Exercise 10.1:

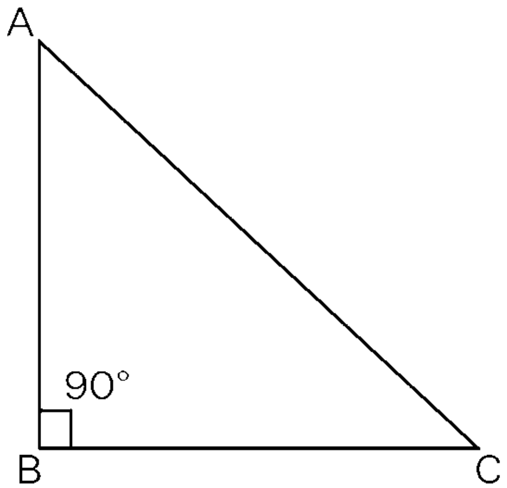
1. An isosceles right triangle has area 8 cm^2. The length of its hypotenuse is

(A) √32 cm (B) √16 cm (C) √48 cm (D) √24 cm

Answer: (A) √32 cm

Given, an isosceles right triangle has area 8 cm^2.

We have to find the length of its hypotenuse.



We know that in an isosceles triangle two sides are of equal length.

Here, AB = BC

AC is the hypotenuse

Area of triangle = 1/2 × base × height

8 = 1/2 × AB × BC

$8 = 1/2 \times AB^2$

$AB^2 = 8 \times 2$

$AB^2 = 16$

Taking square root,

AB = 4 cm

In triangle ABC,

By using Pythagorean theorem,

$AB^2 + BC^2 = AC^2$

$(4)^2 + (4)^2 = AC^2$

$AC^2 = 16 + 16$

$AC^2 = 32$

Taking square root,

AC = √32 cm

Therefore, the length of the hypotenuse is √32 cm.

2. The perimeter of an equilateral triangle is 60 m. The area is

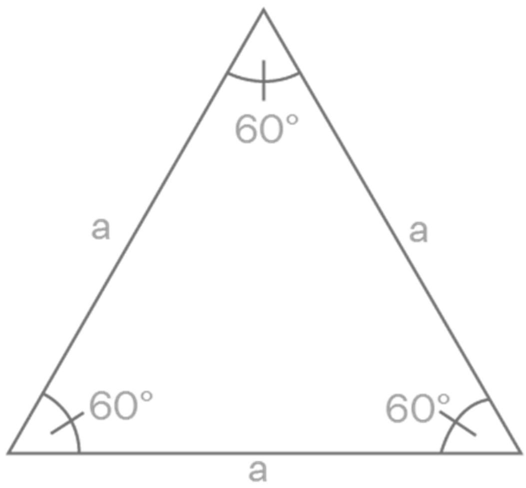
(A) $10\sqrt{3}\ m^2$ (B) $15\sqrt{3}\ m^2$ (C) $20\sqrt{3}\ m^2$ (D) $100\sqrt{3}\ m^2$

Answer: (D) 100√3 m²

Given, the perimeter of an equilateral triangle is 60 m.

We have to find the area.

We know that in an equilateral triangle all the sides are equal.



Let the side of the equilateral triangle be a cm.

Perimeter = side + side + side

So, 60 = a + a + a

3a = 60

a = 60/3

a = 20 m

Therefore, the length of the side is 20 m.

Area of an equilateral triangle = $\sqrt{3}/4\ (\text{side})^2$

$= \sqrt{3}/4\ (20)^2$

$= \sqrt{3}/4\ (400)$

$= 100\sqrt{3}\ m^2$

Therefore, the area of equilateral triangle is $100\sqrt{3}\ m^2$

3. The sides of a triangle are 56 cm, 60 cm and 52 cm long. Then the area of the triangle is

(A) 1322 cm^2 (B) 1311 cm^2 (C) 1344 cm^2 (D) 1392 cm^2

Answer: (C) 1344 cm²

Given, the sides of a triangle are 56 cm, 60 cm and 52 cm long.

We have to find the area of the triangle

Given, a = 56 cm

b = 60 cm

c = 52 cm

By Heron's formula,

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Where s = semiperimeter

s = (a + b + c)/2

Now, s = (56 + 60 + 52)/2

= 168/2

s = 84 cm

Area of triangle = √[84(84 - 56)(84 - 60)(84 - 52)]

= √[84(28)(24)(32)]

= √[12 × 7 × 7 × 4 × 12 × 2 × 16 × 2]

= 12 × 7 × 2 × 2 × 4

= 12 × 7 × 4 × 4

= 12 × 7 × 16

= 1344 cm^2

Therefore, area of triangle is 1344 cm^2

4. The area of an equilateral triangle with side 2√3 cm is

(A) 5.196 cm^2 (B) 0.866 cm^2 (C) 3.496 cm^2 (D) 1.732 cm^2

Answer: (A) 5.196 cm^2

Given, the side of an equilateral triangle = 2√3 cm

We have to find the area of the equilateral triangle.

Area of an equilateral triangle = √3/4 $(\text{side})^2$

= √3/4 $(2\sqrt{3})^2$

= √3/4 (4 × 3)

$= 3\sqrt{3}$

Consider $\sqrt{3} = 1.732$

$= 3 \times 1.732$

$= 5.196\ cm^2$

Therefore, the area of an equilateral triangle is $5.196\ cm^2$

5. The length of each side of an equilateral triangle having an area of $9\sqrt{3}\ cm^2$ is

(a) 8 cm (b) 36 cm (c) 4 cm (d) 6 cm

Answer: (d) 6 cm

Given, the area of an equilateral triangle is $9\sqrt{3}\ cm^2$

We have to find the length of each side of an equilateral triangle

Area of an equilateral triangle = $\sqrt{3}/4\ (side)^2$

$9\sqrt{3} = \sqrt{3}/4\ (side)^2$

$9 = 1/4\ (side)^2$

$(side)^2 = 9(4)$

$(side)^2 = 36$

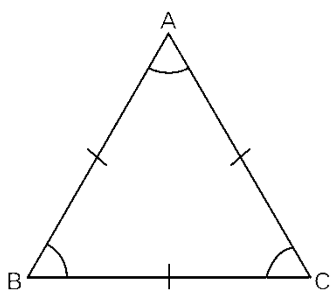
Taking square root, Side = 6 cm

Therefore, the length of each side of an equilateral triangle is 6 cm.

6. If the area of an equilateral triangle is $16\sqrt{3}\ cm^2$, then the perimeter of the triangle is

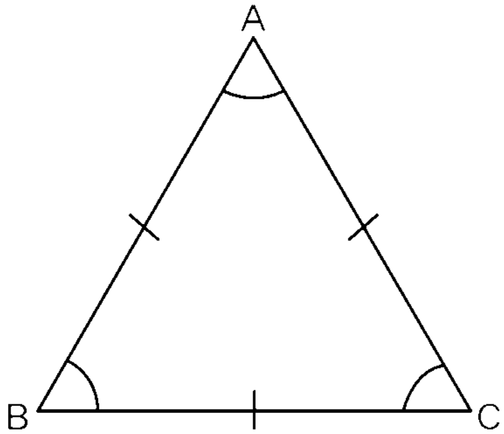
(a) 48 cm (b) 24 cm (c) 12 cm (d) 36 cm

Answer: (b) 24 cm



Given, the area of an equilateral triangle is $16\sqrt{3}\ cm^2$

We have to find the perimeter of the triangle



Area of an equilateral triangle = √3/4 (side)²

16√3 = √3/4 (side)²

16 = 1/4 (side)²

(side)² = 16(4)

(side)² = 64

Taking square root,

Side = 8 cm

The length of each side of an equilateral triangle is 8 cm.

Perimeter of triangle = side + side + side

We know that in an equilateral triangle all the sides are equal

Perimeter = 8 + 8 + 8

= 16 + 8

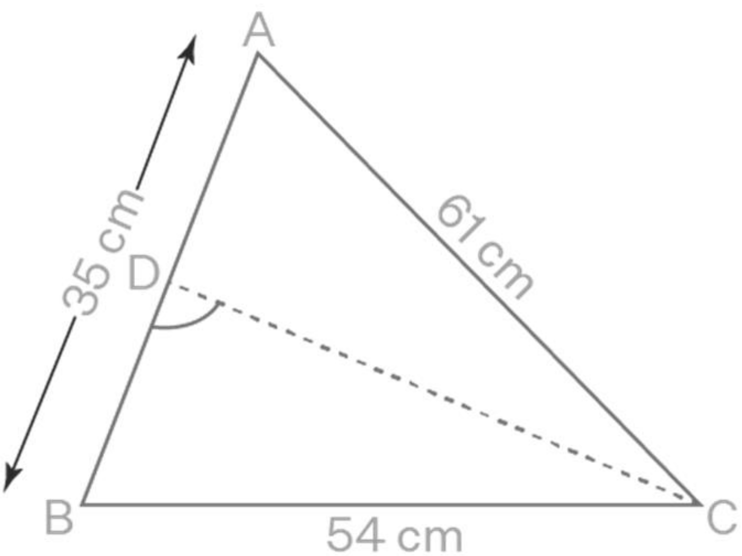
= 24 cm

Therefore, the perimeter of the triangle is 24 cm.

