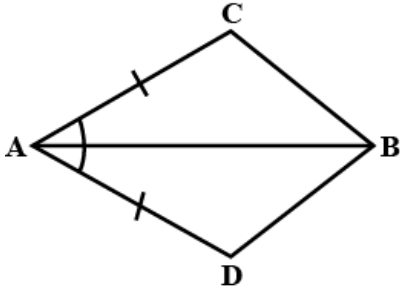




Exercise: 7.1

Q1. In quadrilateral ACBD, $AC = AD$ and AB bisects $\angle A$ (see Fig. 7.16). Show that $\triangle ABC \cong \triangle ABD$. What can you say about BC and BD ?



Answer:

Given: $AC = AD$ and AB bisects $\angle A$

To Prove: $\triangle ABC \cong \triangle ABD$

We can show two sides and the included angle of $\triangle ABC$ is equal to the corresponding sides and included angle of $\triangle ABD$.

In $\triangle ABC$ and $\triangle ABD$,

$AC = AD$ (Given)

$\angle CAB = \angle DAB$ (AB bisects $\angle A$)

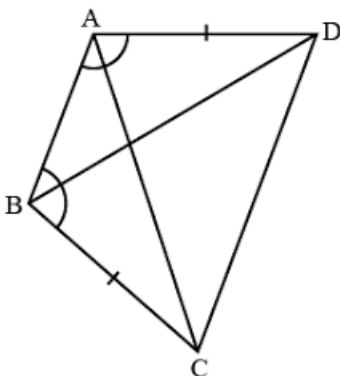
$AB = AB$ (Common)

$\therefore \triangle ABC \cong \triangle ABD$ (By SAS congruence rule)

$\therefore BC = BD$ (By CPCT)

Therefore, BC and BD are of equal lengths.

Q2. $ABCD$ is a quadrilateral in which $AD = BC$ and $\angle DAB = \angle CBA$ (see Fig. 7.17). Prove that





- (i) $\triangle ABD \cong \triangle BAC$
- (ii) $BD = AC$
- (iii) $\angle ABD = \angle BAC$.

Answer:

The parameters given by the questions are $\angle DAB = \angle CBA$ and $AD = BC$.

(i) $\triangle ABD$ and $\triangle BAC$ are similar by SAS congruency as

$AB = BA$ (It is the common arm)

$\angle DAB = \angle CBA$ and $AD = BC$ (These are given in the question)

So, triangles ABD and BAC are similar i.e. $\triangle ABD \cong \triangle BAC$. (Hence proved).

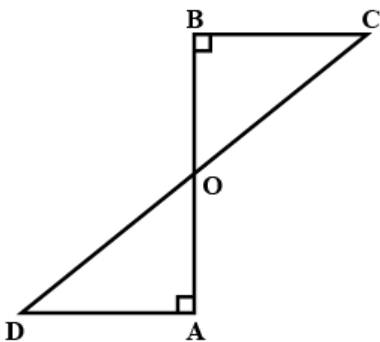
(ii) It is now known that $\triangle ABD \cong \triangle BAC$ so,

$BD = AC$ (by the rule of CPCT).

(iii) Since $\triangle ABD \cong \triangle BAC$ so,

Angles $\angle ABD = \angle BAC$ (by the rule of CPCT).

Q3. AD and BC are equal perpendiculars to a line segment AB (see Fig. 7.18). Show that CD bisects AB.

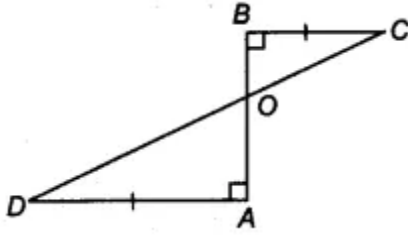


Answer:

Given: $AD \perp AB$, $BC \perp AB$, and $AD = BC$

To Prove: CD bisects AB or $OA = OB$

We can show that the two triangles OBC and OAD are congruent by using AAS congruency rule and then we can say corresponding parts of congruent triangles will be equal.



Consider two triangles $\triangle BOC$ and $\triangle AOD$,

In $\triangle BOC$ and $\triangle AOD$,

$\angle BOC = \angle AOD$ (Vertically opposite angles)

$\angle CBO = \angle DAO$ (Each 90° , since AD and BC are \perp to AB)

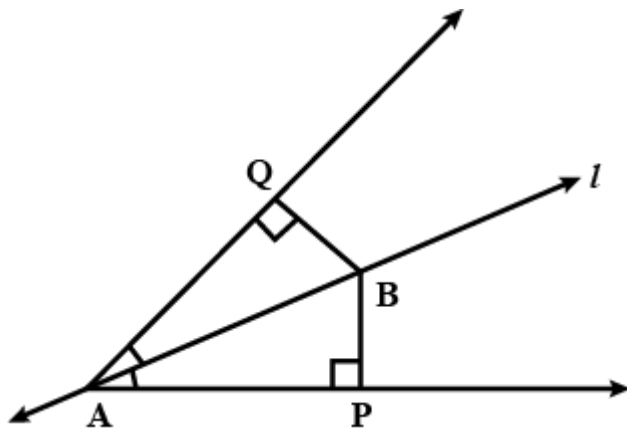
$BC = AD$ (Given)

$\therefore \triangle BOC \cong \triangle AOD$ (AAS congruence rule)

$\therefore BO = AO$ (By CPCT)

Thus, CD bisects AB , and O is the mid-point of AB .

Q5. Line l is the bisector of an angle $\angle A$ and B is any point on l . BP and BQ are perpendiculars from B to the arms of $\angle A$ (see Fig. 7.20). Show that:



Answer:

Given: l is the bisector of an angle $\angle A$ and $BP \perp AP$ and $BQ \perp AQ$

To Prove: $\triangle APB \cong \triangle AQB$ and $BP = BQ$

i) We can show two triangles APB and AQB are congruent by using AAS congruency rule and then show that the corresponding parts of congruent triangles will be equal.

