a polynomial of degree 5.

Exercise 2.3:

1. **Classify the following polynomials as polynomials in one variable, two variables etc.**

(i) $x^2 + x + 1$
(ii) $y^3 – 5y$
(iii) $xy + yz + zx$
(iv) $x^2 – 2xy + y^2 + 1$

Solution:

(i) $x^2 + x + 1$

Here, the polynomial contains only one variable, i.e., x.

Hence, the given polynomial is a polynomial in **one** variable.

(ii) $y^3 – 5y$

Here, the polynomial contains only one variable, i.e., y.

Hence, the given polynomial is a polynomial in **one** variable.

(iii) $xy + yz + zx$

Here, the polynomial contains three variables, i.e., x, y and z.

Hence, the given polynomial is a polynomial in **three** variables.

(iv) $x^2 – 2xy + y^2 + 1$

Here, the polynomial contains two variables, i.e., x and y.

Hence, the given polynomial is a polynomial in **two** variables.

2. Determine the degree of each of the following polynomials:

(i) $2x – 1$
(ii) $–10$
(iii) $x^3 – 9x + 3x^5$
(iv) $y^3 (1 – y^4)$

Answer:

Degree of a polynomial in one variable = highest power of the variable in an algebraic expression

(i) $2x – 1$

Power of x = 1

The highest power of the variable x in the given expression = 1

Hence, the degree of the polynomial $2x – 1 = 1$

(ii) $–10$

There is no variable in the given term.

Let us assume that the variable in the given expression is x.

$– 10 = –10x^0$

Power of x = 0

The highest power of the variable x in the given expression = 0

Hence, the degree of the polynomial $– 10 = 0$

(iii) $x^3 – 9x + 3x^5$

Powers of x = 3, 1 and 5, respectively.

The highest power of the variable x in the given expression = 5

Hence, the degree of the polynomial $x^3 – 9x + 3x^5= 5$

(iv) $y^3 (1 – y^4)$

The equation can be written as,

$y^3 (1 – y^4) = y^3 – y^7$

Powers of y = 3 and 7, respectively.

The highest power of the variable y in the given expression = 7

Hence, degree of the polynomial $y^3 (1 – y^4) = 7$

3. For the polynomial

$$\frac{x^3+2x+1}{5}-\frac{7}{2}x^2-x^6, \text{write}$$ **, write**

(i) the degree of the polynomial
(ii) the coefficient of x^3
(iii) the coefficient of x^6
(iv) the constant term

Answer:

The given polynomial is

$$\frac{(x^3 + 2x + 1)}{5}-\frac{7}{2}x^2-x^6$$

(i)Powers of x = 3, 1, 2 and 6, respectively.

The highest power of the variable x in the given expression = 6

Hence, the degree of the polynomial = 6

(ii) The given equation can be written as,

$$\frac{(x^3 + 2x + 1)}{5}-\frac{7}{2}x^2-x^6 = \frac{1}{5}x^3+\frac{2}{5}x+\frac{1}{5}-\frac{7}{2}x^2-x^6$$

Hence, the coefficient of x^3 in the given polynomial is 1/5.

(iii) The coefficient of x^6 in the given polynomial is – 1

(iv) Since the given equation can be written as,

$$\frac{(x^3 + 2x + 1)}{5}-\frac{7}{2}x^2-x^6 = \frac{1}{5}x^3+\frac{2}{5}x+\frac{1}{5}-\frac{7}{2}x^2-x^6$$

The constant term in the given polynomial is 1/5 as it has no variable x associated with it.

4. Write the coefficient of x^2 in each of the following:

(i) $(\pi/6)x + x^2 - 1$
(ii) $3x - 5$
(iii) $(x - 1)(3x - 4)$
(iv) $(2x - 5)(2x^2 - 3x + 1)$

Solution:

(i) $(\pi/6)x + x^2 - 1$

$(\pi/6)x + x^2 - 1 = (\pi/6)x + (1)x^2 - 1$

The coefficient of x^2 in the polynomial $(\pi/6)x + x^2 - 1 = 1$.

(ii) $3x - 5$

$3x - 5 = 0x^2 + 3x - 5$

The coefficient of x^2 in the polynomial $3x - 5 = 0$, zero.

(iii) $(x - 1)(3x - 4)$

$(x - 1)(3x - 4) = 3x^2 - 4x - 3x + 4$

$= 3x^2 - 7x + 4$

The coefficient of x^2 in the polynomial $3x^2 - 7x + 4 = 3$.

(iv) $(2x - 5)(2x^2 - 3x + 1)$

$(2x - 5)(2x^2 - 3x + 1)$

$= 4x^3 - 6x^2 + 2x - 10x^2 + 15x - 5$

$= 4x^3 - 16x^2 + 17x - 5$

The coefficient of x^2 in the polynomial $(2x - 5)(2x^2 - 3x + 1) = -16$

5. Classify the following as a constant, linear, quadratic and cubic polynomial:

(i) $2 - x^2 + x^3$

(ii) $3x^3$
(iii) $5t - \sqrt{7}$

(iv) $4 - 5y^2$

(v) 3

(vi) 2 + x

(vii) $y^3 - y$

(viii) $1 + x + x^2$

(ix) t^2

(x) $\sqrt{2}x - 1$

Answer:

Constant polynomials: The polynomial of the degree zero.

Linear polynomials: The polynomial of degree one.

Quadratic polynomials: The polynomial of degree two.

Cubic polynomials: The polynomial of degree three.

(i) $2 - x^2 + x^3$

Powers of x = 2 and 3, respectively.

The highest power of the variable x in the given expression = 3

Hence, the degree of the polynomial = 3

Since it is a polynomial of degree 3, it is a cubic polynomial.

(ii) $3x^3$

Power of x = 3.

The highest power of the variable x in the given expression = 3

Hence, the degree of the polynomial = 3

Since it is a polynomial of degree 3, it is a cubic polynomial.

(iii) $5t - \sqrt{7}$

Power of t = 1.

The highest power of the variable t in the given expression = 1

Hence, the degree of the polynomial = 1

Since it is a polynomial of degree 1, it is a linear polynomial.

(iv) $4 – 5y^2$

Power of y = 2.

The highest power of the variable y in the given expression = 2

Hence, the degree of the polynomial = 2

Since it is a polynomial of degree 2, it is a quadratic polynomial.

(v) 3

There is no variable in the given expression.

Let us assume that x is the variable in the given expression.

3 can be written as $3x^0$.

i.e., $3 = x^0$

Power of x = 0.

The highest power of the variable x in the given expression = 0

Hence, the degree of the polynomial = 0

Since it is a polynomial of the degree 0, it is a constant polynomial.

(vi) 2 + x

Power of x = 1.

