1: Which of the following expressions are polynomials in one variable, and which are not? State reasons for your answer:

(i) $3x^2 – 4x + 15$

(ii) $y^2 + 2\sqrt{3}$

(iii) $3\sqrt{x} + \sqrt{2}x$

(iv) $x – 4/x$

(v) $x^{12} + y^3 + t^{50}$

Answer:

(i) $3x^2 – 4x + 15$

It is a polynomial of x.

(ii) $y^2 + 2\sqrt{3}$

It is a polynomial of y.

(iii) $3\sqrt{x} + \sqrt{2}x$

It is not a polynomial since the exponent of $3\sqrt{x}$ is a rational term.

(iv) $x – 4/x$

It is not a polynomial since the exponent of $– 4/x$ is not a positive term.

(v) $x^{12} + y^3 + t^{50}$

It is a three-variable polynomial, x, y and t.

2: Write the degrees of each of the following polynomials:

(i) $7x^3 + 4x^2 – 3x + 12$

(ii) $12 – x + 2x^3$

(iii) $5y – \sqrt{2}$

(iv) 7

(v) 0

Answer:

As we know, degree is the highest power in the polynomial

(i) Degree of the polynomial $7x^3 + 4x^2 - 3x + 12$ is *3*

(ii) Degree of the polynomial $12 - x + 2x^3$ is *3*

(iii) Degree of the polynomial $5y - \sqrt{2}$ is *1*

(iv) Degree of the polynomial 7 is *0*

(v) Degree of the polynomial 0 is *undefined.*

3: Classify the following polynomials as linear, quadratic, cubic and biquadratic polynomials:

(i) $x + x^2 + 4$

(ii) $3x - 2$

(iii) $2x + x^2$

(iv) $3y$

(v) $t^2 + 1$

(vi) $7t^4 + 4t^3 + 3t - 2$

Answer:

(i) $x + x^2 + 4$: It is a quadratic polynomial as its degree is *2*.

(ii) $3x - 2$: It is a linear polynomial as its degree is *1*.

(iii) $2x + x^2$: It is a quadratic polynomial as its degree is *2*.

(iv) $3y$: It is a linear polynomial as its degree is *1*.

(v) $t^2 + 1$: It is a quadratic polynomial as its degree is *2*.

(vi) $7t^4 + 4t^3 + 3t - 2$: It is a biquadratic polynomial as its degree is *4*.

4. Write the coefficient of x in $\sqrt{3}-2\sqrt{2}x+6x^2$.

Answer: The coefficient of x is $=-2\sqrt{2}$

5. State whether the following expression is polynomial or not. In the case of a polynomial, write its degree.
$x^4-x^{3/2}+x-3$.

Answer: The power of x in the given expression is not a whole number.

It is not a polynomial.

6. State whether the following expression is polynomial or not. In the case of a polynomial, write its degree - $1/\sqrt{2}x^2-\sqrt{2}x+2$.

Answer: Given expression is $1/\sqrt{2}x^2-\sqrt{2}x+2$.

∵ All the exponents of x are whole numbers and the highest exponent of x is 3.

It is a polynomial of degree 2.

7. Write the coefficient of x^3 **in** $x+3x^2-5x^3+x4$**.**

Answer: A coefficient is a multiplicative factor in some term of a polynomial, a series, or any expression; it is usually a number but may be any expression.
The coefficient of x3 in $x+3x^2-5x^3+x^4$ is -5

In each of the following, use the factor theorem to find whether polynomial g(x) is a factor of polynomial f(x) or, not:

8: $f(x) = 2x^3 - 9x^2 + x + 12$, $g(x) = 3 - 2x$

Answer:

If g(x) is a factor of f(x), then the remainder will be zero that is g(x) = 0.

$g(x) = 3 - 2x = 0$, then $x = 3/2$

Remainder = f(3/2)

Now,

$f(3/2) = 2(3/2)^3 - 9(3/2)^2 + (3/2) + 12$

$= 2 \times 27/8 - 9 \times 9/4 + 3/2 + 12$

$= 27/4 - 81/4 + 3/2 + 12$

= 0/4

= 0

Therefore, g(x) is a factor of f(x).