In $\triangle APB$ and $\triangle AQB$,



$\angle BAP = \angle BAQ$ (l is the angle bisector of $\angle A$)

$\angle APB = \angle AQB$ (Each 90°)

$AB = AB$ (Common)

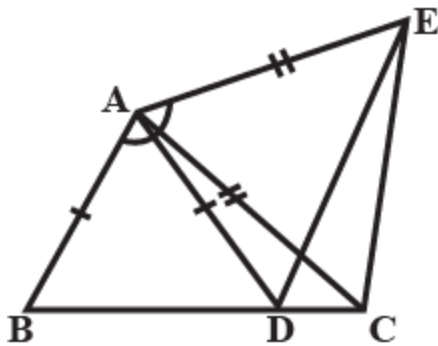
$\therefore \triangle APB \cong \triangle AQB$ (By AAS congruence rule)

ii) Since, $\triangle APB \cong \triangle AQB$

$\therefore BP = BQ$ (By CPCT)

Or, it can be said that point B is equidistant from the arms of $\angle A$.

6. In Fig. 7.21, $AC = AE$, $AB = AD$ and $\angle BAD = \angle EAC$. Show that $BC = DE$.



Answer:

Given: $AC = AE$, $AB = AD$ and $\angle BAD = \angle EAC$

To Prove: $BC = DE$

We can show two triangles BAC and DAE are congruent triangles by using SAS congruency rule and then we can say corresponding parts of congruent triangles will be equal.

It is given that $\angle BAD = \angle EAC$

Thus, by adding $\angle DAC$ to both sides of this equation, we get

$\angle BAD + \angle DAC = \angle EAC + \angle DAC$ ($\angle DAC$ is common)

$\angle BAC = \angle DAE$

In $\triangle BAC$ and $\triangle DAE$,

$AB = AD$ (Given)

$\angle BAC = \angle DAE$ (Proven above)



$AC = AE$ (Given)

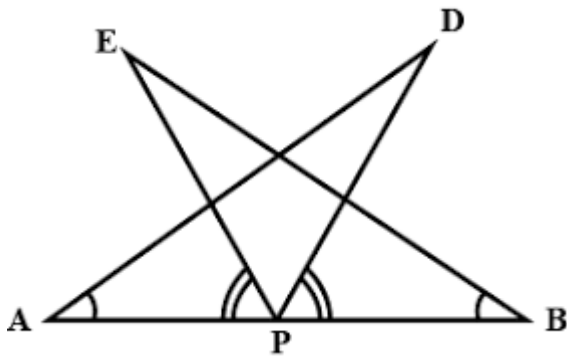
$\therefore \triangle BAC \cong \triangle DAE$ (By SAS congruence rule)

$\therefore BC = DE$ (By CPCT)

Q7. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$ (see Fig. 7.22). Show that

(i) $\triangle DAP \cong \triangle EBP$

(ii) $AD = BE$



Given: P is its mid-point of AB, $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$

To Prove: i) $\triangle DAP \cong \triangle EBP$ and (ii) $AD = BE$.

Reasoning: We can show two triangles DAP and EBP congruent by using ASA congruency rule and then we can say corresponding parts of congruent triangles will be equal.

i) It is given that $\angle EPA = \angle DPB$

$\angle EPA + \angle DPE = \angle DPB + \angle DPE$ ($\angle DPE$ is common)

$\therefore \angle DPA = \angle EPB$

In $\triangle DAP$ and $\triangle EBP$,

$\angle DAP = \angle EBP$ (Given)

$AP = BP$ (P is mid-point of AB)

$\angle DPA = \angle EPB$ (Proven above)

$\therefore \triangle DAP \cong \triangle EBP$ (ASA congruence rule)

ii) Since, $\triangle DAP \cong \triangle EBP$



$\therefore AD = BE$ (By CPCT)

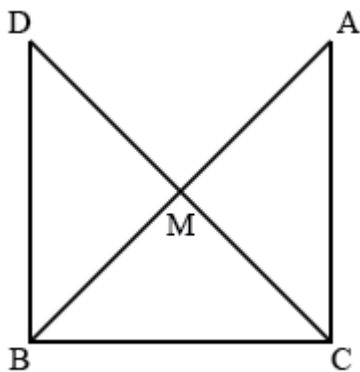
8. In right triangle ABC, right-angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that $DM = CM$. Point D is joined to point B (see Fig. 7.23). Show that:

(i) $\triangle AMC \cong \triangle BMD$

(ii) $\angle DBC$ is a right angle.

(iii) $\triangle DBC \cong \triangle ACB$

(iv) $CM = \frac{1}{2} AB$



Answer:

Given: M is the mid-point of hypotenuse AB, $\angle C = 90^\circ$ and $DM = CM$

To Prove:

i) $\triangle AMC \cong \triangle BMD$

ii) $\angle DBC$ is a right angle.

iii) $\triangle DBC \cong \triangle ACB$

iv) $CM = \frac{1}{2} AB$

i) In $\triangle AMC$ and $\triangle BMD$,

$AM = BM$ (M is the mid - point of AB)

$\angle AMC = \angle BMD$ (Vertically opposite angles)

$CM = DM$ (Given)

$\therefore \triangle AMC \cong \triangle BMD$ (By SAS congruence rule)



$\therefore AC = BD$ (By CPCT)

Also, $\angle ACM = \angle BDM$ (By CPCT)

ii) $\angle DBC$ is a right angle.

We know that, $\angle ACM = \angle BDM$ (proved above)

But, $\angle ACM$ and $\angle BDM$ are alternate interior angles. Since alternate angles are equal, it can be said that $DB \parallel AC$.

$\angle DBC + \angle ACB = 180^\circ$ (Co-interior angles)

$\angle DBC + 90^\circ = 180^\circ$ [Since, $\triangle ACB$ is a right angled triangle]

$\therefore \angle DBC = 90^\circ$

Thus, $\angle DBC$ is a right angle.

iii) In $\triangle DBC$ and $\triangle ACB$,

$DB = AC$ (Already proved)

$\angle DBC = \angle ACB = 90^\circ$ (Proved above)

$BC = CB$ (Common)

$\therefore \triangle DBC \cong \triangle ACB$ (SAS congruence rule)

iv) $CM = \frac{1}{2} AB$

Since $\triangle DBC \cong \triangle ACB$

$AB = DC$ (By CPCT)

$\Rightarrow \frac{1}{2} AB = \frac{1}{2} DC$

It is given that M is the midpoint of DC and AB .