The highest power of the variable x in the given expression = 1

Hence, the degree of the polynomial = 1

Since it is a polynomial of degree 1, it is a linear polynomial.

(vii) $y^3 – y$

Powers of y = 3 and 1, respectively.

The highest power of the variable x in the given expression = 3

Hence, the degree of the polynomial = 3

Since it is a polynomial of degree 3, it is a cubic polynomial.

(viii) $1 + x + x^2$

Powers of x = 1 and 2, respectively.

The highest power of the variable x in the given expression = 2

Hence, the degree of the polynomial = 2

Since it is a polynomial of degree 2, it is a quadratic polynomial.

(ix) t^2

Power of t = 2.

The highest power of the variable t in the given expression = 2

Hence, the degree of the polynomial = 2

Since it is a polynomial of degree 2, it is a quadratic polynomial.

(x) $\sqrt{2}x - 1$

Power of x = 1.

The highest power of the variable x in the given expression = 1

Hence, the degree of the polynomial = 1

Since it is a polynomial of degree 1, it is a linear polynomial.

6.Give an example of a polynomial, which is
(i) monomial of degree 1.
(ii) binomial of degree 20.
(iii) trinomial of degree 2.

Answer:
(i) The example of monomial of degree 1 is 3x.
(ii) The example of binomial of degree 20 is $3x^{20} + x^{10}$
(iii) The example of trinomial of degree 2 is $x^2 - 4x + 3$

7. Find the value of the polynomial $3x^3 - 4x^2 + 7x - 5$, when x = 3 and also when x = –3.

Answer:

Given that,

p(x) = $3x^3 - 4x^2 + 7x - 5$

According to the question,

When x = 3,

p(x) = p(3)

p(x) = $3x^3 - 4x^2 + 7x - 5$

Substituting x = 3,

p(3)= $3(3)^3 - 4(3)^2 + 7(3) - 5$

p(3) = $3(3)^3 - 4(3)^2 + 7(3) - 5$

= 3(27) – 4(9) + 21 – 5

= 81 – 36 + 21 – 5

= 102 – 41

= 61

When x = – 3,

p(x) = p(– 3)

p(x) = $3x^3 - 4x^2 + 7x - 5$

Substituting x = – 3,

p(– 3)= $3(-3)^3 - 4(-3)^2 + 7(-3) - 5$

p(– 3) = $3(-3)^3 - 4(-3)^2 + 7(-3) - 5$

= 3(–27) – 4(9) – 21 – 5

= –81 – 36 – 21 – 5

= –143

8. If p(x) =x^2 – 4x + 3, evaluate: p(2)– p(−1) + p(½).

Given that,

p(x) =x^2 – 4x + 3

According to the question,

When x = 2,

p(x) = p(2)

p(x) =x^2 – 4x + 3

Substituting x = 2,

p(2) = $(2)^2$ – 4(2) + 3

= 4 – 8 + 3

= – 4 + 3

= – 1

When x = – 1,

p(x) = p(– 1)

p(x) =x^2 – 4x + 3

Substituting x = – 1,

p(– 1) = $(– 1)^2$ – 4(– 1) + 3

= 1 + 4 + 3

= 8

When x = ½ ,

p(x) = p(½)

p(x) =x^2 – 4x + 3

Substituting x = ½,

p(½) = $(½)^2$ – 4(½) + 3

= ¼ – 2 + 3

= ¼ + 1

= 5/4

Now,

p(2)– p(−1) + p(½) = – 1 – 8 + (5/4)

= – 9 + (5/4)

= (– 36 + 5)/4

= – 31/4

9. Find p(0), p(1),p(−2) for the following polynomials:

(i) (x)=10x−4x^2 −3

(ii) (y)=(y + 2) (y – 2)

Solution:

(i) According to the question,

p(x) = 10x−4x^2 −3

When x = 0,

p(x) = p(0)

Substituting x = 0,

p(0) = 10(0)−4$(0)^2$ −3

= 0 – 0 – 3

= – 3

When x = 1,

p(x) = p(1)

Substituting x = 1,

p(1) = 10(1)−4$(1)^2$ −3

= 10 – 4 – 3

= 6 – 3

= 3

When x = – 2,

p(x) = p(– 2)

Substituting x = – 2,

$p(-2) = 10(-2) - 4(-2)^2 - 3$

= – 20 – 16 – 3

= – 36 – 3

= – 39

(ii) According to the question,

p(y)=(y + 2) (y – 2)

When y = 0,

p(y) = p(0)

Substituting y = 0,

p(0) =(0 + 2) (0 – 2)

= (2)(– 2)

= – 4

When y = 1,

p(y) = p(1)

Substituting y = 1,

p(1) =(1 + 2) (1 – 2)

=(3) (– 1)

= – 3

When y = – 2,

p(y) = p(– 2)

Substituting y = – 2,

p(– 2) =(– 2 + 2) (– 2 – 2)

= (0) (– 4)

= 0

10. Verify whether the following are true or false.
(i) -3 is a zero of at – 3
(ii) -1/3 is a zero of 3x + 1
(iii) -4/5 is a zero of 4 – 5y
(iv) 0 and 2 are the zeroes of $t^2 - 2t$
(v) -3 is a zero of $y^2 + y - 6$

Answer:
(i) False
Put x – 3 = 0 ⇒ x = 3
Hence, zero of x – 3 is 3.

(ii) True
Put 3x + 1 = 0 ⇒ x = -1/3
Hence, zero of 3x + 1 is -1/5.

(iii) False
Put 4 – 5y = 0 ⇒ y = 4/5
Hence, zero of 4 – 5y is 4/5.

(iv) True
Put $t^2 - 2t = 0 \Rightarrow t(t - 2) = 0$
⇒ t = 0 and t – 2 = 0
⇒ t = 0 and t = 2
Hence, the zeroes of $t^2 - 2t$ are 0 and 2.

(v) True
Put $y^2 + y - 6 = 0 \Rightarrow y^2 + 3y - 2y - 6 = 0$
⇒ y(y + 3) – 2(y + 3) = 0
= (y-2)(y + 3) = 0
⇒ y – 2 = 0 and y + 3 = 0

⇒ $y = 2$ and $y = -3$

Hence, the zeroes of $y^2 + y - 6$ are 2 and – 3.

11. Find the zeroes of the polynomial in each of the following,

(i) $p(x)= x - 4$

(ii) $g(x)= 3 - 6x$

(iii) $q(x) = 2x - 7$

(iv) $h(y) = 2y$

Answer:

(i) Given, polynomial is

$p(x) = x - 4$

For zero of polynomial, put $p(x) = 0$

$\therefore x - 4 = 0 \Rightarrow x = 4$

Hence, zero of polynomial is 4.