7. The sides of a triangle are 35 cm, 54 cm and 61 cm, respectively. The length of its longest altitude

a. 16√5 cm **b. 10√5 cm** **c. 24√5 cm** **d. 28 cm**

Answer: c. 24√5 cm



Consider a triangle ABC

Given, the sides AB = 35 cm

BC = 54 cm

AC = 61 cm

We have to find the length of the longest altitude.

By Heron's formula,

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Where s= semi perimeter

$s = (a + b + c)/2$

Now, $s = (35 + 54 + 61)/2$

$= 150/2$

$s = 75$ cm

Area of triangle $= \sqrt{75(75 - 35)(75 - 54)(75 - 61)}$

$= \sqrt{75(40)(21)(14)}$

$= \sqrt{15 \times 5 \times 8 \times 5 \times 7 \times 3 \times 7 \times 2}$

$= \sqrt{5 \times 3 \times 5 \times 4 \times 2 \times 5 \times 7 \times 3 \times 7 \times 2}$

$= \sqrt{5 \times 5 \times 3 \times 3 \times 4 \times 7 \times 7 \times 2 \times 2}$

$= 5 \times 3 \times 2 \times 7 \times 2 \times \sqrt{5}$

$= 5 \times 4 \times 21 \times \sqrt{5}$

$= 20 \times 21 \times \sqrt{5}$

$= 420\sqrt{5}$ cm²

Area of triangle = 1/2 × base × height

In triangle ABC,

Area of triangle = 1/2 × AB × CD

$420\sqrt{5} = 1/2 \times 35 \times CD$

$CD = (420\sqrt{5} \times 2)/35$

$CD = 12\sqrt{5} \times 2$

$CD = 24\sqrt{5}$ cm

Therefore, the length of the longest altitude is $24\sqrt{5}$ cm

8. The area of an isosceles triangle having base 2 cm and the length of one of the equal sides 4 cm, is

a. $\sqrt{15}$ cm² **b. $\sqrt{15}/2$ cm²** **c. $2\sqrt{15}$ cm²** **d. $4\sqrt{15}$ cm²**

Answer: a. $\sqrt{15}$ cm²

Given, base of an isosceles triangle = 2 cm

Length of one of the equal sides = 4 cm

We have to find the area of an isosceles triangle

Area of an isosceles triangle = $a/4 \sqrt{4b^2 - a^2}$

Here, a = 2 cm and b = 4 cm

Area of triangle = $2/4 \sqrt{4(4)^2 - (2)^2}$

= $2/4 \sqrt{4(16) - 4}$

= $2/4 \sqrt{64 - 4}$

= $2/4 \sqrt{60}$

= $2/4 \sqrt{15 \times 4}$

= $2\sqrt{15}/2$

= $\sqrt{15}$ cm^2

Therefore, the area of isosceles triangle is $\sqrt{15}$ cm^2

9. The edges of a triangular board are 6 cm, 8 cm and 10 cm. The cost of painting it at the rate of 9 paise per cm^2 is

a. Rs 2.00 b. Rs 2.16 c. Rs 2.48 d. Rs 3.00

Answer: b. Rs 2.16

Given, the edges of a triangular board are 6 cm, 8 cm and 10 cm.

We have to find the cost of painting it at the rate of 9 paise per cm^2

Given, a = 6 cm

b = 8 cm

c = 10 cm

By Heron's formula,

Area of triangle = $\sqrt{s(s - a)(s - b)(s - c)}$

Where s= semi perimeter

s = (a + b + c)/2

Now, s = (6 + 8 + 10)/2

= 24/2

s = 12 cm

Area of triangle = $\sqrt{[12(12 - 6)(12 - 8)(12 - 10)]}$

= $\sqrt{[12(6)(4)(2)]}$

= $\sqrt{[12(24)(2)]}$

= $\sqrt{[12 \times 12 \times 2 \times 2]}$

= 12×2

Area = 24 cm^2

The cost of painting area of 1 cm^2 = 9 paise

Cost of painting entire area = 24×9

= 216 paise

= 216/100

= Rs. 2.16/-

Therefore, the cost of painting the triangular board is Rs. 2.16/-

Exercise 10.2:

Write True or False and justify your answer:

1. The area of a triangle with base 4 cm and height 6 cm is 24 cm^2. Is the given statement true or false and justify your answer.

Answer: Given, base of triangle = 4 cm

Height of triangle = 6 cm

We have to determine if the area of triangle is 24 cm^2

Area of triangle = $1/2 \times$ base $\times$ height

= $1/2 \times 4 \times 6$

= 2×6

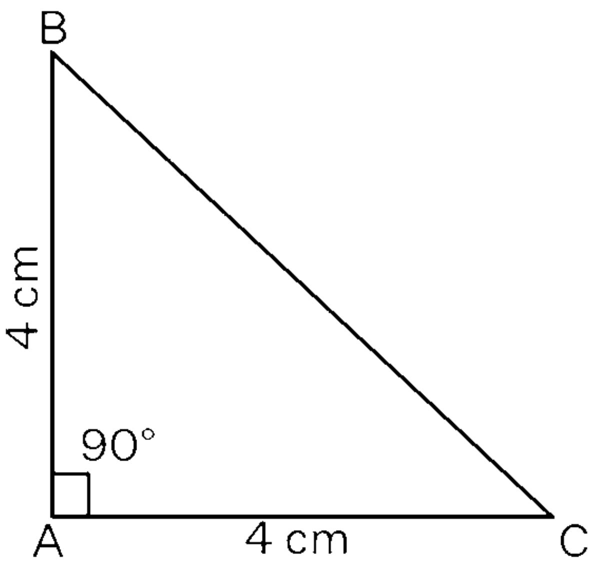
Area = 12 cm^2

2. The area of Δ ABC is 8 cm^2 in which AB = AC = 4 cm and ∠A = 90^0. Is the given statement true or false and justify your answer.

Answer: True

Given, area of triangle ABC = 8 cm^2

AB = AC = 4 cm



$\angle A = 90°$

We have to determine if the given statement is true or false.

ABC is an isosceles triangle right angled at A.

Area of triangle = 1/2 × base × height

Area of triangle ABC = 1/2 × AC × BC

= 1/2 × 4 × 4

= 2 × 4

Area = 8 cm^2

3. The area of the isosceles triangle is 5/4 √11 cm^2 , if the perimeter is 11 cm and the base is 5 cm. Is the given statement true or false and justify your answer.

Answer: True

Given, area of isosceles triangle = 5/4 √11 cm^2

Perimeter = 11 cm

Base = 5 cm

We have to determine if the given statement is true or false.

We know that in an isosceles triangle two sides are of equal length.

So, a = 5 cm

Perimeter = equal side + equal side + base

11 = 2(equal side) + 5

2(equal side) = 11 - 5

2(equal side) = 6

Equal side = 6/2

Equal side = 3 cm

So, b = 3 cm

Area of isosceles triangle = a/4 $\sqrt{4b^2 - a^2}$

$= 5/4 \sqrt{4(3)^2 - (5)^2}$

$= 5/4 \sqrt{4(9) - 25}$

$= 5/4 \sqrt{36 - 25}$

$= 5/4 \sqrt{11}$

Area $= 5/4 \sqrt{11}$ cm²

4. The area of the equilateral triangle is 20√3 cm² whose each side is 8 cm. Is the given statement true or false and justify your answer.

Answer: false

Given, area of the equilateral triangle is $20\sqrt{3}$ cm²

Length of each side = 8 cm

Area of equilateral triangle $= \sqrt{3}/4 \, (\text{side})^2$

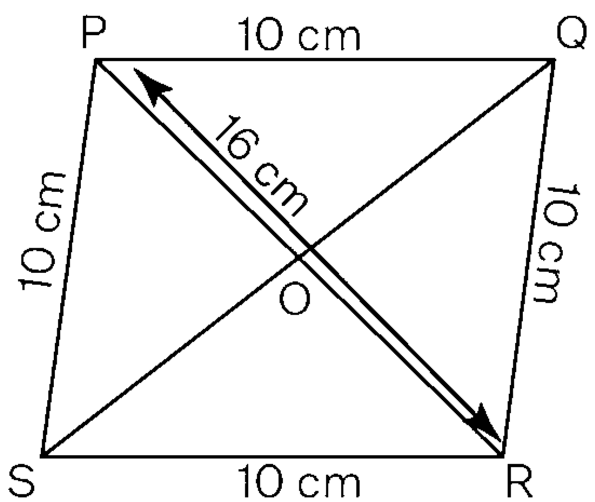
$= \sqrt{3}/4 \, (8)^2$

$= \sqrt{3}/4 \, (64)$

$= \sqrt{3}(16)$

Area $= 16\sqrt{3}$ cm²

5. If the side of a rhombus is 10 cm and one diagonal is 16 cm, the area of the rhombus is 96 cm². Is the given statement true or false and justify your answer.