$CM = \frac{1}{2} DC = \frac{1}{2} AB$

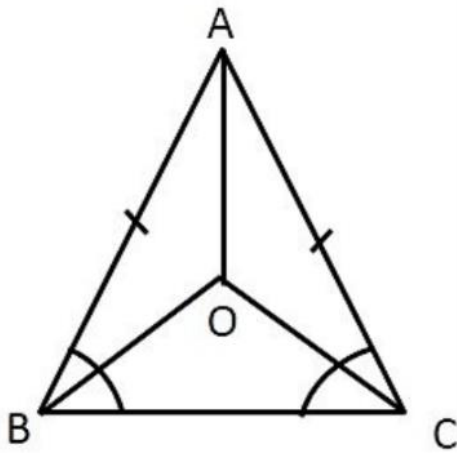
$\therefore CM = \frac{1}{2} AB$



Exercise: 7.3

Q1. In an isosceles triangle ABC, with $AB = AC$, the bisectors of $\angle B$ and $\angle C$ intersect each other at O. Join A to O. Show that:

(i) $OB = OC$ (ii) AO bisects $\angle A$



Answer:

Given: Triangle ABC is isosceles in which $AB = AC$ also OB and OC are bisectors of angle B and angle C

To Prove: i) $OB = OC$ ii) AO bisects $\angle A$

Let's construct a diagram according to the given question.

i) $OB = OC$

It is given that in triangle ABC,

$AB = AC$ (given)

$\angle ACB = \angle ABC$ (Angles opposite to equal sides of an isosceles triangle are equal)

$\frac{1}{2} \angle ACB = \frac{1}{2} \angle ABC$

$\Rightarrow \angle OCB = \angle OBC$ (Since OB and OC are the angle bisectors of $\angle ABC$ and $\angle ACB$)

$\therefore OB = OC$ (Sides opposite to equal angles of an isosceles triangle are also equal)

ii) AO bisects $\angle A$

In $\triangle OAB$ and $\triangle OAC$,

$AO = AO$ (Common)



$AB = AC$ (Given)

$OB = OC$ (Proved above)

Therefore,

$\triangle OAB \cong \triangle OAC$ (By SSS congruence rule)

Also, we can use an alternative approach as shown below,

$\angle OBA = \angle OCA$ (OB and OC bisect angle $\angle B$ and $\angle C$)

$AB = AC$ (Given)

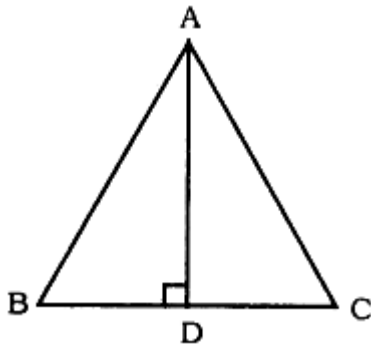
$OB = OC$ (Proved above)

$\triangle OAB \cong \triangle OAC$ (By SAS congruence rule)

$\Rightarrow \angle BAO = \angle CAO$ (CPCT)

\therefore AO bisects $\angle A$ or AO is the angle bisector of $\angle A$.

2. In $\triangle ABC$, AD is the perpendicular bisector of BC (see Fig. 7.30). Show that $\triangle ABC$ is an isosceles triangle in which $AB = AC$.



Answer:

Given: AD is the perpendicular bisector of BC means $\angle ADB = \angle ADC = 90^\circ$ and $BD = DC$

To Prove: $\triangle ABC$ is an isosceles triangle in which $AB = AC$.

In $\triangle ADC$ and $\triangle ADB$,

$AD = AD$ (Common)

$\angle ADC = \angle ADB$ (Each 90°)

$CD = BD$ (AD is the perpendicular bisector of BC)

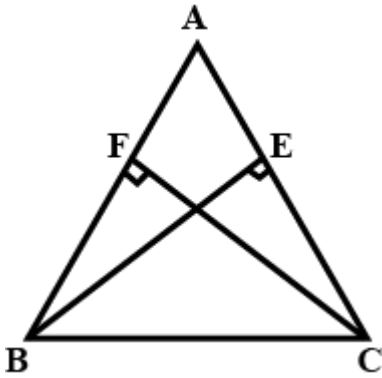
$\therefore \triangle ADC \cong \triangle ADB$ (By SAS congruence rule)



$\therefore AB = AC$ (By CPCT)

Therefore, ABC is an isosceles triangle in which $AB = AC$.

3. ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see Fig. 7.31). Show that these altitudes are equal.



Answer:

Given: $\triangle ABC$ is an isosceles triangle

To prove: $BE = CF$

In $\triangle AEB$ and $\triangle AFC$,

$\angle AEB = \angle AFC$ (Each 90° as BE and CF are altitudes)

$\angle A = \angle A$ (Common angle)

$AB = AC$ (Given $\triangle ABC$ is an isosceles triangle)

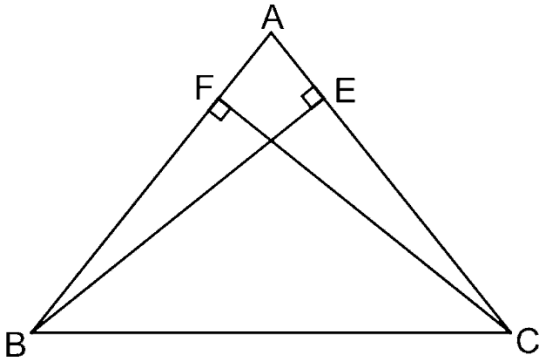
$\therefore \triangle AEB \cong \triangle AFC$ (By AAS congruence rule)

$\therefore BE = CF$ (By CPCT)

4. ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see Fig. 7.32). Show that

(i) $\triangle ABE \cong \triangle ACF$

(ii) $AB = AC$, i.e., ABC is an isosceles triangle.



Answer:

(i) In $\triangle ABE$ and $\triangle ACF$,

$$\angle AEB = \angle AFC \text{ (Each } 90^\circ)$$

$$\angle A = \angle A \text{ (Common angle)}$$

$$BE = CF \text{ (Given)}$$

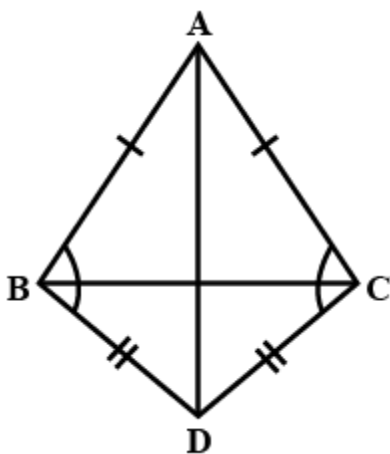
$$\therefore \triangle ABE \cong \triangle ACF \text{ (By AAS congruence rule)}$$

(ii) We have proved above that $\triangle ABE \cong \triangle ACF$

$$\therefore AB = AC \text{ (By CPCT)}$$

Hence, $\triangle ABC$ is an isosceles triangle.

