Section A

Q1. Factorize $10a\,(2p + q)^3 – 15b\,(2p + q)^2 + 35(2p + q)$

Answer:

$10a\,(2p + q)^3 – 15b\,(2p + q)^2 + 35(2p + q)$

⇨ $5\,[\,2a\,(\,(2p + q)^3 – 3b\,(2p + q)^2 + 7\,(2p + q)\,]$

⇨ $5\,(2p + q)\;[\;2a\,(\,(2p + q)^2 – 3b\,(2p + q) + 7\,]$ [$5\,(2p + q)$ - common]

Q2. Factorize

(i) $ab\,(x^2 + y^2) – xy\,(a^2 + b^2)$

(ii) $(ax + by)^2 + (bx – ay)^2$

Answer:

(i) $ab\,(x^2 + y^2) – xy\,(a^2 + b^2)$

⇨ $abx^2 + aby^2 - a^2xy - b^2xy$

⇨ $(abx^2 - b^2xy) + (aby^2 - a^2xy)$

⇨ $bx\,(ax – by) – ay\,(ax – by)$

⇨ $(ax – by)\,(\,bx - ay)$

(ii) $(ax + by)^2 + (bx – ay)^2$

⇨ $(a^2x^2 + b^2y^2 + 2abxy) + (b^2x^2 + a^2y^2 - 2abxy)$

⇨ $a^2x^2 + b^2y^2 + 2abxy + b^2x^2 + a^2y^2 - 2abxy$

⇨ $a^2x^2 + b^2y^2 + b^2x^2 + a^2y^2$

⇨ $a^2x^2 + a^2y^2 + b^2y^2 + b^2x^2$

⇨ $a^2(x^2 + y^2) + b^2(x^2 + y^2)$

⇨ $(a^2 + b^2) + (x^2 + y^2)$

Q3. Factorize

(i) $36p^2 – 60pq + 25q^2$

⇨ $(6p)^2 – 2 \times 6p \times 5q + (5q)^2$

⇨ $(6p – 5q)^2$

(ii) $4x^2 – 169y^2$

⇨ $(2x)^2 – (13y)^2$

⇨ $(2x + 13y)\ (2x – 13y)$

(iii) $(2a + 3b)^2 – 16c^2$

⇨ $(2a + 3b)^2 – (4c)^2$

⇨ $(2a + 3b + 4c)\ (2a + 3b – 4c)$

(iv) $9\ (3x + 2)^2 – 4\ (2x – 1)^2$

⇨ $[3\ (3x + 2)]^2 – [2(2x – 1)]^2$

⇨ $(9x + 6)^2 – (4x – 2)^2$

⇨ $[(9x + 6) + (4x – 2)]\ [(9x + 6) - (4x – 2)]$

⇨ $(9x + 6 + 4x – 2)\ (9x + 6 - 4x – 2)$

⇨ $(13x + 4)\ (5x + 8)$

(v) $9\ (a + b)^3 – 25\ (a + b)$

$= (a + b)\ [9\ (a + b)^2 – 25]$

$= (a + b)\ [(3(a + b)^2 – (5)^2]$

$= (a + b)\ [(3a + 3b + 5)\ (3a + 3b – 5)$

(vi) $x^4 – y^4 + x^2 – y^2$

⇨ $[(x^2)^2 - (y^2)^2] + x^2 – y^2$

$(a^2 – b^2) = (a + b)\ (a – b)$

⇨ $(x^2 + y^2)\ (x^2 - y^2) + 1(x^2 - y^2)$

⇨ $(x^2 - y^2)\ (x^2 + y^2 + 1)$

⇨ $(x + y)\ (x – y)\ (x^2 + y^2 + 1)$

Q4. Factorize :

(i) $7 + 10(2x – 3y) – 8(2x – 3y)^2$

Answer:

$7 + 10(2x – 3y) – 8(2x – 3y)^2$

Let 2x - 3y = z

Then, $7 + 10z – 8z^2$

$\therefore , 7 + 10z – 8z^2 = 7 + 14z – 4z – 8z^2$

⇨ $7 + 14z – 4z – 8z^2$

⇨ $7(1 + 2z) – 4z (1+ 2z)$

⇨ $(7 – 4z) (1+ 2z)$

Substituting z = 2x - 3y, we get

⇨ $(7 – 4(2x - 3y)) (1+ 2(2x - 3y))$

⇨ $(7 – 8x + 12y) (1 + 4x – 6y)$

(ii) $\frac{4x^2}{9} - \frac{9y^2}{9}$

⇨ $\left(\frac{2x}{3}\right)^2 - \left(\frac{3y}{4}\right)^2$

$a^2 – b^2 = (a + b) (a – b)$

⇨ $\left(\frac{2x}{3} - \frac{3y}{4}\right)\left(\frac{2x}{3} + \frac{3y}{4}\right)$