Answer: True

Given, the side of the rhombus = 10 cm

Length of one diagonal = 16 cm

Area of rhombus = 96 cm²

We have to determine if the given statement is true or false.

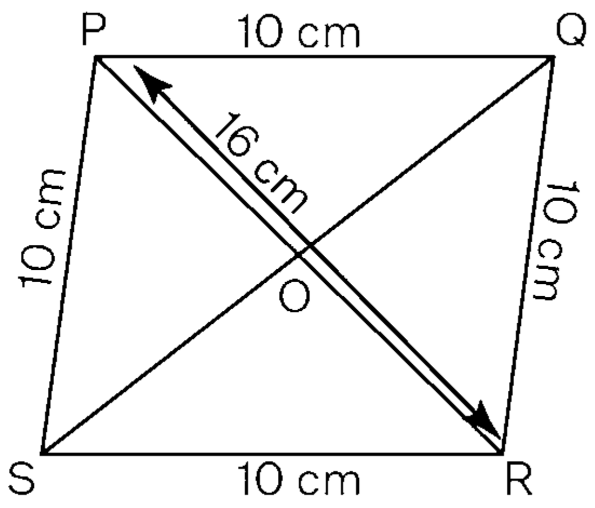
Consider a rhombus PQRS

We know that all the sides of a rhombus are equal in length.

Diagonal PR = 16 cm

We know that the diagonals of a rhombus bisect each other at right angles.

PO = OR



PO = 1/2 PR

PO = 1/2 (16)

PO = 8 cm

In triangle POQ,

By using Pythagorean theorem,

$PQ^2 = OP^2 + OQ^2$

$(10)^2 = (8)^2 + OQ^2$

$100 - 64 = OQ^2$

$OQ^2 = 36$

Taking square root,

OQ = 6 cm

Now, the diagonal SQ = 2(OQ)

= 2(6)

SQ = 12 cm

Area of rhombus = 1/2 × product of diagonals

= 1/2 × PR × SQ

= 1/2 × 16 × 12

= 8 × 12

Area = 96 cm^2

6. The base and the corresponding altitude of a parallelogram are 10 cm and 3.5 cm, respectively. The area of the parallelogram is 30 cm^2. Is the given statement true or false and justify your answer.

Answer: False

Given, the base of a parallelogram = 10 cm

Altitude = 3.5 cm

Area of parallelogram = 30 cm^2

We have to determine if the given statement is true or false.

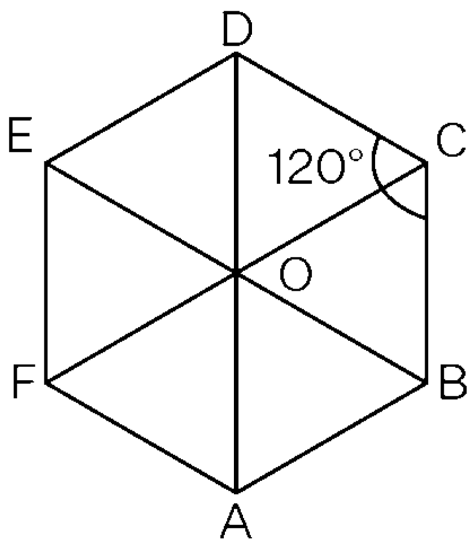
Area of parallelogram = base × height

= 10 × 3.5

= 35 cm²

7. The area of a regular hexagon of side 'a' is the sum of the areas of the five equilateral triangles with side a. Is the given statement true or false and justify your answer.

Answer: False



Given, side of regular hexagon = a

Area of regular hexagon = sum of areas of five equilateral triangles with side a.

We have to determine if the given statement is true or false.

A regular hexagon has 6 identical sides

On joining all the vertex at the centre, we will get 6 identical triangles

So, area of regular hexagon = sum of areas of 6 equilateral triangles

8. The cost of levelling the ground in the form of a triangle having the sides 51 m, 37 m and 20 m at the rate of Rs 3 per m² is Rs 918. Is the given statement true or false and justify your answer.

Answer: True

Given, the ground in the form of a triangle

The sides are 51 m, 37 m and 20 m.

The cost of levelling the ground at the rate of Rs. 3/m² is Rs. 918.

We have to determine if the given statement is true or false.

Let a = 51 m

b = 37 m

c = 20 m

By Heron's formula,

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Where s= semi perimeter

s = (a + b + c)/2

So, s = (51 + 37 + 20)/2

= 108/2

s = 54 m

Now, area = √54(54 - 51)(54 - 37)(54 - 20)

= √54(3)(17)(34)

= √9 × 6 × 3 × 17 × 17 × 2

= √9 × 3 × 3 × 2 × 2 × 17 × 17

= 3 × 3 × 2 × 17

= 18 × 17

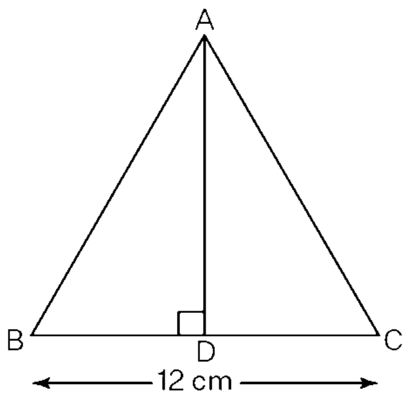
Area = 306 m^2

Cost of leveling 1 m^2 = Rs. 3

Cost of leveling entire area = 306 × 3

Cost of leveling the ground = Rs. 918

9. In a triangle, the sides are given as 11 cm, 12 cm and 13 cm. The length of the altitude is 10.25 cm corresponding to the side having length 12 cm. Is the given statement true or false and justify your answer.



Answer: True

Given, the sides of a triangle are 11 cm, 12 cm and 13 cm

The length of the altitude is 10.25 cm corresponding to the side having length 12 cm.

We have to determine if the given statement is true or false.

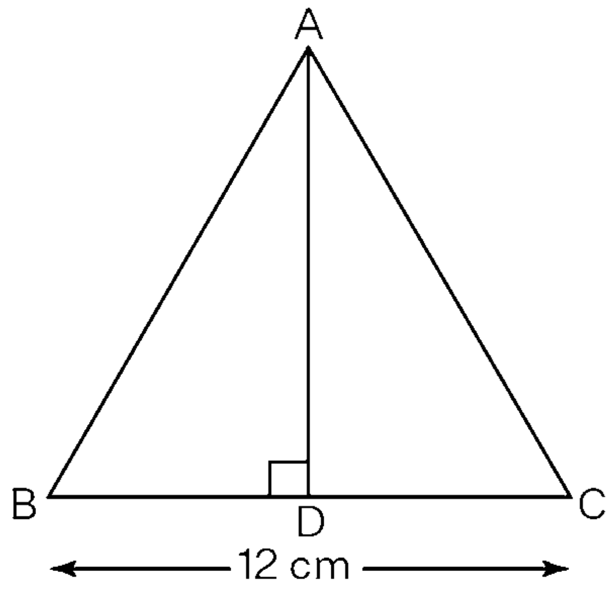
By Heron's formula,

Area of triangle = $\sqrt{s(s - a)(s - b)(s - c)}$

Where s= semi perimeter

s = (a + b + c)/2

So, s = (11 + 12 + 13)/2



= 36/2

s = 18 m

Now, area = √18(18 - 11)(18 - 12)(18 - 13)

= √18(7)(6)(5)

= √6 × 3 × 7 × 6 × 5

= 6√3 × 7 × 5

= 6√105

= 6(10.25)

= 61.5 cm²

Area of triangle = 1/2 × base × height

In triangle ABC,

Area = 1/2 × BC × AD

61.25 = 1/2 × 12 × AD

61.25 = 6 × AD

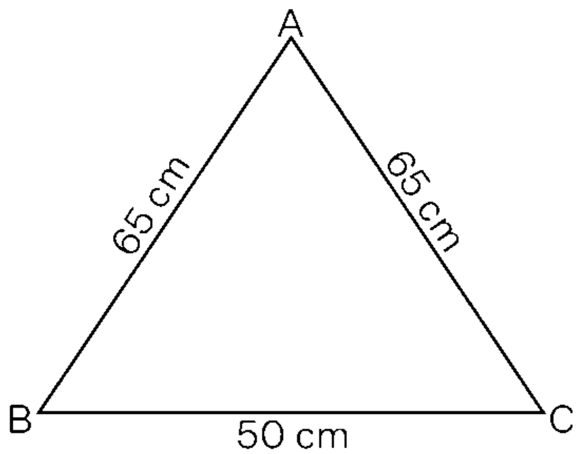
AD = 61.25/6

AD = 10.25 cm

Therefore, the given statement is true.