5. $\triangle ABC$ and $\triangle DBC$ are two isosceles triangles on the same base BC (see Fig. 7.33). Show that $\angle ABD = \angle ACD$.



Answer:

Given: $\triangle ABC$ and $\triangle DBC$ are isosceles triangles

To Prove: $\angle ABD = \angle ACD$



Let's join point A and point B.

In $\triangle DAB$ and $\triangle DAC$,

$AB = AC$ (Given)

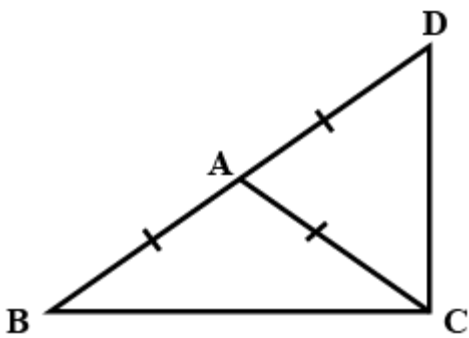
$BD = CD$ (Given)

$AD = AD$ (Common side)

$\therefore \triangle ABD \cong \triangle ACD$ (By SSS congruence rule)

$\therefore \angle ABD = \angle ACD$ (By CPCT)

Q6. $\triangle ABC$ is an isosceles triangle in which $AB = AC$. Side BA is produced to D such that $AD = AB$ (see Fig. 7.34). Show that $\angle BCD$ is a right angle.



Answer:

We can use the property that angles opposite to equal sides are equal and then by the angle sum property in triangle BCD we can show the required result.

In isosceles triangle ABC,

$AB = AC$ (Given)

$\therefore \angle ACB = \angle ABC$ (Angles opposite to equal sides of a triangle are equal)

Let $\angle ACB = \angle ABC$ be x . ----- (1)

In $\triangle ACD$,

$AC = AD$ (Since, $AB = AD$)

$\therefore \angle ADC = \angle ACD$ (Angles opposite to equal sides of a triangle are equal)

Let $\angle ADC = \angle ACD$ be y . ----- (2)



Thus, $\angle BCD = \angle ACB + \angle ACD = x + y$ ----- (3)

In $\triangle BCD$,

$\angle ABC + \angle BCD + \angle ADC = 180^\circ$ (Angle sum property of a triangle)

Substituting the values we get,

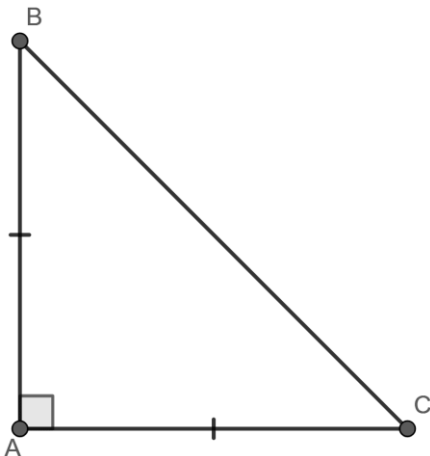
$x + (x + y) + y = 180^\circ$ [From equation (1), (2) and (3)]

$2(x + y) = 180^\circ$

$2(\angle BCD) = 180^\circ$ [From equation(3)]

$\therefore \angle BCD = 90^\circ$

Q7. ABC is a right-angled triangle in which $\angle A = 90^\circ$ and $AB = AC$. Find $\angle B$ and $\angle C$.



Answer:

We can use the property that angles opposite to equal sides are equal and then by using angle sum property in triangle ABC we can find the value of $\angle B$ and $\angle C$.

It is given that,

$AB = AC$

$\therefore \angle C = \angle B$ (Angles opposite to equal sides are also equal)

Let $\angle B = \angle C = x$

In $\triangle ABC$,

$\angle A + \angle B + \angle C = 180^\circ$ (Angle sum property of a triangle)

$90^\circ + x + x = 180^\circ$



$$90^\circ + 2x = 180^\circ$$

$$2x = 90^\circ$$

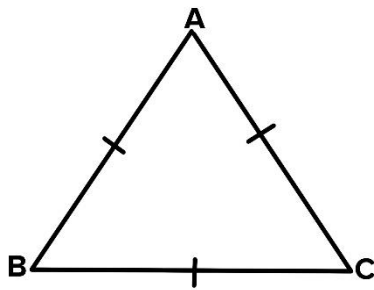
$$x = 45^\circ$$

$$\therefore \angle B = \angle C = 45^\circ$$

8. Show that the angles of an equilateral triangle are 60° each.

Answer:

Let's draw an equilateral triangle ABC as shown below.



Therefore,

$$AB = BC = AC$$

$\therefore \angle C = \angle A = \angle B$ (Angles opposite to equal sides of a triangle are equal)

Let $\angle A = \angle B = \angle C$ be x .

In $\triangle ABC$,

$$\angle A + \angle B + \angle C = 180^\circ \text{ (Angle sum property of a triangle)}$$

$$\Rightarrow x + x + x = 180^\circ$$

$$\Rightarrow 3x = 180^\circ$$

$$\Rightarrow x = 60^\circ$$

$$\therefore \angle A = \angle B = \angle C = 60^\circ$$

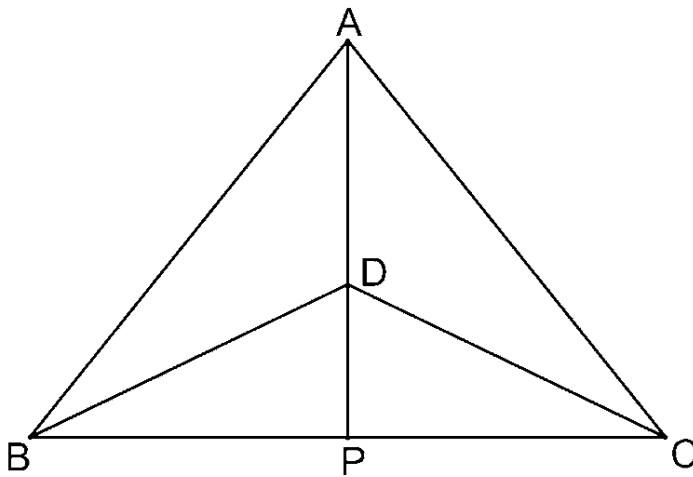
Hence, in an equilateral triangle, all interior angles are of measure 60° .



