Section A:

Q1. $(1/2)^{-5}$ is equal to
(a) 32
(b) 132
(c) -32
(d) −132
Solution:

$\left(\frac{1}{2}\right)^{-5} = (2)^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$ (a)

Question 4.

The value of $\left[\left(\frac{1}{4}\right)^{-2} + \left(\frac{1}{3}\right)^{-2}\right] \div \left(\frac{1}{5}\right)^{-2}$ is

(a) 0 (b) −1

(c) 1 (d) $\frac{7}{5}$

Solution:

$$\left(\left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}\right) \div \left(\frac{1}{5}\right)^{-2}$$

$= [(3)^2 + (4)^2] \div (5^2)$
$= (9 + 16) \div 25$
$= 25 \div 25 = 1$ (c)

Q2. If $2^4 \times 4^3 = 4^x$, then the value of x is
(a) 5
(b) 7
(c) 4
(d) -5

Solution:

$2^4 \times 4^3 = 4^x$

$\Rightarrow 2^4 \times (2^2)^3 = (2^2)^x$

$\Rightarrow 2^4 \times 2^6 = 2^{2x}$

$\Rightarrow 2^{10} = 2^{2x}$

$2x = 10$

$\Rightarrow x = 5$ (a)

Q3. .

$\left(\frac{-4}{3}\right)^7 \times \left(\frac{-4}{3}\right)^8 \div \left(\frac{-3}{4}\right)^{-5}$ is equal to

(a) $\left(\frac{-4}{3}\right)^{20}$ (b) $\left(\frac{-4}{3}\right)^{10}$

(c) $\left(\frac{-3}{4}\right)^{10}$ (d) $\left(\frac{-3}{4}\right)^{20}$

Solution:

$$\left(\frac{-4}{3}\right)^7 \times \left(\frac{-4}{3}\right)^8 \div \left(\frac{-3}{4}\right)^{-5}$$

$$= \left(\frac{-4}{3}\right)^7 \times \left(\frac{-4}{3}\right)^8 \div \left(\frac{4}{-3}\right)^5$$

$$= \left(\frac{-4}{3}\right)^{7+8-5} = \left(\frac{-4}{3}\right)^{10}$$

Q4. Standard form of 0.000000789 is

(a) 78.9×10^{-8}

(b) 789×10^{-9}

(c) 7.89×10^{-7}

(d) None

Solution:

$0.000000789 = 7.89 \times 10^{-7}$ (c)

Q5.

$2^0 + 3^0 + \left(\frac{1}{4}\right)^0$ is equal to

(a) $\frac{21}{4}$ (b) 0

(c) 9 (d) 3

Solution:

$2^0 + 3^0 + \left(\frac{1}{4}\right)^0 = 1 + 1 + 1 = 3 \quad (\because a^0 = 1)$

(d)

Q6.

If $\left(\frac{12}{13}\right)^4 \times \left(\frac{13}{12}\right)^{-8} = \left(\frac{12}{13}\right)^{2x}$, then the value of x is

(a) –2 (b) 6

(c) 2 (d) –6

Solution:

$$\left(\frac{12}{13}\right)^4 \times \left(\frac{13}{12}\right)^{-8} = \left(\frac{12}{13}\right)^{2x}$$

$$\Rightarrow \left(\frac{12}{13}\right)^4 \times \left(\frac{12}{13}\right)^{8} = \left(\frac{12}{13}\right)^{2x}$$

$$\Rightarrow \left(\frac{12}{13}\right)^{4+8} = \left(\frac{12}{13}\right)^{2x} \Rightarrow \left(\frac{12}{13}\right)^{12} = \left(\frac{12}{13}\right)^{2x}$$

Comparing, $2x = 12 \Rightarrow x = \frac{12}{2} = 6$

$\therefore x = 6$ (b)

Q7.

(i) Express 16^{-2} as a power with base 2.

(ii) Express 125^{-4} as a power with base 5.