Exercise 10.3:

1 Find the cost of laying grass in a triangular field of sides 50 m, 65 m and 65 m at the rate of Rs 7 per m².



Answer: Given, the sides of a triangular field are 50 m, 65 m and 65 m.

We have to find the cost of laying grass at the rate of Rs. 7/m² in the field.

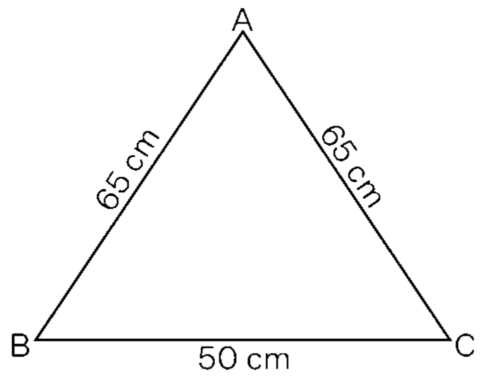
Consider a triangle ABC

The sides AB and AC are equal.

So, ABC is an isosceles triangle

Area of isosceles triangle = a/4 √4b² - a²

Where, a = base



b = length of equal side

Here, a = 50 and b= 65

Area = 50/4 √4(65)² - (50)²

= 50/4 √4(4225) - (2500)

= 50/4 √16900 - 2500

= 50/4 √14400

= 50/4 √144 × 100

= 50/4 (12 × 10)

= 500(3)

Area of triangle ABC = 1500 m²

Cost of laying grass in 1 m² = Rs. 7

Cost of laying grass for entire field = 1500 × 7

= 10500/-

Therefore, the cost of laying grass in a triangular field is Rs. 10500/-

Alternative solution:

According to the question,

Sides of the triangular field are 50 m, 65 m and 65 m.

Cost of laying grass in a triangular field = Rs 7 per m^2

Let a = 50, b = 65, c = 65

s = (a + b + c)/2

⇒ s = (50 + 65 + 65)/2

= 180/2

= 90.

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

= √90(90-50)(90-65)(90-65)

= √90×40×25×25

= 1500m^2

Cost of laying grass = Area of triangle ×Cost per m^2

= 1500×7

= Rs.10500

2. The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 13 m, 14 m and 15 m. The advertisements yield an earning of Rs 2000 per m² a year. A company hired one of its walls for 6 months. How much rent did it pay?

Answer:

Given, the triangular side walls of a flyover have been used for advertisements.

The sides of the walls are 13 m, 14 m and 15 m.

The advertisements yield an earning of Rs. 2000 per m² a year.

A company hired one of its walls for 6 months.

We have to calculate the rent paid by the company for 6 months.

Let a = 13 m

b = 14 m

c = 15 m

By Heron's formula,

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Where s = semi perimeter

s = (a + b + c)/2

So, s = (13 + 14 + 15)/2

= 42/2

s = 21 m

Area = √21(21 - 13)(21 - 14)(21 - 15)

= √21(8)(7)(6)

= √7 × 3 × 4 × 2 × 7 × 3 × 2

= 7 × 3 × 2 × 2

= 21 × 4

Area = 84 m^2

Cost of advertising per year for 1 m^2 = Rs. 2000

Advertisement cost per year for 84 m^2 = 2000 × 84 = Rs.168000

Rent paid by the company for 6 months = 168000/2 = Rs. 84000

Therefore, the rent paid by the company is Rs. 84000/-

3. From a point in the interior of an equilateral triangle, perpendiculars are drawn on the three sides. The lengths of the perpendiculars are 14 cm, 10 cm and 6 cm. Find the area of the triangle.

Answer: According to the question,

The lengths of the perpendiculars are 14 cm, 10 cm and 6 cm.

We know that,

Area of an equilateral triangle of side a = $\sqrt{3}/4\ a^2$

We divide the triangle into three triangles,

Area of triangle = (1/2 × a × 14) + (1/2 × a × 10) + (1/2 × a × 6)

$\sqrt{3}/4\ a^2$ = ½ × a × (14 + 10 + 6)

$\sqrt{3}/4\ a^2$ = ½ × a × 30

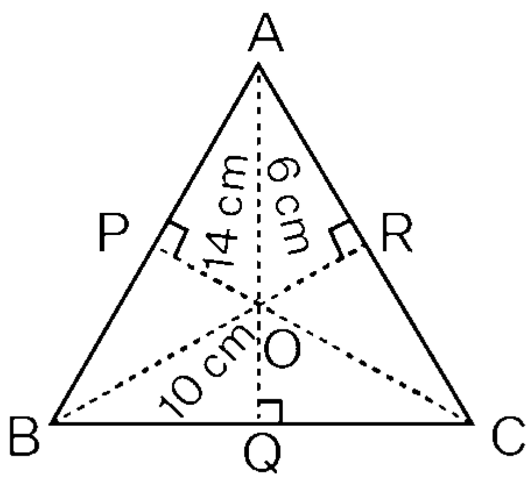
a = 60/√3

= 20√3

Area of the triangle = $\sqrt{3}/4\ a^2$

=$\sqrt{3}/4\ (20\sqrt{3})^2$

= 300√3 cm^2



Alternative Solution:

Consider a equilateral triangle ABC

P, Q and R are the perpendicular drawn on the three sides

The length of the perpendiculars are 14 cm, 10 cm and 6 cm

We have to find the area of the triangle.

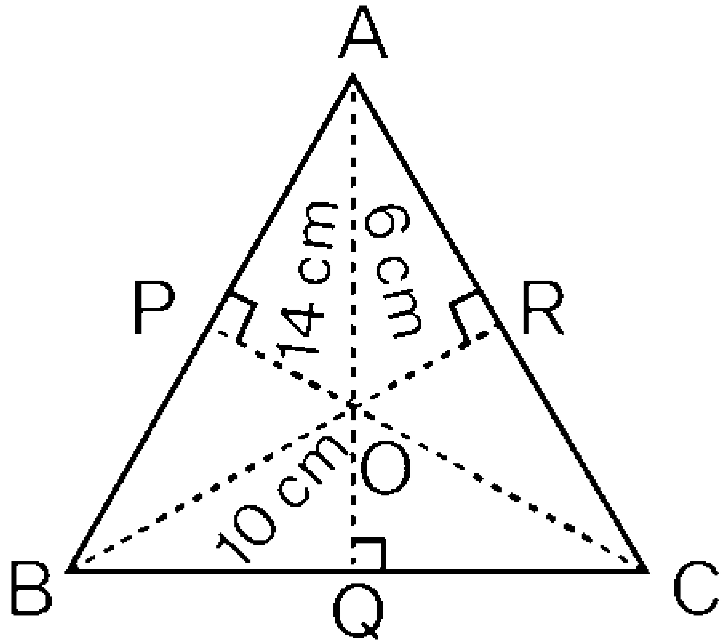
et the sides of an equilateral triangle be a cm

Area of triangle = 1/2 × base × height

Area of triangle OAB = 1/2 × AB × OP

= 1/2 × a × 14 = 7a cm^2

Area of triangle OBC = 1/2 × BC × OQ



= 1/2 × a × 10 = 5a cm^2

Area of triangle OAC = 1/2 × AC × OR

= 1/2 × a × 6 = 3a cm^2

Area of triangle ABC = area of triangle (OAB + OBC + OAC)

= 7a + 5a + 3a = 15a cm^2 ------------------------ (1)

By Heron's formula,

Area of triangle = √s(s - a)(s - b)(s - c)

Where s = semiperimeter

s = (a + b + c)/2

So, s = (a + a + a)/2

= 3a/2

s = 1.5a

Area = √1.5a(1.5a - a)(1.5a - a)(1.5a - a)

= $\sqrt{1.5a(0.5a)^3}$

= $\sqrt{1.5(0.125)a^4}$

= $a^2\sqrt{0.1875}$

= $0.4330a^2$ ------------------------ (2)

Comparing (1) and (2),

15a = 0.4330 a^2

0.433a = 15

a = 15/0.433

a = 34.64 cm

Area of equilateral triangle = √3/4 (side)²

= √3/4 (34.64)²

= √3/4 (1199.99)

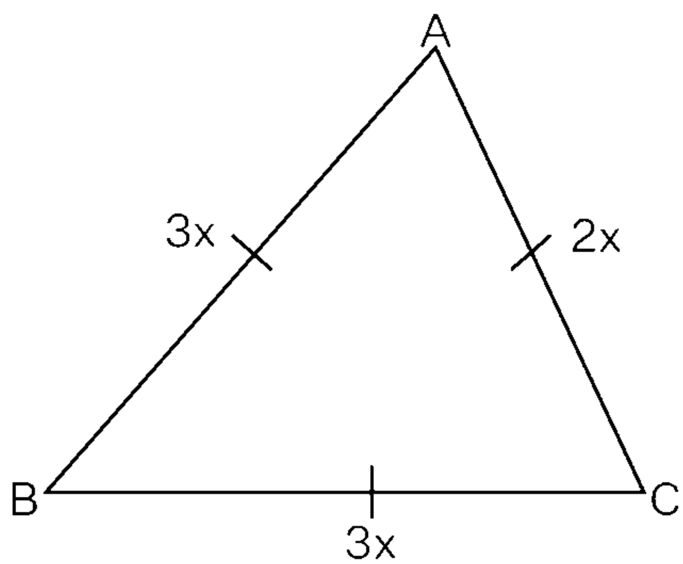
= √3 (299.99)

= 300√3 cm²

Therefore, the area of triangle is 300√3 cm²

4. The perimeter of an isosceles triangle is 32 cm. The ratio of the equal side to its base is 3 : 2. Find the area of the triangle.