(ii) Given, polynomial is

$g(x) = 3 - 6x$

For zero of polynomial, put $g(x) = 0$

$\therefore 3 - 6x = 0 \Rightarrow x = 3/6 = 1/2$.

Hence, zero of polynomial is X

(iii) Given, polynomial is $q(x) = 2x - 7$ For zero of polynomial, put $q(x) = 0$

$\therefore 2x - 7 = 0 \Rightarrow 2x = 7 \Rightarrow x = 7/2$

Hence, zero of polynomial $q(x)$ is $7/2$

(iv) Given polynomial $h(y) = 2y$

For zero of polynomial, put $h(y) = 0$

$\therefore 2y = 0 \Rightarrow y = 0$

Hence, the zero of polynomial $h(y)$ is 0.

12. Find the zeroes of the polynomial:

$p(x)= (x-2)^2-(x+2)^2$

Answer:

$p(x) = (x-2)^2-(x+2)^2$

We know that,

Zero of the polynomial $p(x) = 0$

Hence, we get,

$\Rightarrow (x-2)^2-(x+2)^2 = 0$

Expanding using the identity, $a^2 - b^2 = (a - b)(a + b)$

$\Rightarrow (x - 2 + x + 2)(x - 2 - x - 2) = 0$

$\Rightarrow 2x(-4) = 0$

$\Rightarrow -8x = 0$

Therefore, the zero of the polynomial = 0

13. By actual division, find the quotient and the remainder when the first polynomial is divided by the second polynomial $x^4 + 1$ and $x - 1$.

Answer: Using long division method

$x - 1 \overline{)\, x^4 + 1 \,(} \; x^3 + x^2 + x + 1$

$x^4 - x^3$

$- \quad +$

$x^3 + 1$

$x^3 - x^2$

$- \quad +$

$x^2 + 1$

$x^2 - x$

$- \quad +$

$x + 1$

$x - 1$

$- \quad +$

2

Hence, quotient = $x^3 + x^2 + x + 1$ and remainder = 2

14. By remainder theorem, find the remainder when p(x) is divided by g(x)

(i) $p(x) = x^3 - 2x^2 - 4x - 1$, $g(x) = x + 1$

(ii) $p(x) = x^3 - 3x^2 + 4x + 50$, $g(x) = x - 3$

(iii) $p(x) = x^3 - 12x^2 + 14x - 3$, $g(x) = 2x - 1 - 1$

(iv) $p(x) = x^3 - 6x^2 + 2x - 4$, $g(x) = 1 - (3/2)x$

Answer:

(i) We have, $p(x) = x^3 - 2x^2 - 4x - 1$ and $g(x) = x + 1$

Here, zero of g(x) is -1.

When we divide p(x) by g(x) using remainder theorem, we get the remainder p(-1)
$\therefore$ p(-1) = $(-1)^3 - 2(-1)^2 - 4(-1) - 1$
= -1 – 2 + 4 – 1 = 0
Therefore, remainder is 0.

(ii) We have, p(x) = $x^3 - 3x^2 + 4x + 50$ and g(x) = x – 3
Here, zero of g(x) is 3.
When we divide p(x) by g(x) using remainder theorem, we get the remainder p(3)
$\therefore$ p(3) = $(3)^3 - 3(3)^2 + 4(3) + 50$
= 27 – 27 + 12 + 50 = 62
Therefore, remainder is 62.

(iii) We have, p(x) = $4x^3 - 12x^2 + 14x - 3$ and g(x) = 2x -1
Here, zero of g(x) is 1/2

When we divide $p(x)$ by $g(x)$ using remainder theorem, we get the remainder $p\left(\frac{1}{2}\right)$.

$$\therefore\ p\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + 14\left(\frac{1}{2}\right) - 3$$
$$= 4\times\frac{1}{8} - 12\times\frac{1}{4} + 14\times\frac{1}{2} - 3$$
$$= \frac{1}{2} - 3 + 7 - 3 = \frac{1}{2} + 1 = \frac{3}{2}$$

Therefore, remainder is $\frac{3}{2}$.

15. Check whether p(x) is a multiple of g(x) or not:

(i) p(x) = $x^3 - 5x^2 + 4x - 3$, g(x) = $x - 2$
(ii) p(x)= $2x^3 - 11x^2 - 4x + 5$, $g(x)$= $2x + 1$

Answer:

(i) According to the question,

g(x)=x – 2,

Then, zero of g(x),

g(x) = 0

x – 2 = 0

x = 2

Therefore, zero of g(x) = 2

So, substituting the value of x in p(x), we get,

$p(2) = (2)^3 - 5(2)^2 + 4(2) - 3$

$= 8 - 20 + 8 - 3$

$= -7 \neq 0$

Hence, p(x) is not the multiple of g(x), the remainder ≠ 0.

(ii) According to the question,

$g(x) = 2x + 1$

Then, zero of g(x),

$g(x) = 0$

$2x + 1 = 0$

$2x = -1$

$x = -½$

Therefore, zero of g(x) = – ½

So, substituting the value of x in p(x), we get,

$p(-½) = 2 \times (-½)^3 - 11 \times (-½)^2 - 4 \times (-1/2) + 5$

$= -¼ - 11/4 + 7$

$= 16/4$

$= 4 \neq 0$

Hence, p(x) is not the multiple of g(x), the remainder ≠ 0.

16. Show that:

(i) $x + 3$ is a factor of $69 + 11x - x^2 + x^3$.
(ii) $2x-3$ is a factor of $x + 2x^3 - 9x^2 + 12$

Answer:

(i)According to the question,

Let $p(x) = 69 + 11x - x^2 + x^3$ and $g(x) = x + 3$

$g(x) = x + 3$

zero of $g(x) \Rightarrow g(x) = 0$

$x + 3 = 0$

$x = -3$

Therefore, zero of $g(x) = -3$

So, substituting the value of x in p(x), we get,

$p(-3) = 69 + 11(-3) -(-3)^2 + (-3)^3$

$= 69 - 69$

$= 0$

Since, the remainder = zero,

We can say that,

$g(x) = x + 3$ is factor of $p(x) = 69 + 11x - x^2 + x^3$

(ii) According to the question,

Let $p(x) = x + 2x^3 - 9x^2 + 12$ and $g(x) =2x-3$

$g(x) = 2x - 3$

zero of $g(x) \Rightarrow g(x) = 0$

$2x - 3 = 0$

$x = 3/2$

Therefore, zero of $g(x) = 3/2$

So, substituting the value of x in p(x), we get,

$P(3/2) = 3/2 + 2(3/2)^3 - 9(3/2)^2 + 12$

$= (81 - 81) / 4$

= 0

Since, the remainder = zero,

We can say that,

g(x) = 2x – 3 is factor of p(x) = x + $2x^3 - 9x^2$ + 12

17. Determine which of the following polynomials has x – 2 a factor:

(i) $3x^2 + 6x - 24$.
(ii) $4x^2 + x - 2$.