Exercise: 7.3

1. $\triangle ABC$ and $\triangle DBC$ are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see Fig. 7.39). If AD is extended to intersect BC at P , show that

- (i) $\triangle ABD \cong \triangle ACD$
- (ii) $\triangle ABP \cong \triangle ACP$
- (iii) AP bisects $\angle A$ as well as $\angle D$.
- (iv) AP is the perpendicular bisector of BC .



Answer:

Given: $\triangle ABC$ and $\triangle DBC$ are isosceles triangles.

(i) In $\triangle ABD$ and $\triangle ACD$,

$AB = AC$ (Equal sides of isosceles $\triangle ABC$)

$BD = CD$ (Equal sides of isosceles $\triangle DBC$)

$AD = AD$ (Common)

$\triangle ABD \cong \triangle ACD$ (By SSS congruence rule)

By CPCT, we get

$\angle BAD = \angle CAD$

$\angle BAP = \angle CAP$ (1)

$\angle ADB = \angle ADC$ (2)



(ii) In $\triangle ABP$ and $\triangle ACP$,

$AB = AC$ (Given)

$\angle BAP = \angle CAP$ [From equation (1)]

$AP = AP$ (Common)

$\therefore \triangle ABP \cong \triangle ACP$ (By SAS congruence rule)

$\therefore BP = CP$ (By CPCT).... (3)

(iii) From Equation (1) we know that $\angle BAP = \angle CAP$

Hence, AP is the angle bisector of $\angle A$.

From equation (2), we know that $\angle ADB = \angle ADC$

$\Rightarrow 180^\circ - \angle ADB = 180^\circ - \angle ADC$

$\Rightarrow \angle BDP = \angle CDP$ (4)

Hence, AP is the bisector of $\angle D$.

(iv) In $\triangle BDP$ and $\triangle CDP$,

$DP = DP$ (Common side)

$\angle BDP = \angle CDP$ (Using (4))

$DB = DC$ (Equal sides of isosceles $\triangle DBC$)

$\therefore \triangle BDP \cong \triangle CDP$ (By SAS congruence rule)

$\therefore \angle BPD = \angle CPD$ (By CPCT).... (5)

$\angle BPD + \angle CPD = 180^\circ$ (Linear pair angles)

$\angle BPD + \angle BPD = 180^\circ$ [From Equation (5)]

$\angle BPD = 90^\circ$... (6)

Also, $BP = CP$ [From equation (3)]

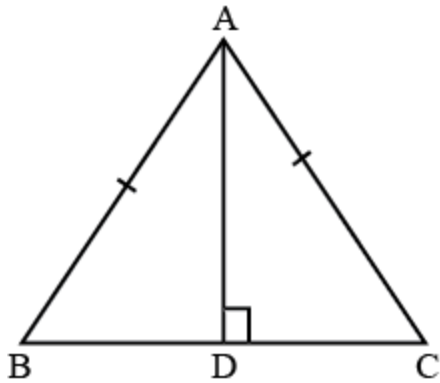
Thus, AP is the perpendicular bisector of BC.



Q2. AD is the altitude of an isosceles triangle ABC in which $AB = AC$. Show that

(i) AD bisects BC (ii) AD bisects $\angle A$.

Answer:



(i) In $\triangle BAD$ and $\triangle CAD$,

$\angle ADB = \angle ADC$ (Each 90° as AD is an altitude)

$AB = AC$ (Given)

$AD = AD$ (Common)

$\therefore \triangle BAD \cong \triangle CAD$ (By RHS Congruence rule)

$\therefore BD = CD$ (By CPCT)

Hence, AD bisects BC.

(ii) Since, $\triangle BAD \cong \triangle CAD$

By CPCT, $\angle BAD = \angle CAD$

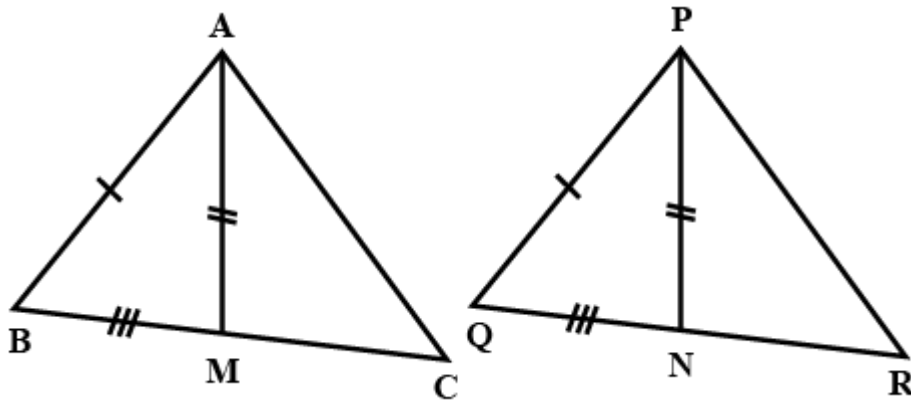
Hence, AD bisects $\angle A$.



Q3. Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of ΔPQR (see Fig. 7.40). Show that:

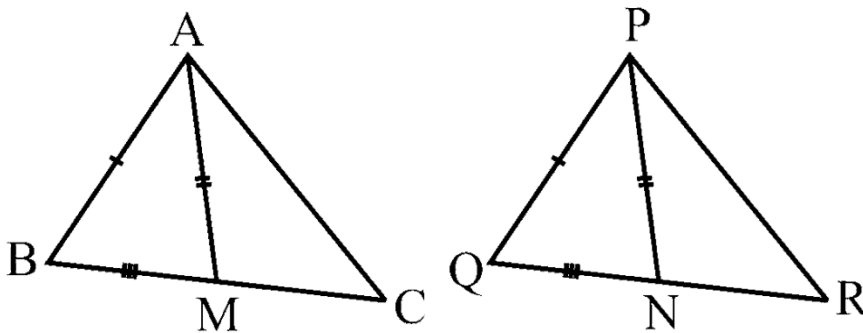
(i) $\Delta ABM \cong \Delta PQN$

(ii) $\Delta ABC \cong \Delta PQR$



Answer:

Given: $AB = PQ$, $AM = PN$, $BM = QN$



(i) In ΔABC , AM is the median to BC.

$$\therefore BM = \frac{1}{2} BC$$

In ΔPQR , PN is the median to QR.