Answer:



Given, the perimeter of an isosceles triangle is 32 cm

The ratio of the equal side to its base is 3 : 2

We have to find the area of the triangle

We know that an isosceles triangle has two equal sides.

Le the sides be AB = BC = 3x and AC = 2x

Perimeter = equal side + equal side + base

32 = 3x + 3x + 2x

32 = 8x

x = 32/8

x = 4 cm

So, AB = BC = 3(4) = 12 cm

AC = 2(4) = 8 cm

Area of isosceles triangle = a/4 √4b² - a²

Where, a = base

b = equal side

Here, a = 8 cm and b = 12 cm

Area = $8/4\ \sqrt{4(12)^2-(8)^2}$

= $2\sqrt{4(144) - 64}$

= $2\sqrt{576 - 64}$

= $2\sqrt{512}$

= $2\sqrt{64 \times 8}$

= $2 \times 8(\sqrt{8})$

= $16(2\sqrt{2})$

= $32\sqrt{2}$ cm²

Therefore, the area of isosceles triangle is $32\sqrt{2}$ cm²

Alternate Solution:

According to the question,

Perimeter of the isosceles triangle = 32 cm

It is also given that,

Ratio of equal side to base = 3 : 2

Let the equal side = 3x

So, base = 2x

Perimeter of the triangle = 32

⇒ 3x + 3x + 2x = 32

⇒ 8x = 32

⇒ x = 4.

Equal side = 3x = 3×4 = 12

Base = 2x = 2×4 = 8

The sides of the triangle = 12cm, 12cm and 8cm.

Let a = 12, b = 12, c = 8

s = (a + b + c)/2

⇒ s = (12 + 12 + 8)/2

= 32/2

= 16.

Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

= √16(16-12)(16-12)(16-8)

= √16 × 4 × 4 × 8

= 32√2 cm^2

5. Find the area of a parallelogram given in Fig. 12.2. Also find the length of the altitude from vertex A on the side DC.

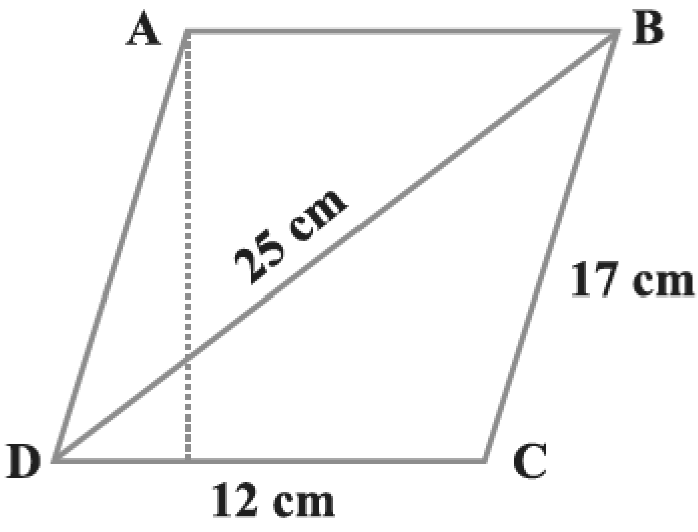


Fig. 12.2

Answer: Given, ABCD is a parallelogram

We have to find the area of the parallelogram

We know that area of parallelogram = 2 × area of triangle BCD

In triangle BCD,

a = 25 cm

b = 12 cm

c = 17 cm

By Heron's formula,

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Where s = semi perimeter

s = (a + b + c)/2

So, s = (25 + 12 + 17)/2

= 54/2

s = 27 cm

Area of triangle BCD = √27(27 - 25)(27 - 17)(27 - 12)

= √27(2)(10)(15)

= √9 × 3 × 2 × 5 × 2 × 5 × 3

= 3 × 3 × 5 × 2

= 9 × 10

Area of triangle BCD = 90 cm²

Area of ABCD = 2(90)

Area of parallelogram = 180 cm²

We have to find the length of the altitude from vertex A on the side DC.

Let the length of the altitude be h cm

Area of parallelogram = base × height

180 = 12 × h

h = 180/12

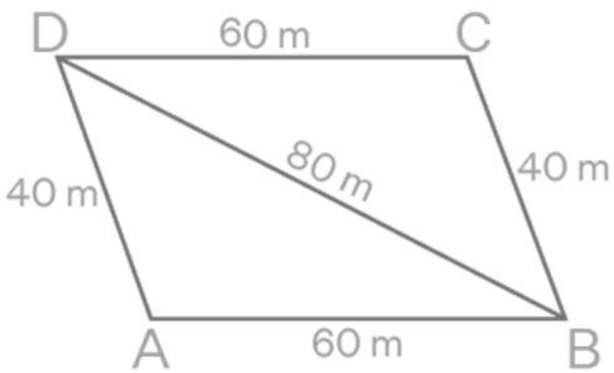
h = 90/6

h = 30/2

h = 15 cm

Therefore, the altitude is 15 cm

6. A field in the form of a parallelogram has sides 60 m and 40 m and one of its diagonals is 80 m long. Find the area of the parallelogram.



Answer: Given, the field is in the form of parallelogram

The sides are 60 m and 40 m

One of its diagonals is 80 m

We have to find the area of parallelogram

Consider a parallelogram ABCD

AB = CD = 60 m

BC = AD = 40 m

BD = 80 m

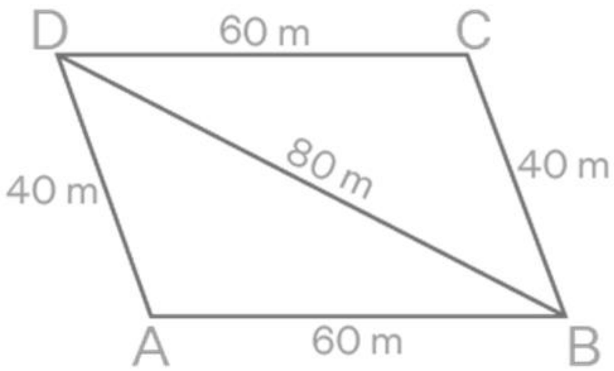
We know that area of parallelogram ABCD = 2 (area of triangle BCD)

In triangle BCD,

a = 60 m

b = 40 m

c = 80 m



By Heron's formula,

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Where s = semiperimeter

s = (a + b + c)/2

So, s = (60 + 40 + 80)/2

= 180/2

s = 90 cm

Area = √90(90 - 60)(90 - 40)(90 - 80)

= √90(30)(50)(10)

= √100 × 5 × 100 × 27

= 100√9 × 3 × 5

= 300√15 cm²

Area of triangle BCD = 300√15 cm²

Area of parallelogram ABCD = 2(300√15)

= 600√15

Therefore, area of parallelogram is 600√15 cm²

7. The perimeter of a triangular field is 420 m and its sides are in the ratio 6 : 7 : 8. Find the area of the triangular field.

Answer: Given, the perimeter of the triangular field is 420 m

The sides are in the ratio 6 : 7 : 8

We have to find the area of the triangular field.