Answer:

(i) According to the question,

Let p(x) =$3x^2 + 6x - 24$ and g(x) = x – 2

g(x) = x – 2

zero of g(x) ⇒ g(x) = 0

x – 2 = 0

x = 2

Therefore, zero of g(x) = 2

So, substituting the value of x in p(x), we get,

p(2) = $3(2)^2$ + 6 (2) – 24

= 12 + 12 – 24

= 0

Since, the remainder = zero,

We can say that,

g(x) = x – 2 is factor of p(x) = $3x^2 + 6x - 24$

(ii) According to the question,

Let p(x) = $4x^2 + x - 2$ and g(x) = x – 2 g(x) = x – 2

zero of g(x) ⇒ g(x) = 0

x – 2 = 0

x = 2

Therefore, zero of g(x) = 2

So, substituting the value of x in p(x), we get,

p(2) = $4(2)^2$ + 2−2

= 16 ≠ 0

Since the remainder ≠ zero,

We can say that,

g(x) = x – 2 is not a factor of p(x) = $4x^2 + x - 2$

18. Show that p – 1 is a factor of p^{10} – 1 and also of p^{11} – 1.

Answer:

According to the question,

Let h(p) = p^{10} – 1,and g(p) = p – 1

zero of g(p) ⇒ g(p) = 0

p – 1 = 0

p = 1

Therefore, zero of g(x) = 1

We know that,

According to factor theorem if g(p) is a factor of h(p) , then h(1) should be zero

So,

h(1) = $(1)^{10}$ − 1 = 1 − 1 = 0

⟹ g (p) is a factor of h(p).

Now, we have h(p) = $p^{11} - 1$, g (p) = $p - 1$

Putting g (p) = 0 ⟹ $p - 1 = 0$ ⟹ $p = 1$

According to the factor theorem, if g (p) is a factor of h(p),

Then h(1) = 0

⟹ $(1)^{11} - 1 = 0$

Therefore, g(p) = $p - 1$ is the factor of h(p) = $p^{10} - 1$

19. For what value of m is $x^3 - 2mx^2 + 16$ divisible by x + 2?

Answer:

According to the question,

Let p(x) = $x^3 - 2mx^2 + 16$, and g(x) = x + 2

g(x) = 0

⟹ x + 2 = 0

⟹ x = – 2

Therefore, zero of g(x) = – 2

We know that,

According to the factor theorem,

if p(x) is divisible by g(x), then the remainder p(−2) should be zero.

So, substituting the value of x in p(x), we get,

p(– 2) = 0

⟹ $(-2)^3 - 2m(-2)^2 + 16 = 0$

⟹ 0 – 8 – 8m + 16 = 0

⟹ 8m = 8

⟹ m = 1

Q20. If x + 2a is a factor of $a^5 – 4a^2x^3 + 2x + 2a + 3$, then find the value of a.

Answer:

Let $p(x) = a^5 – 4a^2x^3 + 2x + 2a + 3$
Since, x + 2a is a factor of p(x), then put p(-2a) = 0
$\therefore (-2a)^5 – 4a^2 (-2a)^3 + 2(-2a) + 2a + 3 = 0$
$\Rightarrow -32a^5 + 32a^5 – 4a + 2a + 3 = 0$
$\Rightarrow -2a + 3 = 0$
2a = 3
a = 3/2.
Hence, the value of a is 3/2.

21. Find the value of m, so that 2x -1 be a factor of
$8x^4 + 4x^3 – 16x^2 + 10x + m$

Answer:

Let $p(x) = 8x^4 + 4x^3 – 16x^2 + 10x + m$
Since 2x – 1 is afactor of p(x) then p(1/2) = 0

$$\therefore\ 8\left(\frac{1}{2}\right)^4 + 4\left(\frac{1}{2}\right)^3 - 16\left(\frac{1}{2}\right)^2 + 10\left(\frac{1}{2}\right) + m = 0$$

$$\Rightarrow 8\times\frac{1}{16} + 4\times\frac{1}{8} - 16\times\frac{1}{4} + 10\left(\frac{1}{2}\right) + m = 0$$

$$\Rightarrow \frac{1}{2} + \frac{1}{2} - 4 + 5 + m = 0$$

$$\Rightarrow 6 - 4 + m = 0$$

$$\therefore\ m = -2$$

22. If x + 1 is a factor of $ax^3 + x^2 – 2x + 4o – 9$, find the value of a.

Answer:
Let $p(x) = ax^3 + x^2 – 2x + 4a – 9$
Since, x + 1 is a factor of p(x), then p(-1) = 0
$a(-1)^3 + (-1)^2 – 2(-1) + 4a – 9 = 0$
$\Rightarrow -a + 1 + 2 + 4a – 9 = 0$
$\Rightarrow 3a = 6$
$\Rightarrow a = 2$

Q23. Factorise:

(i) $x^2 + 9x + 18$

(ii) $6x^2 + 7x - 3$

(iii) $2x^2 - 7x - 15$

(iv) $84 - 2r - 2r^2$

Answer:

(i) We have, $x^2 + 9x + 18 = x^2 + 6x + 3x + 18$

$= x(x + 6) + 3(x + 6) = (x + 3)(x + 6)$

(ii) We have, $6x^2 + 7x - 3 = 6x^2 + 9x - 2x - 3$

$= 3x(2x + 3) - 1(2x + 3) = (3x - 1)(2x + 3)$

(iii) We have, $2x^2 - 7x - 15 = 2x^2 - 10x + 3x - 15$

$= 2x(x - 5) + 3(x - 5) = (2x + 3)(x - 5)$

(iv) We have, $84 - 2r - 2r^2 = -2(r^2 + r - 42)$

$= -2(r^2 + 7r - 6r - 42)$

$= -2[r(r + 7) - 6(r + 7)]$

$= 2(6 - r)(r + 7)$ or $2(6 - r)(7 + r)$

Q24.Factorise:

(i) $2x^3 - 3x^2 - 17x + 30$

(ii) $x^3 - 6x^2 + 11x - 6$

(iii) $x^3 + x^2 - 4x - 4$

(iv) $3x^3 - x^2 - 3x + 1$

Answer:

(i) We have, $2X^3 - 3x^2 - 17x + 30$

$= 2x^3 - 4x^2 + x^2 - 2x - 15x + 30$

$= 2x^2(x - 2) + x(x - 2) - 15(x - 2)$

$= (x - 2)(2x^2 + x - 15)$

$= (x - 2)(2x^2 + 6x - 5x - 15)$

$= (x - 2)[2x(x + 3) - 5(x + 3)]$

$= (x - 2)(x + 3)(2x - 5)$

(ii)We have, $x^3 - 6x^2 + 11x - 6$

$= x^3 - x^2 - 5x^2 + 5x + 6x - 6$

$= x^2(x - 1) - 5x (x - 1) + 6(x - 1)$
$= (x -1) (x^2 - 5x + 6)$
$= (x - 1) (x^2 - 3x - 2x + 6)$
$= (x - 1) [x(x - 3) - 2(x - 3)]$
$= (x-1)(x-2)(x-3)$