$$\therefore QN = \frac{1}{2} QR$$

It is given that $BC = QR$

$$\therefore \frac{1}{2} BC = \frac{1}{2} QR$$

$$\therefore BM = QN \dots (1)$$

In ΔABM and ΔPQN ,



$$AB = PQ \text{ (Given)}$$

$$BM = QN \text{ [From equation (1)]}$$

$$AM = PN \text{ (Given)}$$

$$\therefore \triangle ABM \cong \triangle PQN \text{ (Using SSS congruence criterion)}$$

$$\Rightarrow \angle ABM = \angle PQN \text{ (By CPCT)}$$

$$\Rightarrow \angle ABC = \angle PQR \dots (2)$$

(ii) In $\triangle ABC$ and $\triangle PQR$,

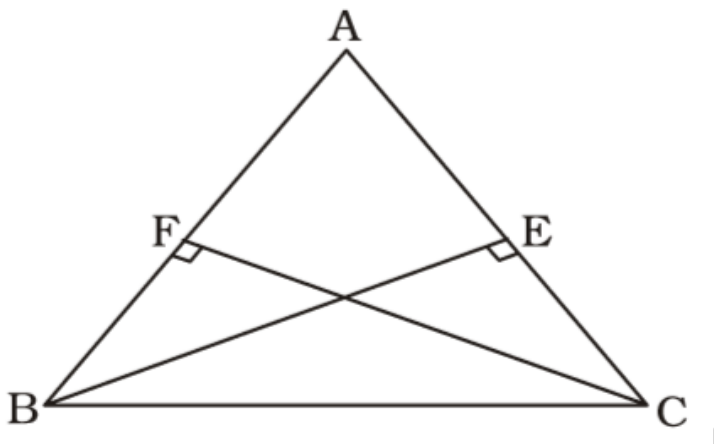
$$AB = PQ \text{ (Given)}$$

$$\angle ABC = \angle PQR \text{ [From Equation (2)]}$$

$$BC = QR \text{ (Given)}$$

$$\therefore \triangle ABC \cong \triangle PQR \text{ (By SAS congruence rule)}$$

Q4. BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.



Answer:

It is known that BE and CF are two equal altitudes.

Now, in $\triangle BEC$ and $\triangle CFB$,

$$\angle BEC = \angle CFB = 90^\circ \text{ (Same Altitudes)}$$

$$BC = CB \text{ (Common side)}$$

$$BE = CF \text{ (Common side)}$$



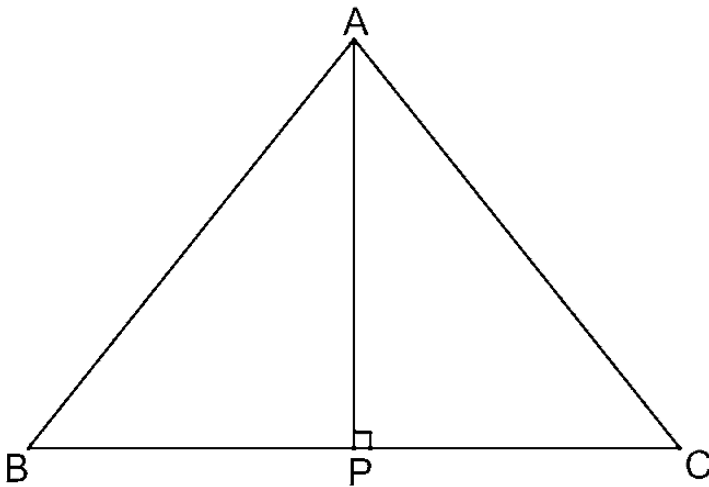
So, $\triangle BEC \cong \triangle CFB$ by RHS congruence criterion.

Also, $\angle C = \angle B$ (by CPCT)

Therefore, $AB = AC$ as sides opposite to the equal angles is always equal.

Q5. ABC is an isosceles triangle with $AB = AC$. Draw $AP \perp BC$ to show that $\angle B = \angle C$.

Answer:



In triangles APB and APC,

$\angle APB = \angle APC$ (Each 90°)

$AB = AC$ (Since ABC is an isosceles triangle)

$AP = AP$ (Common)

$\triangle APB \cong \triangle APC$ (Using RHS congruence rule)

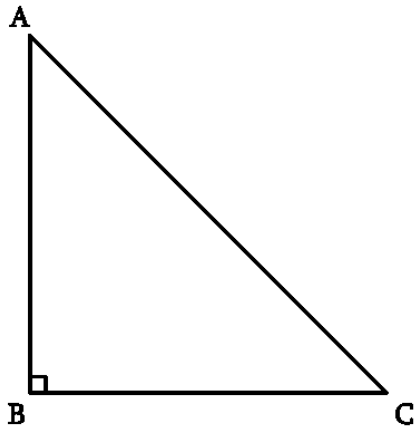
Thus, $\angle B = \angle C$ (CPCT)



Exercise: 7.4

Q1. Show that in a right-angled triangle, the hypotenuse is the longest side.

Answer:



Let us consider a right-angled triangle ABC, right-angled at B.

In $\triangle ABC$,

$$\angle A + \angle B + \angle C = 180^\circ \text{ (Angle sum property of a triangle)}$$

$$\angle A + 90^\circ + \angle C = 180^\circ$$

$$\angle A + \angle C = 90^\circ$$

Hence, the other two angles have to be acute (i.e., less than 90°).

Thus, $\angle B$ is the largest angle in $\triangle ABC$.

So, $\angle B > \angle A$ and $\angle B > \angle C$

Therefore, $AC > BC$ and $AC > AB$ [Using theorem 7.7 of triangles, in any triangle, the side opposite to the larger (greater) angle is longer.]

Therefore, AC is the largest side in $\triangle ABC$.

However, AC is the hypotenuse of $\triangle ABC$.

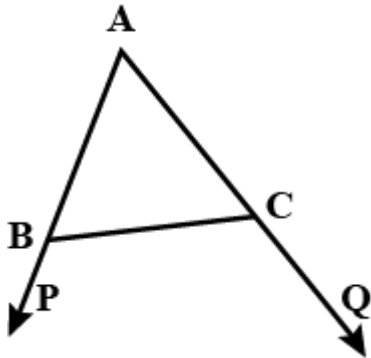
Therefore, the hypotenuse is the longest side in a right-angled triangle.



Q2. In Fig. 7.48, sides AB and AC of $\triangle ABC$ are extended to points P and Q respectively. Also, $\angle PBC < \angle QCB$. Show that $AC > AB$.

Answer:

Let's look into the solution below.