9: $f(x) = x^3 – 6x^2 + 11x – 6$; $g(x) = x – 3$

Answer:

If g(x) is a factor of f(x), then the remainder will be zero that is g(x) = 0.

g(x) = x -3 = 0

or x = 3

Remainder = f(3)

Now,

$f(3) = (3)^3 – 6(3)^2 +11 \times 3 – 6$

= 27 – 54 + 33 – 6

= 60 – 60

= 0

Therefore, g(x) is a factor of f(x)

Using the factor theorem, factorize each of the following polynomials:

10: $x^3 – 6x^2 + 3x + 10$

Answer:

Let $f(x) = x^3 – 6x^2 + 3x + 10$

Constant term = 10

Factors of 10 are ±1, ±2, ±5, ±10

Let x + 1 = 0 or x = -1

$f(-1) = (-1)^3 – 6(-1)^2 + 3(-1) + 10 = 10 – 10 = 0$

f(-1) = 0

Let x + 2 = 0 or x = -2

$f(-2) = (-2)^3 - 6(-2)^2 + 3(-2) + 10 = -8 - 24 - 6 + 10 = -28$

$f(-2) \neq 0$

Let x – 2 = 0 or x = 2

$f(2) = (2)^3 - 6(2)^2 + 3(2) + 10 = 8 - 24 + 6 + 10 = 0$

f(2) = 0

Let x – 5 = 0 or x = 5

$f(5) = (5)^3 - 6(5)^2 + 3(5) + 10 = 125 - 150 + 15 + 10 = 0$

f(5) = 0

Therefore, (x + 1), (x – 2) and (x-5) are factors of f(x)

Hence f(x) = (x + 1) (x – 2) (x-5)

11: $x^4 - 7x^3 + 9x^2 + 7x - 10$

Answer:

Let $f(x) = x^4 - 7x^3 + 9x^2 + 7x - 10$

Constant term = -10

Factors of -10 are ±1, ±2, ±5, ±10

Let x – 1 = 0 or x = 1

$f(1) = (1)^4 - 7(1)^3 + 9(1)^2 + 7(1) - 10 = 1 - 7 + 9 + 7 - 10 = 0$

f(1) = 0

Let x + 1 = 0 or x = -1

$f(-1) = (-1)^4 - 7(-1)^3 + 9(-1)^2 + 7(-1) - 10 = 1 + 7 + 9 - 7 - 10 = 0$

f(-1) = 0

Let x – 2 = 0 or x = 2

$f(2) = (2)^4 - 7(2)^3 + 9(2)^2 + 7(2) - 10 = 16 - 56 + 36 + 14 - 10 = 0$

f(2) = 0

Let x – 5 = 0 or x = 5

$f(5) = (5)^4 - 7(5)^3 + 9(5)^2 + 7(5) - 10 = 625 - 875 + 225 + 35 - 10 = 0$

f(5) = 0

Therefore, (x – 1), (x + 1), (x – 2) and (x-5) are factors of f(x)

Hence f(x) = (x – 1) (x + 1) (x – 2) (x-5)

12: If x = 1/2 is a zero of the polynomial $f(x) = 8x^3 + ax^2 - 4x + 2$, find the value of a.

Answer:

If x = 1/2 is a zero of the polynomial f(x), then f(1/2) = 0

$8(1/2)^3 + a(1/2)^2 - 4(1/2) + 2 = 0$

8 x 1/8 + a/4 – 2 + 2 = 0

1 + a/4 = 0

a = -4

13: If x+1 is a factor of $x^3 + a$, then write the value of a.