Let the sides be

a = 6x

b = 7x

c = 8x

Perimeter = a + b + c

$420 = 6x + 7x + 8x$

$21x = 420$

$x = 420/21$

$x = 20$

$a = 6(20) = 120$

$b = 7(20) = 140$

$c = 8(20) = 160$

By Heron's formula,

Area of triangle = $\sqrt{s(s - a)(s - b)(s - c)}$

Where s = semi perimeter

$s = (a + b + c)/2$

So, $s = (120 + 140 + 160)/2$

$= 420/2$

s = 210 cm

Area = $\sqrt{210(210 - 120)(210 - 140)(210 - 160)}$

$= \sqrt{210(90)(70)(50)}$

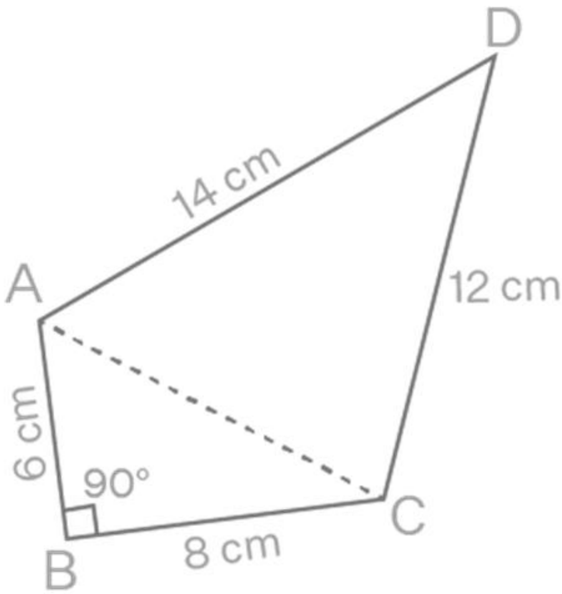
$= \sqrt{7 \times 3 \times 9 \times 7 \times 5 \times 10000}$

$= 100 \times 3 \times 7 (\sqrt{3 \times 5})$

$= 2100\sqrt{15}$ cm²

Therefore, the area of the triangular field is $2100\sqrt{15}$ cm²

8. The sides of a quadrilateral ABCD are 6 cm, 8 cm, 12 cm and 14 cm (taken in order) respectively, and the angle between the first two sides is a right angle. Find its area.



Solution: Given, ABCD is a quadrilateral

The sides of the quadrilateral are 6 cm, 8 cm, 12 cm and 14 cm (taken in order)

The angle between the first two sides is a right angle.

We have to find the area of the quadrilateral.

Given, ABC is a right triangle with B at right angle.

By using Pythagorean theorem,

$AC^2 = AB^2 + BC^2$

$AC^2 = (6)^2 + (8)^2$

$AC^2 = 36 + 64$

$AC^2 = 100$

Taking square root,

AC = 10 cm

Area of quadrilateral ABCD = area of triangle ABC + area of triangle ACD

Area of triangle = 1/2 × base × height

Area of triangle ABC = 1/2 × BC × AB

= 1/2 × 8 × 6

= 4 × 6

Area of triangle ABC = 24 cm^2

Considering triangle ACD,

a = 10 cm

b = 12 cm

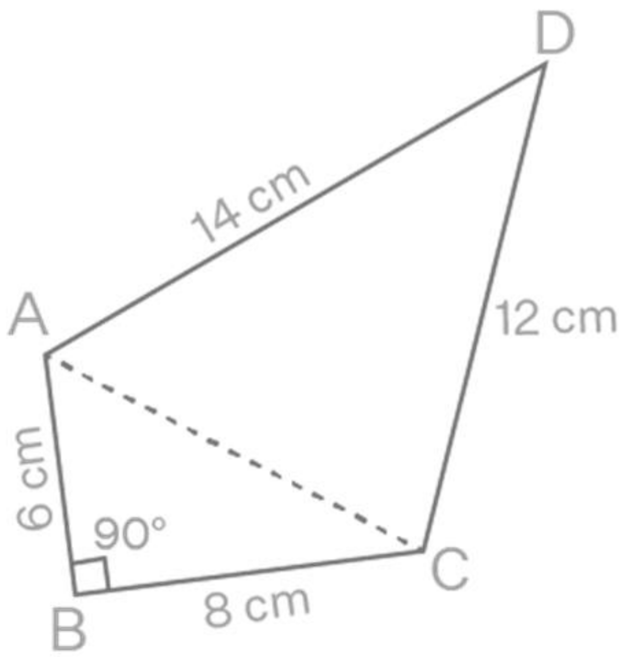
c = 14 cm

By Heron's formula,

Area of triangle = $\sqrt{s(s - a)(s - b)(s - c)}$

Where s = semi perimeter

s = (a + b + c)/2



So, s = (10 + 12 + 14)/2

= 36/2

s = 18 cm

Area of triangle ACD = $\sqrt{18(18 - 10)(18 - 12)(18 - 14)}$

= $\sqrt{18(8)(6)(4)}$

= $\sqrt{9 \times 2 \times 4 \times 2 \times 3 \times 2 \times 4}$

= $(3 \times 2 \times 4)\sqrt{3 \times 2}$

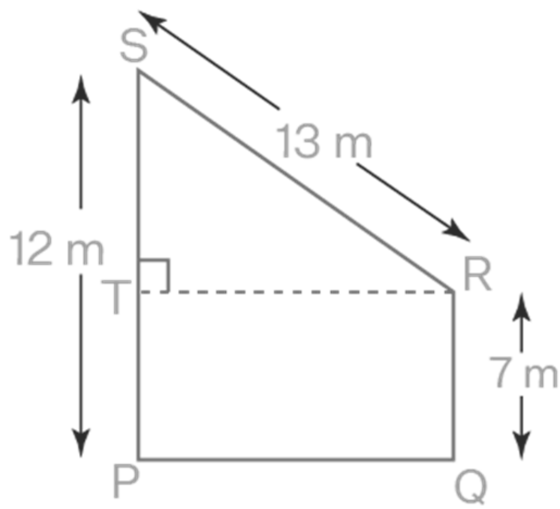
Area of triangle ACD = $24\sqrt{6}$ cm²

Area of ABCD = $24 + 24\sqrt{6}$

= $24(1 + \sqrt{6})$ cm²

Therefore, the area of quadrilateral ABCD is$24(1 + \sqrt{6})$ cm²

9. Find the area of the trapezium PQRS with height PQ given in Fig. 12.3



Answer: Given, PQRS is a trapezium with height PQ.

We have to find the area of trapezium PQRS.

Draw RT perpendicular to PS

Now, ST = PS - PT

= 12 - 7

ST = 5 m

Also, PT = RQ

Area of trapezium PQRS = area of triangle STR + area of rectangle PQRT

Considering triangle STR,

STR is a right angle with T at right angle.

By using Pythagorean theorem,

$SR^2 = ST^2 + TR^2$

$(13)^2 = (5)^2 + TR^2$

$169 = 25 + TR^2$

$TR^2 = 169 - 25$

$TR^2 = 144$

Taking square root,

TR = 12 cm

Area of triangle = 1/2 × base × height

Area of triangle STR = 1/2 × TR × ST

= 1/2 × 12 × 5

= 6 × 5

Area of triangle STR = 30 cm^2

Area of rectangle = length × width

Area of rectangle PQRT = PQ × RQ

= 12 × 7

Area of rectangle PQRS = 84 cm^2

Now, area of trapezium PORS = 30 + 84

= 114 cm^2

Therefore, the area of trapezium is 114 cm^2

Exercise 10.4:

1. How much paper of each shade is needed to make a kite given in Fig. 12.4, in which ABCD is a square with diagonal 44 cm.

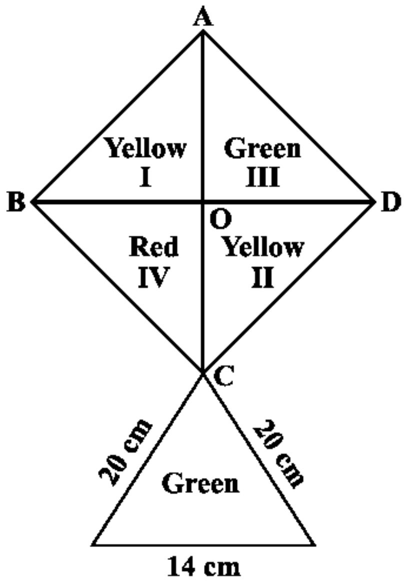


Fig. 12.4

Answer: Given, ABCD is a square with diagonal 44 cm

We have to find the quantity of paper of each shade needed to make a kite as shown in

the given figure.

We know that all the sides of a square are equal.

AB = BC = CD = AD

In triangle ACD,

Given, AC = 44 cm

By using Pythagorean theorem,

$AC^2 = AD^2 + DC^2$

Since AD = DC

$(44)^2 = AD^2 + AD^2$

$2AD^2 = 1936$

$AD^2 = 1936/2$

$AD^2 = 968$

Taking square root,

$AD = \sqrt{22 \times 44}$

$= \sqrt{2 \times 11 \times 4 \times 11}$

$= (2 \times 11)\sqrt{2}$

$AD = 22\sqrt{2}$ cm

Area of square = side × side

Area of square ABCD = AB × CD

$= 22\sqrt{2} \times 22\sqrt{2}$

$= 968\ cm^2$

From the figure,

We observe that the square is divided into four equal parts.

2 yellow parts, 1 green part and 1 red part.