(iii) We have, $x^3 + x^2 - 4x - 4$
$= x^2(x + 1) - 4(x + 1)$
$= (x + 1)(x^2 - 4)$
$= (x + 1) (x - 2) (x + 2)[\therefore a^2 - b^2 = (a - b) (a + b)]$

(iv) We have, $3x^3 - x^2 - 3x + 1 = 3x^3 - 3x^2 + 2x^2 - 2x - x + 1$
$= 3x^2 (x - 1) + 2x(x - 1) -1(x - 1)$
$= (x - 1)(3x^2 + 2x - 1)$
$= (x - 1) (3x^2 + 3x - x - 1)$
$= (x - 1) [3x(x + 1) - 1(x + 1)]$
$= (x - 1) (x + 1)(3x -1)$

25. Using suitable identity, evaluate the following:
(i) 103^3
(ii) 101×102
(iii) 999^2

Answer:
(i) We have, $103^3 = (100 + 3)^3$
$= (100)^3 + (3)^3 + 3(100)(3)(100 + 3)$
$[\therefore (a + b)^3 = a^3 + b^3 + 3ab(a + b)]$
$= 1000000 + 27 + 900(103)$
$= 1000027 + 92700 = 1092727$
(ii) We have, $101 \times 102 = (100 + 1) (100 + 2)$
$= (100)^2 + (1 + 2)100 + (1)(2)$
$[\therefore (x + a)(x + b) = x^2 + (a + b)x + ab]$
$= 10000 + 300 + 2 = 10302$

(iii) We have, $(999)^2 = (1000 -1)^2$
$= (1000)^2 + (1)^2 - 2(1000)(1)$
$[\therefore (a - b)^2 = a^2 + b^2 - 2ab]$
$= 1000000 + 1 - 2000 = 998001$

26.

(i) $4x^2 + 20x + 25$ **(ii)** $9y^2 - 66yz + 121z^2$

(iii) $\left(2x + \frac{1}{3}\right)^2 - \left(x - \frac{1}{2}\right)^2$

Answer:

(i) We have, $4x^2 + 20x + 25$

$= (2x)^2 + 2 \times 2x \times 5 + (5)^2$

$= (2x + 5)^2$ $[\because a^2 + 2ab + b^2 = (a + b)^2]$

(ii) We have, $9y^2 - 66yz + 121z^2$

$= (3y)^2 - 2 \times 3y \times 11z + (11z)^2$

$= (3y - 11z)^2$ $[\because a^2 - 2ab + b^2 = (a - b)^2]$

(iii) We have, $\left(2x + \frac{1}{3}\right)^2 - \left(x - \frac{1}{2}\right)^2$

$= \left[\left(2x + \frac{1}{3}\right) - \left(x - \frac{1}{2}\right)\right]\left[\left(2x + \frac{1}{3}\right) + \left(x - \frac{1}{2}\right)\right]$

$[\because a^2 - b^2 = (a - b)(a + b)]$

$= \left(2x - x + \frac{1}{3} + \frac{1}{2}\right)\left(2x + x + \frac{1}{3} - \frac{1}{2}\right)$

$= \left(x + \frac{5}{6}\right)\left(3x - \frac{1}{6}\right)$

27. Factorise the following:

(i) $9x^2 - 12x + 3$

(ii) $9x^2 - 12x + 4$

Answer:

(i) We have, $9x^2 - 12x + 3 = 3(3x^2 - 4x + 1)$

$= 3(3x^2 - 3x - x + 1)$

$= 3[3x(x - 1) - 1(x - 1)] = 3(3x - 1)(x - 1)$

(ii) We have, $9x^2 - 12x + 4$

$= (3x)^2 - 2 \times 3x \times 2 + (2)^2$

$= (3x - 2)^2$ $[\therefore a^2 - 2ab + b^2 = (a - b)^2]$

$= (3x - 2)(3x - 2)$

28. Expand the following:
(i) $(4a – b + 2c)^2$
(ii) $(3a - 5b -c)^2$
(ii) $(-x + 2y – 3z)^2$

Answer:
We know that
$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
(i) We have, $(4a – b + 2c)^2 = (4a)^2 + (-b)^2 + (2c)^2 + 2(4a)(-b) + 2(-b)(2c) + 2(2c)(4a)$
$= 16a^2 + b^2 + 4c^2 – 8ab – 4bc + 16ac$
(ii)We have, $(3a – 5b – cf = (3a)^2 + (-5b^2 + (- c^2) + 2(3a)(-5b) + 2(-5b)(-c) + 2(-c)(3a)$
$= 9a^2 + 25b^2 + c^2 – 30ab + 10bc – 6ac$

(iii) We have, $(- x + 2y – 3z)^2 = (- x)^2 + (2y)^2 + (-3z)^2 + 2(-x)(2y) + 2(2y)(- 3z) + 2(- 3z)(- x)$
$= x^2 + 4y^2 + 9z^2 – 4xy – 12yz + 6xz$

29. Factorise the following:
(i) $9x^2 + 4y^2+16z^2+12xy-16yz-24xz$
(ii) $25x^2 + 16y^2 + 4Z^2 – 40xy +16yz – 20xz$
(iii) $16x^2 + 4)\wedge + 9z^2-\wedge 6xy – 12yz + 24xz$

Answer:
(i) We have,
$9x^2 + 4y^2 + 16z^2 + 12xy – 16yz – 24xz$
$= (3x)^2 + (2y)^2 + (-4z)^2 + 2(3x)(2y) + 2(2y)(-4z) + 2(-4z)(3x)$
$= (3x + 2y – 4z)^2$
$[\therefore a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2]$
$= (3x + 2y- 4z) (3x + 2y – 4z)$

(ii)We have, $25X^2 + 16y^2 + 4z^2 – 40xy + 16yz – 20xz$
$=(-5x)^2 + (4y)^2 + (2z)^2 + 2(-5x)(4y) + 2(4y)(2z) + 2(2z)(-5x)$
$= (- 5x + 4y + 2z)^2$
$[\therefore a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2]$
$= (- 5x + 4y + 2z)(- 5x + 4y + 2z)$

(iii) We have, $16x^2 + 4y^2 + 9z^2 – 16xy – 12yz + 24xz$
$= (4x)^2 + (- 2y)^2 + (3z)^2 + 2(4x)(-2y) + 2(-2y)(3z) + 2(3z)(4x)$
$= (4x – 2y + 3z)^2$
$[\therefore a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2]$
$= (4x – 2y + 3z)(4x -2y + 3z)$