Solution:

$$(i)\ 16^{-2} = (2^4)^{-2} = 2^{4 \times (-2)} = 2^{-8} = \frac{1}{2^8}$$

$$(ii)\ 125^{-4} = (5^3)^{-4} = (5)^{-3 \times 4} = 5^{-12} = \frac{1}{5^{12}}$$

Q8. Write the following numbers in expanded form using exponents:

(i) 2789.453

(ii) 3007.805

Solution:

(i) $2789.453 = 2 \times 10^3 + 7 \times 10^2 + 8 \times 10^1 + 9 \times 10^0 + 4 \times 10^{-1} + 5 \times 10^{-2} + 3 \times 10^{-3}$

(ii) $3007.805 = 3 \times 10^3 + 7 \times 10^0 + 8 \times 10^{-1} + 5 \times 10^{-3}$

Section B

Q9.

$$(i)\ \left[\left(\frac{1}{2}\right)^{-3} + \left(\frac{1}{3}\right)^{-3} + \left(\frac{1}{4}\right)^{-3}\right] \div (3)^2$$

$$(ii)\ \left[3^3 - \left(\frac{1}{2}\right)^{-3} \times \frac{1}{19}\right]$$

$$(iii)\ [2^{-1} + 3^\circ + 5^1 + 7^2 + 9^3] \div \left(\frac{2}{3}\right)^{-1}$$

Solution:

(i) $\left[\left(\frac{1}{2}\right)^{-3}+\left(\frac{1}{3}\right)^{-3}+\left(\frac{1}{4}\right)^{-3}\right] \div (3)^2$

$= (2^3 + 3^3 + 4^3) \div (3)^2$

$= (8 + 27 + 64) \div 9 = \frac{99}{9} = 11$

(ii) $\left[3^3 - \left(\frac{1}{2}\right)^{-3} \times \frac{1}{19}\right]$

$= [3^3 - (2)^3] \times \frac{1}{19}$

$= (27 - 8) \times \frac{1}{19} = 19 \times \frac{1}{19} = 1$

(iii) $[2^{-1} + 3^0 + 5^1 + 7^2 + 9^3] \div \left(\frac{2}{3}\right)^{-1}$

$= \left(\frac{1}{2}+1+5+49+729\right) \div \left(\frac{3}{2}\right)$

$= \left(\frac{1}{2}+784\right) \div \frac{3}{2} = \frac{1569}{2} \div \frac{3}{2}$

$= \frac{1569}{2} \times \frac{2}{3} = \frac{1569}{3} = 523$

Q10. Simplify:

$$\frac{5^{n+2}-6+5^{n+1}}{13\times5^n-2\times5^{n+1}}$$

Solution:

$$\frac{5^{n+2}-6+5^{n+1}}{13\times5^n-2\times5^{n+1}}$$

$$= \frac{5^n\times5^2-6\times5\times5^n}{13\times5^n-2\times5\times5^n}$$

$$= \frac{5^n(5^2-6\times5)}{5^n(13-2\times5)}$$

$$= \frac{25-30}{13-10} = \frac{-5}{3} = -\frac{5}{3}$$

Q11. Find the value of m for which $9^m \div 3^{-2} = 9^4$.

Solution:

$9^m \div 3^{-2} = 9^4$

$\Rightarrow (3^2)^m \div 3^{-2} = (3^2)^4$

$\Rightarrow 3^{2m} \div 3^{-2} = 3^8$

$\Rightarrow 3^{2m+2} = 3^8$

Comparing, we get

$2m + 2 = 8$

$\Rightarrow 2m = 8 - 2 = 6$

$\Rightarrow m = \frac{6}{2} = 3$

Section C

Q12. Simplify:

(*i*) $[(2)^{-1} + (4)^{-1} + (3)^{-1}]^{-1}$

(*ii*) $[(4)^{-1} - (5)^{-1}]^2 \times \left(\frac{5}{8}\right)^{-1}$

(*iii*) $[4^0 + 4^2 - 2^3] \times 3^{-2}$

(*iv*) $\left[(5)^2 - \left(\frac{1}{4}\right)^{-2}\right] \times \left(\frac{3}{4}\right)^{-2}$

Solution:

(*i*) $[(2)^{-1} + (4)^{-1} + (3)^{-1}]^{-1}$

$= \left(\frac{1}{2}+\frac{1}{4}+\frac{1}{3}\right)^{-1}$

$= \left(\frac{6+3+4}{12}\right)^{-1} = \left(\frac{13}{12}\right)^{-1} = \frac{12}{13}$

(*ii*) $[(4)^{-1} - (5)^{-1}]^2 \times \left(\frac{5}{8}\right)^{-1}$

$= \left(\frac{1}{4}-\frac{1}{5}\right)^2 \times \left(\frac{8}{5}\right)^1 = \frac{5-4}{20} \times \left(\frac{8}{5}\right)^1$