(iii) $(a – b)^2 – (b – c)^2$

$a^2 – b^2 = (a + b) (a – b)$

$(a – b)^2 – (b – c)^2 = (a - b + (b – c)) (a – b – (b - c))$

$(a – b)^2 – (b – c)^2 = (a - b + b – c) (a – b – b + c)$

$(a - b)^2 - (b - c)^2 = (a-c)\ (a - 2b + c)$

Section B

Q5. The area of a square is $9x^2 + 24xy + 16y^2$. Find the side of the square.

Answer:

Area of a square = (side) × (side) = $(side)^2$

Given area of the square is $9x^2 + 24xy + 16y^2$

∴ Area of the square, $9x^2 + 24xy + 16y^2 = (side)^2$

Side = $\sqrt{(9x^2 + 24xy + 16y^2)}$

= $\sqrt{[(3x)^2 + 2(3x)(4y) + (4y)^2]}$

= $\sqrt{(3x + 4y)^2}$

= $(3x + 4y)$

∴ Side of the square is $(3x + 4y)$.

Q6. The area of a circle is given by the expression $\pi x^2 + 6\pi x + 9\pi$. Find the radius of the circle.

Answer:

Given, Area of a circle = $\pi x^2 + 6\pi x + 9\pi$

Area of a circle = πr^2

$\Rightarrow \pi x^2 + 6\pi x + 9\pi = \pi r^2$

$\Rightarrow \pi\ (x^2 + 6x + 9) = \pi r^2$

$\Rightarrow x^2 + 6x + 9 = r^2$

$\Rightarrow x^2 + 3x + 3x + 9 = r^2$

$\Rightarrow x(x + 3) + 3(x + 3) = r^2$

$\Rightarrow (x + 3)^2 = r^2$

$\Rightarrow r = (x + 3)$

$\therefore$ Radius of the circle = $(x + 3)$

Q7. The base of a parallelogram is (2x + 3 units) and the corresponding height is (2x - 3 units). Find the area of the parallelogram in terms of x. What will be the area of parallelogram of x = 30 units?

Answer:

Area of the parallelogram = Base × Height

Given, base of parallelogram = (2x + 3) units and

Height of parallelogram = (2x - 3) units

$\therefore$ Area of the parallelogram = Base × Height

$= (2x + 3) \times (2x - 3)$

Using the standard identity: $a^2 - b^2 = (a + b)(a - b)$

$= (2x)^2 - (3)^2$

$= 4x^2 - 9$

Given x = 30,

Area of the parallelogram = $4x^2 - 9$

$= 4(30)^2 - 9$

= 3591 square units

Q8. The radius of a circle is 7ab - 7bc - 14ac. Find the circumference of the circle. (π = 22/7)

Answer:

Circumference of the circle = 2πr

Given, radius of circle = 7ab - 7bc - 14ac

⇒ Circumference of the circle = 2πr

= 2 × π × (7ab - 7bc - 14ac)

= 2 × (22/7) × (7ab - 7bc - 14ac)

= 2 × (22/7) × 7(ab - bc - 2ac)

= 44(ab - bc - 2ac)

Q9. Match the expressions of column I with that of column II:

Column I	Column II
(1) $(21x + 13y)^2$	**(a) $441x^2 - 169y^2$**
(2) $(21x - 13y)^2$	**(b) $441x^2 + 169y^2 + 546xy$**
(3) (21x - 13y) (21x + 13y)	**(c) $441x^2 + 169y^2 - 546xy$**
	(d) $441x^2 - 169y^2 + 546xy$

Answer:

Given (1) $(21x + 13y)^2$

Using standard identity: $(a + b)^2 = a^2 + 2ab + b^2$

$(21x + 13y)^2 = (21x)^2 + 2(21x)(13y) + (13y)^2$

$= 441x^2 + 546xy + 169y^2 \Rightarrow$ option (b) of column II

(2) $(21x - 13y)^2$

Using standard identity: $(a - b)^2 = a^2 - 2ab + b^2$

$(21x - 13y)^2 = (21x)^2 - 2(21x)(13y) + (13y)^2$

$= 441x^2 - 546xy + 169y^2 \Rightarrow$ option (c) of column II

(3) $(21x - 13y)(21x + 13y)$

Using standard identity: $(a - b)(a + b) = a^2 - b^2$

$(21x - 13y)(21x + 13y) = (21x)^2 - (13y)^2$

$= 441x^2 - 169y^2 \Rightarrow$ option (a) of column II

Column I	Column II
(1) $(21x + 13y)^2$	**(b) $441x^2 + 169y^2 + 546xy$**
(2) $(21x - 13y)^2$	**(c) $441x^2 + 169y^2 - 546xy$**
(3) $(21x - 13y)(21x + 13y)$	**(a) $441x^2 - 169y^2$**

Section C

Q11. $16(2l - 3m)^2 - 12(3m - 2l)$

Answer:

We have,

$16 (2l – 3m)^2 – 12 (3m – 2l)$

By taking (-1) as common, we get,

$16 (2l – 3m)^2 + 12 (2l – 3m)$

By taking $4(2l – 3m)$ as common we get,

$4(2l – 3m) [4 (2l – 3m) + 3]$

$4(2l – 3m) (8l – 12m + 3)$

Q12. $4(x + y) (3a – b) + 6(x + y) (2b – 3a)$

Answer:

We have,

$4(x + y) (3a – b) + 6(x + y) (2b – 3a)$

By taking $(x + y)$ as common, we get,

$(x + y) [4(3a – b) + 6(2b – 3a)]$

$(x + y) [12a – 4b + 12b – 18a]$

$(x + y) [-6a + 8b]$

$(x + y) 2(-3a + 4b)$

$(x + y) 2(4b – 3a)$

Q13. $25(2x–y)^2–16(x–2y)^2$

Answer:

$25(2x – y)^2 – 16(x – 2y)^2$

⇨ $(5 (2x – y))^2–(4(x – 2y))^2$

⇨ $[5 (2x – y) –4 (x – 2y)] [5 (2x – y) + 4(x – 2y)]$

⇨ [10x – 5y – 4x + 8y] [10x – 5y + 4x – 8y]

[$a^2 - b^2 = (a - b)(a + b)$]

Q14. 25 (x – 2y)^2 – 4

Answer:

Using the formula

[∵ $(x^2 - y^2) = (x + y)(x - y)$]

= $[5(x - 2y)]^2 - (2)^2$

= (5(x - 2y) - 2) (5(x - 2y) + 2)

= (5x - 10y - 2)(5x - 10y + 2)

Q15. 12(a + b)^2 – (a + b) - 35

Answer:

=12 (a + b) 2 – 21 (a + b) + 20 (a + b) – 35

– 21 (a + b) + 20 (a + b) = (a + b)

= 4 (a + b) (3 (a + b) + 5) – 7 (3 (a + b) + 5)

= (4a + 4b – 7) (3a + 3b + 5)

1 – 3x – 3y - 4 $(x + y)^2$

Q16. 1 - 3x - 3y - 4(x + y)^2

Answer:

1 - 3x - 3y - 4$(x + y)^2$ (1 can be presented as 4 – 3)

⇨ 4 – 3 - 3x - 3y - 4$(x + y)^2$

⇨ 4 - 4 $(x + y)^2$ – 3 - 3x - 3y

⇨ $4 [1 – (x + y)^2] – 3 (1 + x + y)$

⇨ $4 [1^2 – (x + y)^2] – 3 (1 + x + y)$

⇨ $(1 + x + y) (4(1 – x – y) -3]$

⇨ $(1 + x + y) (4 – 4x – 4y - 3)$

⇨ $(1 + x + y) (1 – 4x – 4y)$

Q17. Factorise $xy^2 – xz^2$, Hence, find the value of

(i) $9 \times 8^2 – 9 \times 2^2$

(ii) $40 \times 5.5^2 – 40 \times 4.5^2$

Answer:

$xy^2 – xz^2 = x (y^2 – z^2) = x (y – z) (y + z)$ $(a^2 – b^2) = (a + b) (a – b)$

(i) $9 \times 8^2 – 9 \times 2^2 = 9 (8^2 - 2^2)$

⇨ $9 (8 – 2) (8 + 2)$

⇨ $9 (6) (10) = 9 \times 6 \times 10 = 540$

(ii) $40 \times 5.5^2 – 40 \times 4.5^2$

⇨ $40 (5.5^2 - 4.5^2)$

⇨ $40 (5.5 – 4.5) (5.5 + 4.5)$

⇨ $40 (1) (10) = 40 \times 1 \times 10 = 400$