30. If a+b+c=9 and ab + bc + ca = 26, find $a^2 + b^2 + c^2$.

Answer:

We have, a + b + c = 9

$\Rightarrow (a + b + c)^2 = (9)^2$ [Squaring on both sides]

$\Rightarrow a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = 81$

$\Rightarrow a^2 + b^2 + c^2 + 2(ab + bc + ca) = 81$

$\Rightarrow a^2 + b^2 + c^2 + 2(26) = 81$ [∴ ab + bc + ca = 26]

$\Rightarrow a^2 + b^2 + c^2 = 81 - 52 = 29$

31. Expand the following

(i) $(3a - 2b)^3$ (ii) $\left(\frac{1}{x} + \frac{y}{3}\right)^3$

(iii) $\left(4 - \frac{1}{3x}\right)^3$

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Answer:

(i) We have, $(3a - 2b)^3$

$= (3a)^3 - (2b)^3 - 3(3a)(2b)(3a - 2b)$

[∴ $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$]

$= 27a^3 - 8b^3 - 18ab(3a - 2b)$

$= 27a^3 - 8b^3 - 54a^2b + 36ab^2$

$= 27a^3 - 54a^2b + 36ab^2 - 8b^3$

(ii) We have, $\left(\frac{1}{x}+\frac{y}{3}\right)^3$

$$=\left(\frac{1}{x}\right)^3+\left(\frac{y}{3}\right)^3+3\left(\frac{1}{x}\right)\left(\frac{y}{3}\right)\left(\frac{1}{x}+\frac{y}{3}\right)$$

$[\because (a+b)^3 = a^3+b^3+3ab(a+b)]$

$$=\frac{1}{x^3}+\frac{y^3}{27}+\frac{y}{x}\left(\frac{1}{x}+\frac{y}{3}\right)$$

$$=\frac{1}{x^3}+\frac{y^3}{27}+\frac{y}{x^2}+\frac{y^2}{3x}$$

$$=\frac{1}{x^3}+\frac{y}{x^2}+\frac{y^2}{3x}+\frac{y^3}{27}$$

(iii) We have, $\left(4-\frac{1}{3x}\right)^3$

$$=(4)^3-\left(\frac{1}{3x}\right)^3-3(4)\left(\frac{1}{3x}\right)\left(4-\frac{1}{3x}\right)$$

$[\because (a-b)^3 = a^3-b^3-3ab(a-b)]$

$$=64-\frac{1}{27x^3}-\frac{4}{x}\left(4-\frac{1}{3x}\right)$$

$$=64-\frac{1}{27x^3}-\frac{16}{x}+\frac{4}{3x^2}$$

$$=64-\frac{16}{x}+\frac{4}{3x^2}-\frac{1}{27x^3}$$

32. Factorise the following:

(i) $1-64a^3-12a+48a^2$

(ii) $8p^3+\frac{12}{5}p^2+\frac{6}{25}p+\frac{1}{125}$

Answer:

(i) We have, $1-64a^3-12a+48a^2$

$=(1)^3-(4a)^3-3(1)^2(4a)+3(1)(4a)^2$

$=(1-4a)^3$ $[\because a^3-b^3-3a^2b+3ab^2=(a-b)^3]$

$=(1-4a)(1-4a)(1-4a)$

(ii) We have, $8p^3+\frac{12}{5}p^2+\frac{6}{25}p+\frac{1}{125}$

$$=8p^3+\frac{1}{125}+\frac{12}{5}p^2+\frac{6}{25}p$$

$$= (2p)^3 + \left(\frac{1}{5}\right)^3 + 3(2p)^2\left(\frac{1}{5}\right) + 3(2p)\left(\frac{1}{5}\right)^2$$

$$= \left(2p + \frac{1}{5}\right)^3 \quad [\because a^3 + b^3 + 3a^2 b + 3ab^2 = (a+b)^3]$$

$$= \left(2p + \frac{1}{5}\right)\left(2p + \frac{1}{5}\right)\left(2p + \frac{1}{5}\right)$$

33. Find the following products:

(i) $\left(\frac{x}{2} + 2y\right)\left(\frac{x^2}{4} - xy + 4y^2\right)$

(ii) $(x^2 - 1)(x^4 + x^2 + 1)$

Answer:

(i) We have, $\left(\frac{x}{2} + 2y\right)\left(\frac{x^2}{4} - xy + 4y^2\right)$

$$= \frac{x}{2}\left(\frac{x^2}{4} - xy + 4y^2\right) + 2y\left(\frac{x^2}{4} - xy + 4y^2\right)$$

$$= \frac{x^3}{8} - \frac{x^2y}{2} + 2xy^2 + \frac{x^2y}{2} - 2xy^2 + 8y^3$$

$$= \frac{x^3}{8} + 8y^3$$

(ii) We have, $(x^2 – 1)(x^4 + x^2 + 1)$

$= x^2(x^4 + x^2 + 1) – 1(x^4 + x^2 + 1)$

$= x^6 + x^4 + x^2 – x^4 – x^2 – 1 = x^6 – 1$

34. Factorise:

(i) $1 + 64x^3$

(ii) $a^3 – 2\sqrt{2}b^3$

Answer:

(i) We have, $1 + 64x^3 = (1)^3 + (4x)^3$

$= (1 + 4x)[(1)^2 – (1)(4x) + (4x)^2]$

$[\therefore a^3 + b^3 = (a + b)(a^2 – ab + b^2)]$

$= (1 + 4x)(1 – 4x + 16x^2)$

(ii) We have, $a^3 – 2\sqrt{2}b^3 = (a)^3 – (\sqrt{2}b)^3$
$= (a – \sqrt{2}b)[a^2 + a(\sqrt{2}b) + (\sqrt{2}b)^2]$
$[\therefore a^3 – b^3 = (a – b)(a^2 + ab + b^2)]$
$= (a – \sqrt{2}b)(a^2 + \sqrt{2}ab + 2b^2)$

35. Find $(2x – y + 3z)(4x^2 + y^2 + 9z^2 + 2xy + 3yz – 6xz)$.

Answer:

We have, $(2x – y + 3z)(4x^2 + y^2 + 9z^2 + 2xy + 3yz – 6xz)$
$= 2x(4x^2 + y^2 + 9z^2 + 2xy + 3yz – 6xz) – y(4x^2 + y^2 + 9z^2 + 2xy + 3yz – 6xz) + 3z(4x^2 + y^2 + 9z^2 + 2xy + 3yz – 6xz)$
$= 8x^3 + 2xy^2 + 18xz^2 + 4x^2y + 6xyz – 12x^2z – 4x^2y – y^3 – 9yz^2 – 2xy^2 – 3y^2z + 6xyz + 12x^2z + 3y^2z + 27z^3 + 6xyz + 9yz^2 – 18xz^2$
$= 8X^3 – y^3 + 27z^3 + 18xyz$