Area of green region = 968/4 = 242 cm^2

Area of red region = 968/4 = 242 cm^2

Area of 2 yellow regions = 968/2 = 484 cm^2

Area of green part = area of triangle PCQ

In triangle PCQ,

PC = QC = 20 cm

PQ = 14 cm

This implies PCQ is an isosceles triangle

Area of isosceles triangle = $a/4 \sqrt{4b^2 - a^2}$

Here, a = 14 cm and b = 20 cm

$= 14/4 \sqrt{4(20)^2 - (14)^2}$

$= 7/2 \sqrt{4(400) - 196}$

$= 7/2 \sqrt{1600 - 196}$

$= 7/2 \sqrt{1404}$

= 7/2 (37.46)

= 131.04 cm^2

Area of green part = 131.04 cm^2

Therefore, area of green part = 242 + 131.04

= 373.04 cm^2

Therefore, the paper required for each shade to make the kite is

Yellow shade = 484 cm^2

Red shade = 242 cm^2

Green shade = 373.04 cm^2

2. The perimeter of a triangle is 50 cm. One side of a triangle is 4 cm longer than the smaller side and the third side is 6 cm less than twice the smaller side. Find the area of the triangle.

Answer: Given, perimeter of a triangle = 50 cm

One side of a triangle is 4 cm longer than the smaller side

The third sides is 6 cm less than twice the smaller side

We have to find the area of the triangle.

Let the smaller side = x cm

One side = x + 4 cm

Third side = 2x - 6 cm

Perimeter of triangle = sum of all the sides of triangle

50 = x + 4 + x + 2x - 6

50 = 4x - 2

4x = 50 + 2

4x = 52

x = 52/4

x = 13 cm

So, smaller side = 13 cm

One side = 13 + 4 = 17 cm

Third side = 13(2) - 6 = 26 - 6 = 20 cm

By Heron's formula,

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Where s = semiperimeter

s = (a + b + c)/2

Here, a = 13 cm, b = 17 cm and c = 20 cm

So, s = (13 + 17 + 20)/2

= 50/2

s = 25 cm

Area = $\sqrt{25(25 - 13)(25 - 17)(25 - 20)}$

= $\sqrt{25(12)(8)(5)}$

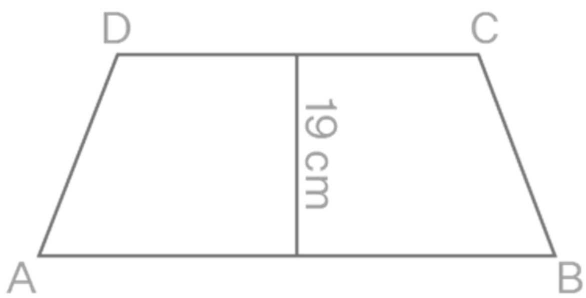
= $\sqrt{5 \times 5 \times 4 \times 3 \times 4 \times 2 \times 5}$

= $(5 \times 4)\sqrt{5 \times 3 \times 2}$

= $20\sqrt{30}$ cm²

Therefore, the area of the triangle is $20\sqrt{30}$ cm².

3. The area of a trapezium is 475 cm² and the height is 19 cm. Find the lengths of its two parallel sides if one side is 4 cm greater than the other.



Answer: Given, area of a trapezium is 475 cm²

Height = 19 cm

We have to find the lengths of its two parallel sides if one side is 4 cm greater than the other.

Consider the trapezium ABCD

Let the side CD = x cm

Given, AB = x+4 cm

Area of trapezium = 1/2 × (sum of parallel sides) × distance between parallel sides

475 = 1/2 × (AB + CD) × 19

475 = 1/2 × (x + x + 4) × 19

475 × 2 = (2x + 4) × 19

2x + 4 = (475 × 2)/19

2x + 4 = 25 × 2

2x + 4 = 50

2x = 50 - 4

2x = 46

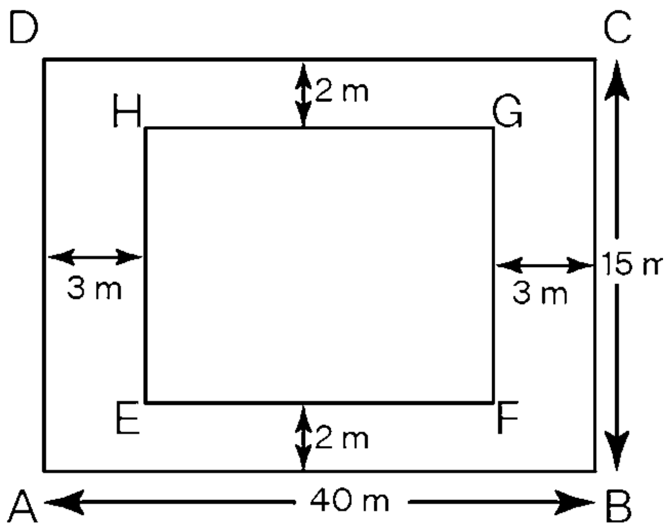
x = 23 cm

So, CD = 23 cm

AB = 23 + 4 = 27 cm

Therefore, the lengths of the parallel sides are 23 cm and 27 cm.

4. A rectangular plot is given for constructing a house, having a measurement of 40 m long and 15 m in the front. According to the laws, a minimum of 3 m, wide space should be left in the front and back each and 2 m wide space on each of other sides. Find the largest area where house can be constructed.



Answer: Given, a rectangular plot for constructing a house

The dimension of the plot is 40 m long and 15 m in the front.

According to the laws, a minimum of 3 m wide space should be left in the front and back each and 2 m wide space on each of the other sides.

We have to find the largest area where houses can be constructed.

Let ABCD be the rectangular plot

AB = CD = 40 m

BC = AD = 15 m

Let the inner rectangle be EFGH

EF = GH = 40 - 3 - 3

So, EF = GH = 34 cm

EH = FG = 15 - 2 - 2

So, EH = FG = 11 cm

The largest area to construct the house = area of rectangle EFGH

Area of rectangle = length × breadth

Area of rectangle EFGH = EF × GH

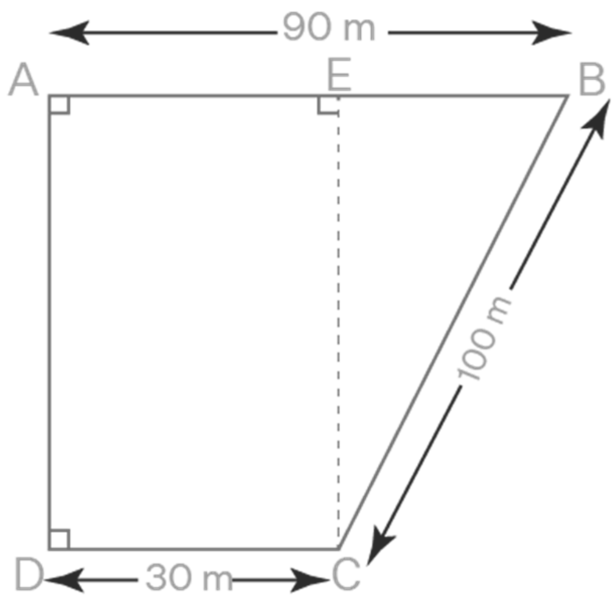
= 34 × 11

= 374 cm²

Therefore, the largest area where the house can be constructed is 374 cm²

5. A field is in the shape of a trapezium having parallel sides 90 m and 30 m. These sides meet the third side at right angles. The length of the fourth side is 100 m. If it Rs. 4 costs to plough 1 m^2 of the field, find the total cost of ploughing the field.

Answer: Given, the field is in the shape of a trapezium



The parallel sides are 90 m and 30 m

The parallel sides meet the third side at right angles

The length of the fourth side is 100 m

We have to find the cost of ploughing the entire field at the rate of Rs. 4/m^2

Consider a trapezium ABCD

Draw CE perpendicular to AB

DC = AE = 30 m

EB = AB - AE

EB = 90 - 30

EB = 60 m

Consider right angled triangle BEC,

By using Pythagorean theorem,

$BC^2 = BE^2 + EC^2$

$(100)^2 = (60)^2 + EC^2$

$EC^2 = 10000 - 3600$

$EC^2 = 6400$

Taking square root,

EC = 80 m

Area of trapezium = 1/2 × sum of parallel sides × distance between parallel sides

Area of trapezium ABCD = 1/2 × (AB + CD) × EC

= 1/2 × (90 + 30) × 80

= 1/2 × 120 × 80

= 60 × 80

= 4800 m²

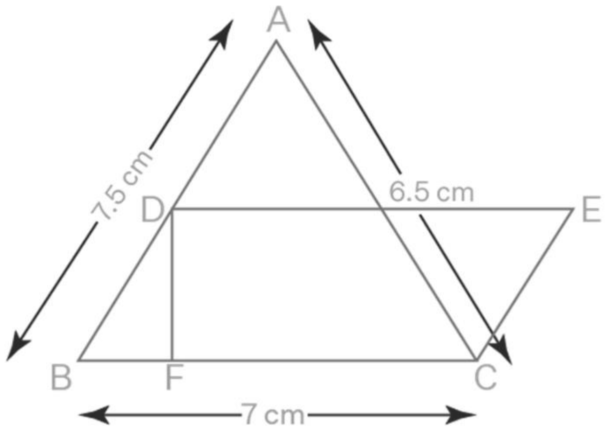
Cost of ploughing 1 m² of field = Rs. 4

Cost of ploughing 4800 m² of field = 4800 × 4

= Rs. 19200

Therefore, the cost of ploughing the field is Rs. 19200/-

6. In Fig. 12.5, Δ ABC has sides AB = 7.5 cm, AC = 6.5 cm and BC = 7 cm. On base BC a parallelogram DBCE of same area as that of Δ ABC is constructed. Find the height DF of the parallelogram.