$= \left(\frac{1}{20}\right)^2 \times \frac{8}{5}$

$= \frac{1}{20} \times \frac{1}{20} \times \frac{8}{5} = \frac{1}{250}$

(*iii*) $[4^0 + 4^2 - 2^3] \times 3^{-2}$

$= (1 + 16 - 8) \times \frac{1}{3^2}$

$= 9 \times \frac{1}{9} = 1$

(*iv*) $\left[(5)^2 - \left(\frac{1}{4}\right)^{-2}\right] \times \left(\frac{3}{4}\right)^{-2}$

$= [25 - (4)^2] \times \left(\frac{4}{3}\right)^2$

$= (25 - 16) \times \frac{16}{9}$

$= 9 \times \frac{16}{9} = 16$

Q13. Simplify and write in exponential form with a positive exponent:

(*i*) $\left[\left\{\left(\frac{5}{7}\right)^2\right\}^{-1}\right]^{-3}$

(*ii*) $\left(\frac{2}{7}\right)^2 \times \left(\frac{7}{2}\right)^{-3} \div \left\{\left(\frac{7}{5}\right)^{-2}\right\}^{-4}$

(*i*) $\left[\left\{\left(\frac{5}{7}\right)^2\right\}^{-1}\right]^{-3}$

(*ii*) $\left(\frac{2}{7}\right)^2 \times \left(\frac{7}{2}\right)^{-3} \div \left\{\left(\frac{7}{5}\right)^{-2}\right\}^{-4}$

Solution:

$$= \frac{2^{2+3}}{7^{2+3}} \times \frac{5^8}{7^8} = \frac{2^5}{7^5} \times \frac{5^8}{7^8}$$

$$= \frac{2^5 \times 5^8}{7^{5+8}} = \frac{2^5 \times 5^8}{7^{13}}$$

(iii) $\left(\frac{4}{5}\right)^2 \times 5^4 \times \left(\frac{2}{5}\right)^{-2} \div \left(\frac{5}{2}\right)^{-3}$

$$= \frac{4^2}{5^2} \times 5^4 \times \frac{2^{-2}}{5^{-2}} \div \frac{5^{-3}}{2^{-3}}$$

$$= \frac{(2^2)^2 \times 5^4 \times 2^{-2}}{5^2 \times 5^{-2}} \times \frac{2^{-3}}{5^{-3}}$$

$$= \frac{2^4 \times 5^4 \times 2^{-2} \times 2^{-3}}{5^2 \times 5^{-2} \times 5^{-3}}$$

$$= 2^{4-2-3} \times 5^{4-2+2+3} = 2^{-1} \times 5^7$$

$$= \frac{5^7}{2^1}$$

(iv) $\frac{8^{-1} \times 5^3}{2^{-4}} = \frac{(2^3)^{-1} \times 5^3}{2^{-4}} = \frac{2^{-3} \times 5^3}{2^{-4}}$

$$= 2^{-3+4} \times 5^3 = 2^1 \times 5^3$$

$$= 2 \times 5 \times 5 \times 5 = 250$$

(i) $\left[\left\{\left(\frac{5}{7}\right)^2\right\}^{-1}\right]^{-3} = \left\{\left(\frac{5}{7}\right)^2\right\}^{-1\times(-3)}$

$$= \left(\frac{5}{7}\right)^{2\times3} = \left(\frac{5}{7}\right)^6$$

(ii) $\left(\frac{2}{7}\right)^2 \times \left(\frac{7}{2}\right)^{-3} \div \left\{\left(\frac{7}{5}\right)^{-2}\right\}^{-4}$

$$= \left(\frac{2}{7}\right)^2 \times \left(\frac{2}{7}\right)^3 \div \left(\frac{7}{5}\right)^{-2\times(-4)}$$

$$= \left(\frac{2}{7}\right)^2 \times \left(\frac{2}{7}\right)^3 \div \left(\frac{7}{5}\right)^8$$

$$= \left(\frac{2}{7}\right)^2 \times \left(\frac{2}{7}\right)^3 \times \left(\frac{5}{7}\right)^8$$

$$= \frac{2^2}{7^2} \times \frac{2^3}{7^3} \times \frac{5^8}{7^8}$$

Section D

Q14. The number of red blood cells per cubic millimeter of blood is approximately 5.5 million. If the average body contains 5 liters of blood, what is the total number of red cells in the body? (1 litre = 1,00,000 mm^3)

Solution:

Red blood per cubic millimeter = 5.5 million = 5.5×10^6

Red blood in 5 litres of blood

= $5.5 \times 10^6 \times 5 \times 10^5$ (1 litre = 10^5 mm)

= $27.5 \times 10^{6+5}$

= 27.5×10^{11}

= $2.75 \times 10 \times 10^{11}$

= 2.75×10^{12}

Q15.Mass of Mars is 6.42×10^{29} kg and the mass of the sun is 1.99×10^{30} kg. What is the total mass?