Answer:

Let $f(x) = x^3 + a$

If x+1 is a factor of $x^3 + a$ then f(-1) = 0

$(-1)^3 + a = 0$

-1 + a = 0

or a = 1

14. Evaluate the following by using factors:

(i) $(979)^2 - (21)^2$

(ii) $(99.9)^2 - (0.1)^2$

Answer:

(i) $(979)^2 - (21)^2$

We know that

= (979 + 21) (979 – 21)

So we get

= 1000 ×958

= 958000

(ii) $(99.9)^2 - (0.1)^2$

We know that

= (99.9 + 0.1) (99.9 – 0.1)

So we get

= 100 × 99.8

= 9980

15. Question 1: If $f(x) = 2x^3 - 13x^2 + 17x + 12$, find

(i) f (2)

(ii) f (-3)

(iii) f(0)

Answer:

$f(x) = 2x^3 - 13x^2 + 17x + 12$

(i) $f(2) = 2(2)^3 - 13(2)^2 + 17(2) + 12$

$= 2 \times 8 - 13 \times 4 + 17 \times 2 + 12$

$= 16 - 52 + 34 + 12$

$= 62 - 52$

= 10

(ii) $f(-3) = 2(-3)^3 - 13(-3)^2 + 17 \times (-3) + 12$

$= 2 \times (-27) - 13 \times 9 + 17 \times (-3) + 12$

$= -54 - 117 - 51 + 12$

$= -222 + 12$

= -210

(iii) $f(0) = 2 \times (0)^3 - 13(0)^2 + 17 \times 0 + 12$

$= 0 - 0 + 0 + 12$

= 12

16. If a + b = 8 and ab = 15, find the value of $a^4 + a^2b^2 + b^4$.

Answer:-

$a^4 + a^2b^2 + b^4$

Above terms can be written as,

$a^4 + 2a^2b^2 + b^4 - a^2b^2$

$(a^2)^2 + 2a^2b^2 + (b^2)^2 - (ab)^2$

$(a^2 + b^2)^2 - (ab)^2$

$(a^2 + b^2 + ab)(a^2 + b - ab)$

$a + b = 8$, $ab = 15$

So, $(a + b)^2 = 8^2$

$a^2 + 2ab + b^2 = 64$

$a^2 + 2(15) + b^2 = 64$

$a^2 + b^2 + 30 = 64$

By transposing,

$a^2 + b^2 = 64 – 30$

$a^2 + b^2 = 34$

Then, $a^4 + a^2b^2 + b^4$

$= (a^2 + b^2 + ab) (a^2 + b^2 – ab)$

$= (34 + 15) (34 – 15)$

$= 49 \times 19$

$= 931$

17. If $p(x)=x^3−5x^2+4x−3$ and $g(x)=x−2$, show that p(x) is not a multiple of g(x).

Answer: Given, $P(x)=x^3−5x^2+4x−3$

$g(x)=x−2$

Put $g(x)=0$

$\Longrightarrow x=2$

Now, p(2) should be 0 if p(x) is a multiple of g(x)

so,

$p(2)=2^3−5(2)^2+4(2)−3$

$p(2)=8−20+8−3$

$p(2)=16−23$

$p(2)=−7$

since the remainder is not zero, p(x) is not a multiple of g(x).