Answer:

Given, ABC is a triangle

The sides are AB = 7.5 cm

AC = 6.5 cm

BC = 7 cm

On base BC a parallelogram DBCE of the same area as that of triangle ABC is constructed.

We have to find the height DF of the parallelogram

In triangle ABC,

By Heron's formula,

Area of triangle = $\sqrt{s(s - a)(s - b)(s - c)}$

Where s = semiperimeter

s = (a + b + c)/2

Here, a = 7.5 cm, b = 7 cm and c = 6.5 cm

So, s = (7.5 + 7 + 6.5)/2

= 21/2

s = 10.5 cm

Area = $\sqrt{10.5(10.5 - 7.5)(10.5 - 7)(10.5 - 6.5)}$

= $\sqrt{10.5(3)(3.5)(4)}$

= $\sqrt{36.75 \times 12}$

= $\sqrt{441}$

Area of triangle ABC = 21 cm²

Area of parallelogram = base × height

Area of parallelogram DBCE = BC × DF

Given, area of parallelogram DBCE = area of triangle ABC

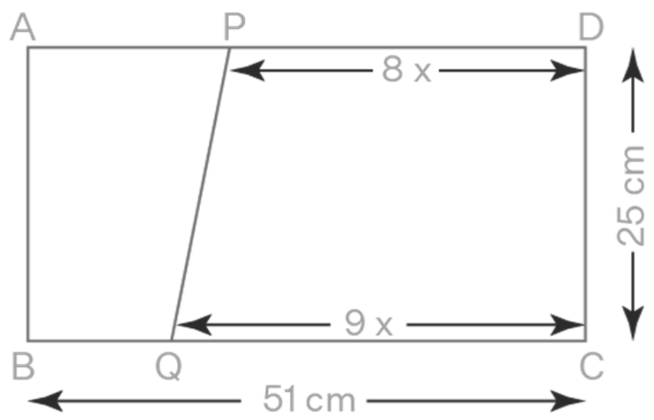
So, 21 = 7 × DF

DF = 21/7

DF = 3 cm

Therefore, the height of the parallelogram is 3 cm.

7. The dimensions of a rectangle ABCD are 51 cm × 25 cm. A trapezium PQCD with its parallel sides QC and PD in the ratio 9 : 8, is cut off from the rectangle as shown in the Fig. 12.6. If the area of the trapezium PQCD is 5/6 th part of the area of the rectangle, find the lengths QC and PD.



Answer:

Given, ABCD is a rectangle

The dimensions are 51 cm × 25 cm

A trapezium PQCD is cut off from the rectangle

The parallel sides of the trapezium QC and PD are in the ratio 9 : 8

The area of the trapezium PQCD is $5/6^{th}$ part of the area of the rectangle.

We have to find the lengths QC and PD.

Given, QC : PD = 9 : 8

Let the length of the parallel sides be

QC = 9x

PD = 8x

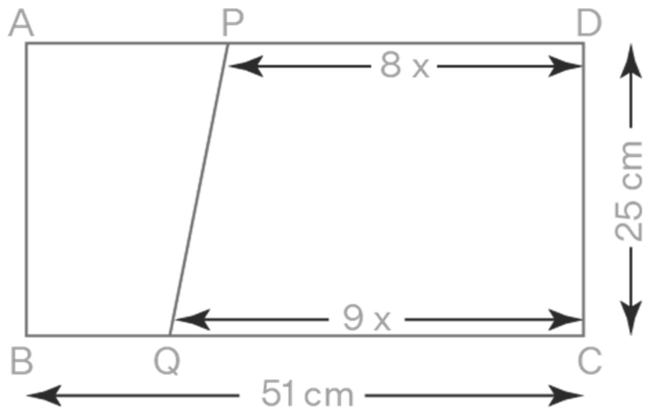
Area of rectangle = length × breadth

Area of rectangle ABCD = AD × AB

= 51 × 25

= 1275 cm²

Area of trapezium = 1/2 × sum of parallel sides × distance between parallel sides



Area of trapezium PQCD = 1/2 × (QC + PD) × DC

= 1/2 × (9x + 8x) × 25

= 1/2 × 17x × 25

Given, area of trapezium PQCD = 5/6 × area of rectangle ABCD

1/2 × 17x × 25 = 5/6 × 1275

1/2 × 17x = (5 × 1275)/(6 × 25)

1/2 × 17x = (5 × 51)/6

17x = (5 × 51 × 2)/6

17x = (5 × 51)/3

17x = 5 × 17

x = 5

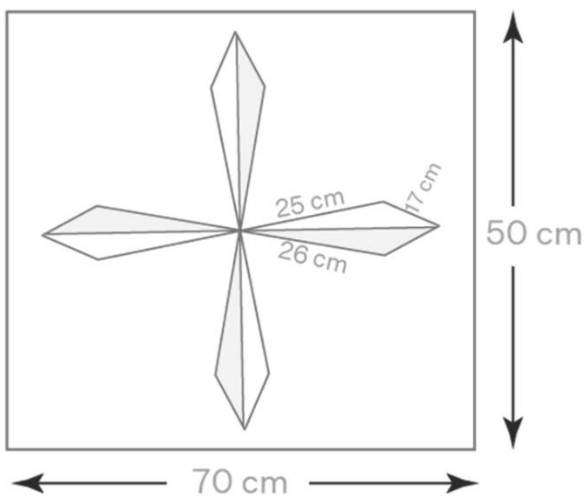
So, QC = 9x = 9(5) = 45 cm

PD = 8x = 8(5) = 40 cm

Therefore, the lengths of QC and PD are 45 cm and 40 cm.

8. A design is made on a rectangular tile of dimensions 50 cm × 70 cm as shown in Fig. 12.7. The design shows 8 triangles, each side 26 cm, 17 cm and 25 cm. Find the total area of the design and the remaining area of the tile.

Answer: Given, a design is made on a rectangular tile of dimension 50 cm × 70 cm



The design shows 8 triangles with dimensions 26 cm, 17 cm and 25 cm.

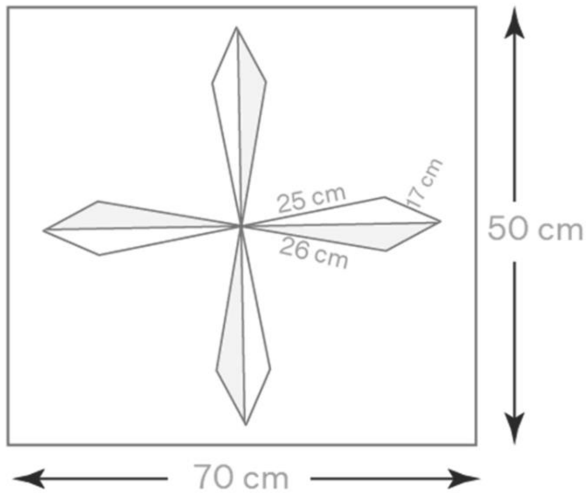
We have to find the total area of the design and the remaining area of the tile.

Area of rectangle = length × breadth

Area of rectangular tile = 50 × 70

= 3500 cm²

Consider one triangle out of 8 triangles,



By Heron's formula,

Area of triangle = $\sqrt{s(s - a)(s - b)(s - c)}$

Where s = semi perimeter

s = (a + b + c)/2

Here, a = 26 cm, b = 17 cm and c = 25 cm

So, s = (26 + 17 + 25)/2

= 68/2

s = 34 cm

Area = $\sqrt{34(34 - 26)(34 - 17)(34 - 25)}$

= $\sqrt{34(8)(17)(9)}$

= $\sqrt{17 \times 2 \times 2 \times 4 \times 17 \times 9}$

= 17 × 2 × 2 × 3

Area of one triangle = 204 cm^2

Area of 8 triangles = 8(204) = 1632 cm^2

Therefore, total area of the design is 1632 cm^2

Remaining area of the tile = area of rectangular tile - area of 8 triangles

= 3500 - 1632

= 1868 cm^2

Therefore, the remaining area of the tiles is 1868 cm^2