36. Factorise
(i) $a^3 -8b^3 -64c^3 -2Aabc$
(ii) $2\sqrt{2}a^3 +8b^3 -27c^3 +18\sqrt{2}abc$

Answer:

(i) We have, $a^3 – 8b^3 – 64c^3 – 24abc$
$= (a)^3 + (– 2b)^3 + (– 4c)^3 – 3(a)(– 2b)(– 4c)$
$= (a – 2b – 4c)\ [(a)^2 + (– 2b)^2 + (– 4c)^2 – a(– 2b) – (– 2b)(– 4c) – (– 4c)(a)]$
$[\because a^3 + b^3 + c^3 – 3abc = (a + b + c)(a^2 + b^2 + c^2 – ab – bc – ca)]$
$= (a – 2b – 4c)\ (a^2 + 4b^2 + 16c^2 + 2ab – 8bc + 4ac)$

(ii) We have, $2\sqrt{2}a^3 + 8b^3 – 27c^3 + 18\sqrt{2}abc$
$= (\sqrt{2}a)^3 + (2b)^3 + (–3c)^3 – 3(\sqrt{2}a)(2b)(–3c)$
$= \left(\sqrt{2}a + 2b – 3c\right)\left[(\sqrt{2}a)^2 + (2b)^2 + (–3c)^2 –(\sqrt{2}a)(2b) – (2b)(–3c) – (–3c)(\sqrt{2}a)\right]$
$[\because a^3 + b^3 + c^3 – 3abc = (a + b + c)(a^2 + b^2 + c^2 – ab – bc – ca)]$
$= (\sqrt{2}a + 2b – 3c)[2a^2 + 4b^2 + 9c^2 – 2\sqrt{2}ab + 6bc + 3\sqrt{2}ac]$

37. Without actually calculating the cubes, find the value

(i) $\left(\frac{1}{2}\right)^3+\left(\frac{1}{3}\right)^3-\left(\frac{5}{6}\right)^3$

(ii) $(0.2)^3-(0.3)^3+(0.1)^3$

Answer:

(i) We have, $\left(\frac{1}{2}\right)^3+\left(\frac{1}{3}\right)^3-\left(\frac{5}{6}\right)^3$

$=\left(\frac{1}{2}\right)^3+\left(\frac{1}{3}\right)^3+\left(-\frac{5}{6}\right)^3$

Since, $\frac{1}{2}+\frac{1}{3}-\frac{5}{6}=\frac{3+2-5}{6}=0$

$\therefore \left(\frac{1}{2}\right)^3+\left(\frac{1}{3}\right)^3-\left(\frac{5}{6}\right)^3=3\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)\left(-\frac{5}{6}\right)=-\frac{5}{12}$

[$\because$ If $a+b+c=0$, then $a^3+b^3+c^3=3abc$]

(ii) We have, $(0.2)^3 - (0.3)^3 + (0.1)^3$
$= (0.2)^3 + (-0.3)^3 + (0.1)^3$
Since, $0.2 - 0.3 + 0.1 = 0$,
$\therefore (0.2)^3 + (-0.3)^3 + (0.1)^3 = 3(0.2)(-0.3)(0.1)$
[$\therefore$ If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$] = -0.018

38. Without finding the cubes, factorise $(x-2y)^3 + (2y - 3z)^3 + (3z - x)^3$.
Answer:
we see that $(x - 2y) +(2y - 3z)+ (3z - x) = 0$
Therefore, $(x - 2y)^3 + (2y - 3z)^3 + (3z - x)^3 = 3(x - 2y)(2y - 3z)(3z - x)$.
If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

39. Find the value of
(i) $x^3 + y^3 - 12xy + 64$, when $x + y = -4$.
(ii) $x^3 - 8y^3 - 36xy-216$, when $x = 2y + 6$.

Answer:
(i) Since, $x + y + 4 = 0$, then
$x^3 + y^3 + (4)^3 = 3xy(4)$
[$\therefore$ If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$]
$\Rightarrow x^3 + y^3 + 64 = 12xy$
$\Rightarrow x^3 + y^3 - 12xy + 64 = 0$

(ii) Since, $x – 2y – 6 = 0$, then

$x^3 + (-2y)^3 + (-6)^3 = 3x(-2y)(-6)$

[∴ If $a + b + c = 0$, then $a^3 + b^3 + c^3$ 3abc]

$\Rightarrow x^3 – 8y^3 – 216 = 36xy$

$\Rightarrow x^3 – 8y^3 – 36xy – 216 = 0$

40. Give possible expression for the length and breadth of the rectangle whose area is given by $4a^2 + 4a – 3$.

Answer:

Given, area of rectangle = (Length) × (Breadth)

$= 4a^2 + 4a – 3$

$= 4a^2 + 6a – 2a – 3$

$= 2a(2a + 3) -1 (2a + 3) = (2a – 1)(2a + 3)$

Hence, possible length = 2a -1 and breadth = 2a + 3

Exercise 2.4:

1. If the polynomials $az^3 + 4z^2 + 3z – 4$ and $z^3 – 4z + a$ leave the same remainder when divided by $z – 3$, find the value of a.

Answer:

Zero of the polynomial,

$g_1(z) = 0$

$z-3 = 0$

$z = 3$

Therefore, zero of g(z) = – 2a

Let $p(z) = az^3+4z^2+3z-4$

So, substituting the value of z = 3 in p(z), we get,

$p(3) = a(3)^3+4(3)^2+3(3)-4$

$\Rightarrow p(3) = 27a+36+9-4$

$\Rightarrow p(3) = 27a+41$

Let $h(z) = z^3-4z+a$

So, substituting the value of z = 3 in h(z), we get,

$h(3) = (3)^3-4(3)+a$

$\Rightarrow h(3) = 27-12+a$

$\Rightarrow h(3) = 15+a$

According to the question,

We know that,

The two polynomials, p(z) and h(z), leaves same remainder when divided by z-3

So, h(3)=p(3)

$\Rightarrow 15+a = 27a+41$

$\Rightarrow 15-41 = 27a - a$

$\Rightarrow -26 = 26a$

$\Rightarrow a = -1$

2. The polynomial $p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7$ when divided by $x + 1$ leaves the remainder 19. Find the values of a. Also, find the remainder when $p(x)$ is divided by $x + 2$.

Answer:

$p(x) = x^4 - 2x^3 + 3x^2 - ax + 3a - 7$.

Divisor = x + 1

x + 1 = 0

x = -1

So, substituting the value of x = – 1 in p(x), we get,

$p(-1) = (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + 3a - 7$.