18. Factorize:

(i) $ab(x^2 + y^2) - xy(a^2 + b^2)$

Answer:-

$ab(x^2 + y^2) - xy(a^2 + b^2)$

The above question can be written as,

$abx^2 + aby^2 - xya^2 - xyb^2$

Re-arranging the above we get,

$abx^2 - xyb^2 + aby^2 - xya^2$

Take out common in all terms,

$bx(ax - by) + ay(by - ax)$

$bx(ax - by) - ay (ax - by)$

$(ax - by) (bx - ay)$

(ii) $9x^2 - 4(y + 2x)^2$

Answer:-

$9x^2 - 4(y + 2x)^2$

Above question can be written as,

$(3x)^2 - [2(y + 2x)]^2$

$(3x)^2 - (2y + 4x)^2$

We know that, $a^2 - b^2 = (a + b) (a - b)$

$(3x + 2y + 4x) (3x - 2y - 4x)$

$(7x + 2y) (-x - 2y)$

19. Factorize:

(i) $(x + 4)^2 – 5xy - 20y – 6y^2$

Answer:-

$(x + 4)^2 – 5xy - 20y – 6y^2$

Above terms can be written as,

$(x + 4)^2 – 5y(x + 4) – 6y^2$

$(x + 4)^2 – 6y(x + 4) + y(x + 4) – 6y^2$

Take out common in all terms we get,

$(x + 4)(x + 4 – 6y) + y(x + 4 – 6y)$

$(x – 6y + 4)(x + 4 + y)$

(ii) $(x^2 – 2x^2) – 23(x^2 – 2x) + 120$

Answer:-

$(x^2 – 2x^2) – 23(x^2 – 2x) + 120$

Above terms can be written as,

$(x^2 – 2x)^2 – 15(x^2 – 2x) – 8(x^2 – 2x) + 120$

Take out common in all terms we get,

$(x^2 – 2x)(x^2 – 2x – 15) – 8(x^2 – 2x – 15)$

$(x^2 – 2x – 15)(x^2 – 2x – 8)$

(iii). $4(2a – 3)^2 – 3(2a – 3)(a – 1) – 7(a – 1)^2$

Answer:-

$4(2a – 3)^2 – 3(2a – 3)(a – 1) – 7(a – 1)^2$

Let us assume, $2a – 3 = p$ and $a – 1 = q$

So, $4p^2 – 3pq – 7q^2$

Then, $4p^2 – 7pq + 4pq – 7q^2$

Take out common in all terms we get,

P(4p – 7q) + q(4p – 7q)

(4p – 7q) (p + q)

Now, substitute the value of p and q we get,

(4(2a – 3) – 7(a – 1)) (2a – 3 + a – 1)

(8a – 12 – 7a + 7) (3a – 4)

(a – 5) (3a – 4)

(iv). $(2x^2 + 5x) (2x^2 + 5x – 19) + 84$

Answer:-

$(2x^2 + 5x) (2x^2 + 5x – 19) + 84$

Let us assume, $2x^2 + 5x = p$

So, (p) (p – 19) + 84

$p^2 – 19p + 84$

$p^2 – 12p – 7p + 84$

p(p – 12) – 7(p – 12)

(p – 12) (p – 7)

Now, substitute the value of p we get,

$(2x^2 + 5x – 12) (2x^2 + 5x – 7)$

(v). $27(x + y)^3 + 8(2x – y)^3$

Answer:-

$27(x + y)^3 + 8(2x – y)^3$

Above terms can be written as,

$3^3(x + y)^3 + 2^3(2x – y)^3$

$(3(x + y))^3 + (2(x – y))^3$

We know that, $a^3 + b^3 = (a + b) (a^2 – ab + b^2)$

Where, $a = 3(x + y)$, $b = 2(x – y)$

$= [3(x + y) + 2(2x – y)] [(3(x + y))^3 – (3(x + y) \times 2(2x – y)) + (2(2x – y))^2]$

$= [3x + 3y + 4x – 2y] [9(x + y)^2 – 6(x + y)(2x – y) + 4(2x – y)^2]$

$= (7x – y) [9(x^2 + y^2 + 2xy) – 6(2x^2 – xy + 2xy – y^2) + 4(4x^2 + y^2 – 4xy)]$

$= (7x – y) [9x^2 + 9y^2 + 18xy – 12x^2 – 6xy – 6y^2 + 16x^2 + 4y^2 – 16xy]$

$= (7x – y) [13x^2 – 4xy + 19y^2]$

(vi). $32a^2x^3 – 8b^2x^3 – 4a^2y^3 + b^2y^3$

Answer:-

$32a^2x^3 – 8b^2x^3 – 4a^2y^3 + b^2y^3$

Take out common in all terms we get,

$8x^3(4a^2 – b^2) – y^3(4a^2 – b^2)$

$(4a^2 – b^2) (8x^3 – y^3)$

Above terms can be written as,

$((2a)^2 – b^2) ((2x)^3 – y^3)$

We know that, $a^3 – b^3 = (a – b) (a^2 + ab + b^2)$ and $(a^2 – b^2) = (a + b) (a – b)$

$(2a + b) (2a – b) [(2x – y) ((2x)^2 + 2xy + y^2)]$

$(2a + b) (2a – b) (2x – y) (4x^2 + 2xy + y^2)$