In the given figure,

$$\angle ABC + \angle PBC = 180^\circ \text{ [Linear pair of angles]}$$

$$\text{Also, } \angle ABC = 180^\circ - \angle PBC \quad \dots (1)$$

$$\angle ACB + \angle QCB = 180^\circ \text{ [Linear pair]}$$

$$\angle ACB = 180^\circ - \angle QCB \quad \dots (2)$$

As $\angle PBC < \angle QCB$ (given),

$$180^\circ - \angle PBC > 180^\circ - \angle QCB$$

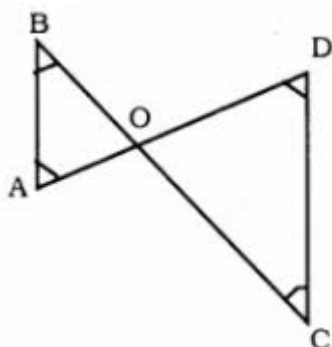
$$\angle ABC > \angle ACB \text{ [From Equations (1) and (2)]}$$

Thus, $AC > AB$ (Side opposite to the larger angle is larger).

Hence proved, $AC > AB$.

Q3. In Fig. 7.49, $\angle B < \angle A$ and $\angle C < \angle D$. Show that $AD < BC$.

Answer:





Given: $\angle B < \angle A$ and $\angle C < \angle D$

To prove: $AD < BC$

We can use the fact that in any triangle, the side opposite to the larger (greater) angle is longer.

In $\triangle AOB$, $\angle B < \angle A$ (given)

$AO < OB$ (The side opposite to the smaller angle is smaller) ... (1)

In $\triangle COD$, $\angle C < \angle D$

$OD < OC$ (The side opposite to the smaller angle is smaller) ... (2)

On adding Equations (1) and (2), we obtain

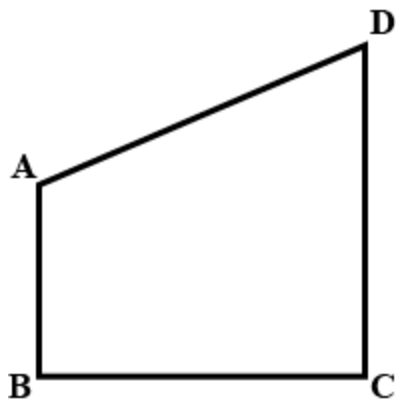
$$AO + OD < BO + OC$$

$$AD < BC$$

Hence, proved

Q4. AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (see Fig. 7.50).

Show that $\angle A > \angle C$ and $\angle B > \angle D$.

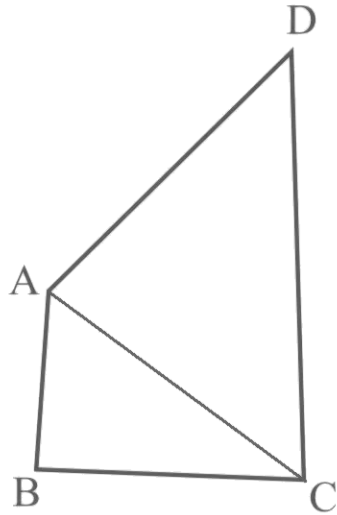


Answer:

Given: AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD

To prove: $\angle A > \angle C$ and $\angle B > \angle D$

Let's join vertex A to C as shown below.



In the above $\triangle ABC$,

$AB < BC$ (given AB is the smallest side of quadrilateral $ABCD$)

$\angle ACB < \angle BAC$ (Angle opposite to the smaller side is smaller) ... (1)

In $\triangle ADC$,

$AD < CD$ (given CD is the largest side of quadrilateral $ABCD$)

$\angle ACD < \angle CAD$ (Angle opposite to the smaller side is smaller) ... (2)

On adding Equations (1) and (2), we obtain

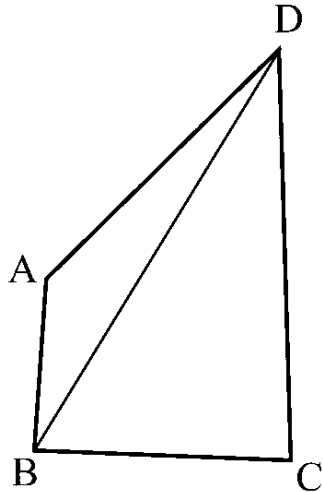
$$\angle ACB + \angle ACD < \angle BAC + \angle CAD$$

$$\angle C < \angle A$$

This means, $\angle A > \angle C$

Hence Proved.

Let's now join BD .



In $\triangle ABD$,

$AB < AD$ (Given AB is the smallest side of quadrilateral $ABCD$)

$\angle ADB < \angle ABD$ (Angle opposite to the smaller side is smaller) ... (3)

In $\triangle BDC$,

$BC < CD$ (Given CD is the largest side of quadrilateral $ABCD$)

$\angle BDC < \angle CBD$ (Angle opposite to the smaller side is smaller) ... (4)

On adding equations (3) and (4), we obtain

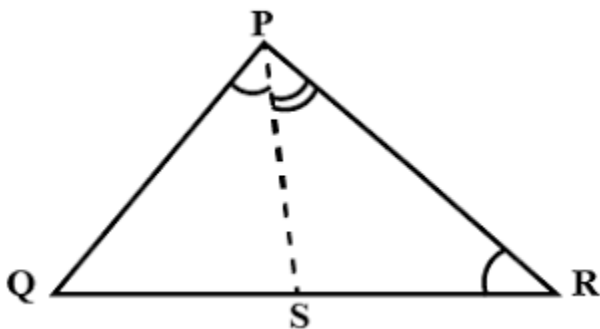
$$\angle ADB + \angle BDC < \angle ABD + \angle CBD$$

$$\angle D < \angle B$$

This means, $\angle B > \angle D$

Hence, proved.

Q5. In Fig 7.51, $PR > PQ$ and PS bisect $\angle QPR$. Prove that $\angle PSR > \angle PSQ$.





Given: $PR > PQ$ and PS bisects $\angle QPR$

To prove: $\angle PSR > \angle PSQ$

As $PR > PQ$, $\angle PQS > \angle PRS$ (Angle opposite to larger side is larger) ...(1)

PS is the angle bisector of $\angle QPR$.

$\angle QPS = \angle RPS$...(2) [Since, PS bisects $\angle QPR$]

$\angle PSR$ is the exterior angle of $\triangle PQS$.

$\angle PSR = \angle PQS + \angle QPS$...(3) [Using exterior angle sum property]

$\angle PSQ$ is the exterior angle of $\triangle PRS$.