$19 = 1 + 2 + 3 + a + 3a - 7$

$19 = 6 - 7 + 4a$

4a – 1 = 19

4a = 20

a = 5

Since a = 5.

We get the polynomial,

$p(x) = x^4 – 2x^3 + 3x^2 – (5)x + 3(5) – 7$

$p(x) = x^4 – 2x^3 + 3x^2 – 5x + 15 – 7$

$p(x) = x^4 – 2x^3 + 3x^2 – 5x + 8$

As per the question,

When the polynomial obtained is divided by (x + 2),

We get,

x + 2 = 0

x = – 2

So, substituting the value of x = – 2 in p(x), we get,

$p(-2) = (-2)^4 – 2(-2)^3 + 3(-2)^2 – 5(-2) + 8$

⇒ p(-2) = 16 + 16 + 12 + 10 + 8

⇒ p(-2) = 62

Therefore, the remainder = 62.

3. If both x – 2 and x -(1/2) are factors of px^2+ 5x+r, then show that p = r.

Answer:

Let $f(x) = px^2 + 5x + r$

Since, $x - 2$ is a factor of $f(x)$, then $f(2) = 0$

$\therefore \ p(2)^2 + 5(2) + r = 0$

$\Rightarrow \ 4p + 10 + r = 0$ ... (i)

Since, $x - \frac{1}{2}$ is a factor of $f(x)$, then $f\left(\frac{1}{2}\right) = 0$

$\therefore \ p\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) + r = 0$

$\Rightarrow \ \frac{p}{4} + \frac{5}{2} + r = 0$

$\Rightarrow \ p + 10 + 4r = 0$ ... (ii)

From (i) and (ii), we get

$4p + 10 + r = p + 10 + 4r$

$\Rightarrow \ 3p = 3r$

$\therefore \ p = r$

4. Without actual division, prove that
$2x^4 – Sx^3 + 2x^2 – x + 2$ is divisible by $x^2 – 3x + 2$. [Hint: Factorise $x^2 – 3x + 2$]

Answer:

Let $p(x) = 2x^4 – 5x^3 + 2x^2 – x + 2$

Now, $x^2-3x + 2 = x^2 – 2x – x + 2$

$= (x-2)(x- 1)$

Hence, zeroes of $x^2 – 3x + 2$ are 1 and 2.

$\Rightarrow$ p(x) is divisible by $x^2 – 3x + 2$ i.e., divisible by $x – 1$ and $x – 2$, if $p(1) = 0$ and $p(2) = 0$

Now, $p(1) = 2(1)^4 – 5(1)^3 + 2(1)^2 -1 + 2$

$= 2 – 5 + 2 – 1 + 2 = 6 – 6 = 0$

and $p(2) = 2(2)^4 – 5(2)^3 + 2(2)^2 -2 + 2$

$= 32 – 40 + 8 = 40 – 40 = 0$

Hence, p(x) is divisible by $x^2 – 3x + 2$.

Q5.Simplify $(2x- 5y)^3 – (2x+ 5y)^3$.

Answer:

We have, $(2x-5y)^3-(2x+5y)^3$

$$= [(2x)^3-(5y)^3-3(2x)(5y)(2x-5y)]-[(2x)^3 + (5y)^3+3(2x)(5y)(2x+5y)]$$

$$\left[\begin{array}{l}\because (a-b)^3 = a^3-b^3-3ab(a-b) \text{ and} \\ (a+b)^3 = a^3+b^3+3ab(a+b)\end{array}\right]$$

$$= (2x)^3-(5y)^3-30xy(2x-5y)-(2x)^3-(5y)^3 - 30xy(2x+5y)$$

$$= -2(5y)^3-30xy(2x-5y+2x+5y)$$

$$= -250y^3-120x^2y$$

Q6. Multiply $x^2 + 4y^2 + z^2 + 2xy + xz – 2yz$ by $(-z + x-2y)$.

Answer: We have,

$(x^2 + 4y^2 + z^2 + 2xy + xz – 2yz)(- z + x – 2y)$

$= x^2 (- z + x – 2y) + 4y^2(- z + x – 2y) + z^2(- z + x – 2y) + 2xy(- z + x – 2y) + xz(- z + x – 2y) – 2yz (- z + x – 2y)$

$= -x^2z + x^3 – 2 x^2y – 4y^2z + 4xy^2 – 81y^3 – z^3 + xz^2 – 2yz^2 – 2xyz + 2x^2y – 4xy^2 – xz^2 + x^2z – 2xyz + 2yz^2 – 2xyz + 4y^2z$

$= x^3 – 8y^3 – z^3 – 6xyz$

7.

If a, b, c are all non-zero and $a + b + c = 0$, prove that $\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}=3$.

Answer:

As we know, if $a+b+c=0$ then

$a^3+b^3+c^3=3abc$

On dividing both sides by abc, we get

$$\frac{a^2}{bc}+\frac{b^2}{ac}+\frac{c^2}{ab}=3$$

8. If a + b + c = 5 and ab + bc + ca = 10, then prove that $a^3 + b^3 + c^3 – 3abc = -25$.

Answer:

We have, a + b + c = 5,ab + bc + ca = 10

Since $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$,

then $(5)^2 = a^2 + b^2 + c^2 + 2(10)$

$\Rightarrow a^2 + b^2 + c^2 = 25 – 20$

$\Rightarrow a^2 + b^2 + c^2 = 5$ ... (i)

L.H.S. = $a^3 + b^3 + c^3 – 3abc$

$= (a + b + c)(a^2 + b^2 + c^2 – ab – be – ca)$

= (5) [5 – (ab + be + ca)] [From (i)]

= 5(5 -10) = 5(-5) = – 25 = R.H.S.

9. Prove that $(a +b +c)^3 -a^3 -b^3 – c^3 =3(a +b)(b +c)(c +a)$.

Answer:

L.H.S. = $[(a + b + c)^3 – a^3] – (b^3 + c^3)$

$= (a + b + c – a)[(a + b + c)^2 + a^2 + (a + b + c)a] – [(b + c) (b^2 + c^2 – be)]$

$x^3 – y^3 = (x – y)(x^2 + y^2 + xy)$ and

$x^3 + y^3 = (x + y)(x^2 + y^2 – xy)$

$= (b + c)[a^2 + b^2 + c^2 + 2 ab + 2 bc + 2 ca + a^2 + a^2 + ab + ac] – (b + c)(b^2 + c^2 – bc)$

$= (b + c)[3a^2 + 3ab + 3ac + 3bc]$

$= (b + c)[3(a^2 + ab + ac + bc)]$

= 3 (b + c)[a(a + b) + c(a + b)]

= 3(a + b)(b + c)(c + a) = R.H.S.