Solution:

Mass of Mars = 6.42×10^{29} kg

and mass of sun = 1.99×10^{30}

Total mass = $6.42 \times 10^{29} + 1.99 \times 10^{30}$

= 10^{29} $(6.42 + 1.99 \times 10)$

= 10^{29} $(6.42 + 19.9)$

= 26.32×10^{29}

Q16. The mass of the Earth is 5.97×10^{24} kg and the mass of the Moon is 7.35×10^{22} kg. What is the difference of their masses?

Solution:

Mass of Earth = 5.97×10^{24} kg

and mass of Moon = 7.35×10^{22} kg

∴ Difference = $(5.97 \times 10^{24}) - (7.35 \times 10^{22})$

= 10^{22} $(5.97 \times 10^2 - 7.35)$

= 10^{22} $(597 - 7.35) = 589.65 \times 10^{22}$

= $5.8965 \times 10^2 \times 10^{22}$

= 5.8965×10^{24}

Section E:

Q17.

(i) $\frac{49 \times z^{-3}}{7^{-3} \times 10 \times z^{-5}} \ (z \neq 0)$

(ii) $\frac{9^3 \times 27 \times t^4}{(3)^2 \times (3)^4 \times t^2}$

(iii) $\frac{(3^{-2})^2 \times (5^2)^{-3} \times (t^{-3})^2}{(3^{-2})^5 \times (5^3)^{-2} \times (t^{-4})^3}$

(iv) $\frac{2^5 \times 15^{-5} \times 500}{5^{-6} \times 6^{-5}}$

Solution:

(i) $\frac{49 \times z^{-3}}{7^{-3} \times 10 \times z^{-5}} \ (z \neq 0)$

$= \frac{7^2 \times z^{-3}}{7^{-3} \times 10 \times z^{-5}} = \frac{7^{2+3} \times z^{-3+5}}{10}$

$= \frac{7^5 \times z^2}{10}$

(ii) $\frac{9^3 \times 27 \times t^4}{(3)^2 \times (3)^4 \times t^2} = \frac{(3^2)^3 \times 3^3 \times t^4}{3^2 \times 3^4 \times t^2}$

$= \frac{3^6 \times 3^3 \times t^4}{3^2 \times 3^4 \times t^2} = 3^{6+3-2-4} \times t^{4-2}$

$= 3^3 \times t^2 = 27t^2$

(iii) $\dfrac{(3^{-2})^2 \times (5^2)^{-3} \times (t^{-3})^2}{(3^{-2})^5 \times (5^3)^{-2} \times (t^{-4})^3}$

$$= \frac{3^{-2\times2} \times 5^{2\times(-3)} \times t^{(-3)\times2}}{(3)^{-2\times5} \times 5^{3\times(-2)} \times t^{-4\times3}}$$

$$= \frac{3^{-4} \times 5^{-6} \times t^{-6}}{3^{-10} \times 5^{-6} \times t^{-12}}$$

$$= 3^{-4+10} \times 5^{-6+6} \times t^{-6+12}$$

$$= 3^6 \times 5^0 \times t^6 = 3^6 \times 1 \times t^6$$

$$= (3t)^6$$

(iv) $\dfrac{2^{-5} \times 15^{-5} \times 500}{5^{-6} \times 6^{-5}} = \dfrac{2^{-5} \times (3\times5)^{-5} \times 2^2 \times 5^3}{5^{-6} \times (2\times3)^{-5}}$

$$= \frac{2^{-5} \times 3^{-5} \times 5^{-5} \times 2^2 \times 5^3}{5^{-6} \times 2^{-5} \times 3^{-5}}$$

$$= 2^{-5+2+5} \times 3^{-5+5} \times 5^{-5+3+6}$$

$$= 2^2 \times 3^0 \times 5^4 = 4 \times 1 \times 625 = 2500$$

Q19.

If $\left(\frac{-5}{7}\right)^{-4} \times \left(\frac{-5}{7}\right)^{12} = \left\{\left(\frac{-5}{7}\right)^3\right\}^x \times \left(\frac{-5}{7}\right)^{-1}$

find the value of x.

Solution:

$$\left(\frac{-5}{7}\right)^{-4} \times \left(\frac{-5}{7}\right)^{12} = \left\{\left(\frac{-5}{7}\right)^{3}\right\}^{x} \times \left(\frac{-5}{7}\right)^{-1}$$

$$\left(\frac{-5}{7}\right)^{-4} \times \left(\frac{-5}{7}\right)^{12} = \left(\frac{-5}{7}\right)^{3x} \times \left(\frac{-5}{7}\right)^{-1}$$

$$\Rightarrow \left(\frac{-5}{7}\right)^{-4+12} = \left(\frac{-5}{7}\right)^{3x-1}$$

$$\Rightarrow \left(\frac{-5}{7}\right)^{8} = \left(\frac{-5}{7}\right)^{3x-1}$$

Comparing, we get

$\Rightarrow 8 = 3x - 1 \Rightarrow 3x = 8 + 1 = 9$

$$x = \frac{9}{3} = 3$$

$\therefore x = 3$

Q19.

Find x if $\left(\frac{-2}{3}\right)^{-13} \times \left(\frac{3}{-2}\right)^{8} = \left(\frac{-2}{3}\right)^{-2x+1}$

Solution:

$$\left(\frac{-2}{3}\right)^{-13} \times \left(\frac{3}{-2}\right)^{8} = \left(\frac{-2}{3}\right)^{-2x+1}$$

$$\Rightarrow \left(\frac{-2}{3}\right)^{-13} \times \left(\frac{-2}{3}\right)^{-8} = \left(\frac{-2}{3}\right)^{-2x+1}$$

$$\Rightarrow \left(\frac{-2}{3}\right)^{-13-8} = \left(\frac{-2}{3}\right)^{-2x+1}$$

$$\Rightarrow \left(\frac{-2}{3}\right)^{-21} = \left(\frac{-2}{3}\right)^{-2x+1}$$

Comparing, we get

$-2x + 1 = -21 \Rightarrow 2x - 1 = 21$

$\Rightarrow 2x = 21 + 1 = 22$

$$x = \frac{22}{2} = 11$$

Hence, $x = 11$

19..

(*i*) If $5^{2x-1} = \frac{1}{(125)^{x-3}}$, find x.

(*ii*) If $\frac{9^n \times 3^5 \times (27)^3}{3 \times (81)^4} = 27$, find n.

Solution:

(*i*) $5^{2x-1} = \frac{1}{(125)^{x-3}}$

$\Rightarrow 5^{2x-1} = \frac{1}{(5^3)^{x-3}} \Rightarrow 5^{2x-1} = \frac{1}{5^{3x-9}}$

$\Rightarrow 5^{2x-1} = 5^{-3x+9}$

Comparing, we get

$2x - 1 = -3x + 9$

$2x + 3x = 9 + 1 \Rightarrow 5x = 10$

$\Rightarrow x = \frac{10}{5} = 2$

$\therefore x = 2$

(*ii*) $\frac{9^n \times 3^5 \times (27)^3}{3 \times (81)^4} = 27$

$\Rightarrow \frac{(3^2)^n \times 3^5 \times (3^3)^3}{3 \times (3^4)^4} = 3^3$

$\Rightarrow \frac{3^{2n} \times 3^5 \times 3^9}{3 \times 3^{16}} = 3^3$

$\Rightarrow 3^{2n+5+9-1-16} = 3^3$

$\Rightarrow 3^{2n-3} = 3^3$

Comparing, we get

$2n - 3 = 3 \Rightarrow 2n = 3 + 3 = 6$

$n = \frac{6}{2} = 3$

$\therefore n = 3$

Q20. If $(9/44)^{-4} \times (2/3)3=(pq)11$, then find the value of $(p/q)^{-2}$.

Solution:

$$\left(\frac{9}{4}\right)^{-4} \times \left(\frac{2}{3}\right)^{3} = \left(\frac{p}{q}\right)^{11}$$

$$\Rightarrow \left[\left(\frac{3}{2}\right)^{2}\right]^{-4} \times \left(\frac{2}{3}\right)^{3} = \left(\frac{p}{q}\right)^{11}$$

$$\Rightarrow \left(\frac{3}{2}\right)^{-8} \times \left(\frac{2}{3}\right)^{3} = \left(\frac{p}{q}\right)^{11}$$

$$\Rightarrow \left(\frac{2}{3}\right)^{8} \times \left(\frac{2}{3}\right)^{3} = \left(\frac{p}{q}\right)^{11}$$

$$\Rightarrow \left(\frac{2}{3}\right)^{8+3} = \left(\frac{p}{q}\right)^{11} \Rightarrow \left(\frac{2}{3}\right)^{11} = \left(\frac{p}{q}\right)^{11}$$

Comparing, we get

$$\frac{p}{q} = \frac{2}{3}$$