$\angle PSQ = \angle PRS + \angle RPS$...(4) [Using exterior angle sum property]

From Equations (1) and (2), we obtain

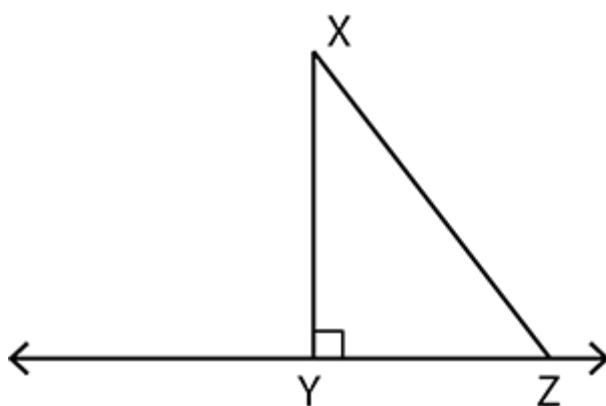
$\angle PQS + \angle QPS > \angle PRS + \angle RPS$

$\angle PSR > \angle PSQ$ [From Equations (3) and (4)]

Q6. Show that the perpendicular line segment is the shortest of all line segments drawn from a given point, not on it.

Answer:

We know that in a triangle if one angle is 90 degrees, then the other angles have to be acute.



Let us take a line l and from point P , that is, not on line l , draw two line segments PN and PM . Let PN be perpendicular to line l and PM is drawn at some other angle.

In $\triangle PNM$, $\angle N = 90^\circ$

$\angle P + \angle N + \angle M = 180^\circ$ (Angle sum property of a triangle)



$$\angle P + \angle M = 90^\circ$$

Clearly, $\angle M$ is an acute angle.

$$\angle M < \angle N$$

$PN < PM$ (The side opposite to the smaller angle is smaller)

Similarly, by drawing different line segments from P to l, it can be proved that PN is smaller in comparison to all of them. Therefore, it can be observed that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

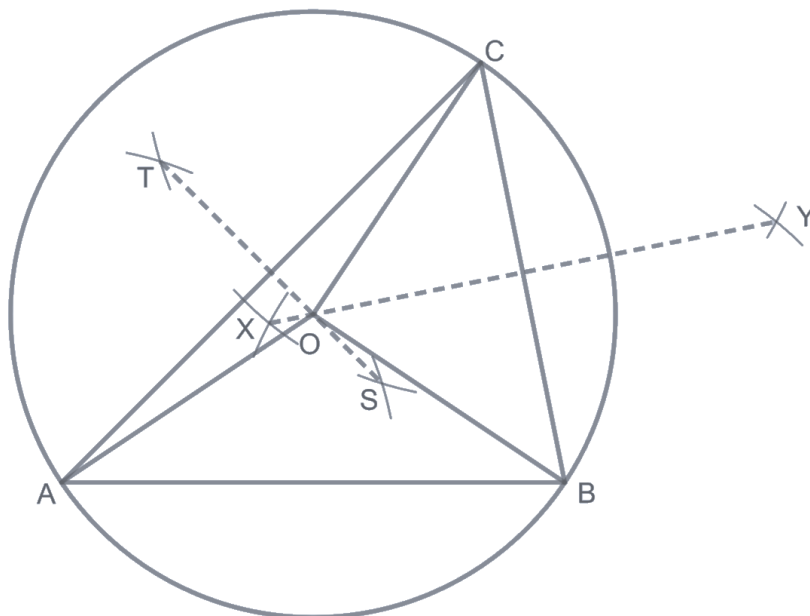
Exercise: 7.5

Q1. ABC is a triangle. Locate a point in the interior of ΔABC that is equidistant from all the vertices of ΔABC .

Answer:

Circumcentre is the point where perpendicular bisectors of the sides of the triangle meet together.

The circumcentre of a triangle is always equidistant from all the vertices of that triangle.





In $\triangle ABC$, we can find the circumcentre by drawing the perpendicular bisectors of sides AC and CB.

As we see that O is the point where these bisectors are meeting.

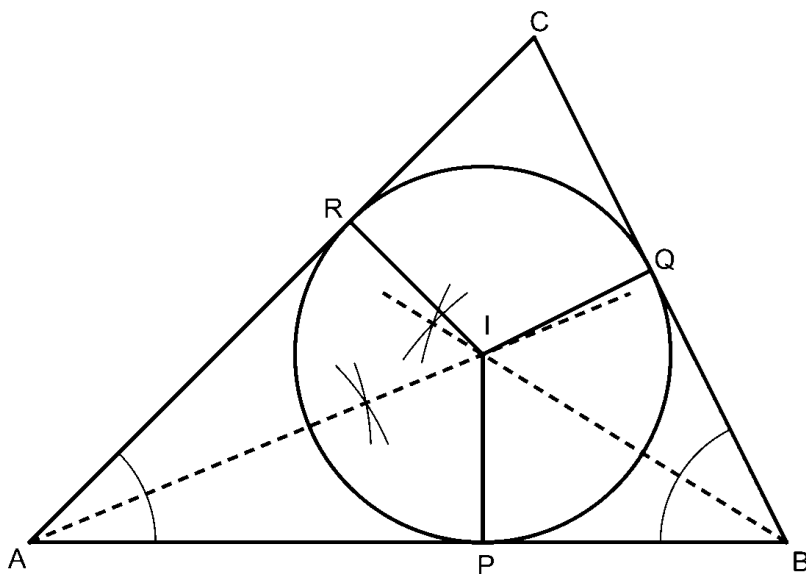
Therefore, O which is called the circumcentre is the point that is equidistant from all the vertices of $\triangle ABC$ that is, $OA = OB = OC$.

Q2. In a triangle locate a point in its interior that is equidistant from all the sides of the triangle.

Answer:

The point which is equidistant from all the sides of a triangle is called the incentre of the triangle.

The incentre of a triangle is the intersection point of the angle bisectors of the interior angles of that triangle.



Here, in $\triangle ABC$, we can find the incentre of this triangle by drawing the angle bisectors of the interior angles of this triangle. I is the point where these angle bisectors are intersecting each other.

Therefore, I is the point which is equidistant from all the sides of $\triangle ABC$ that is $IP = IQ = IR$.



Q3. In a huge park, people are concentrated at three points (see Fig. 7.52):



A: where there are different slides and swings for children,

B: near which a man-made lake is situated,

C: which is near to a large parking and exit.

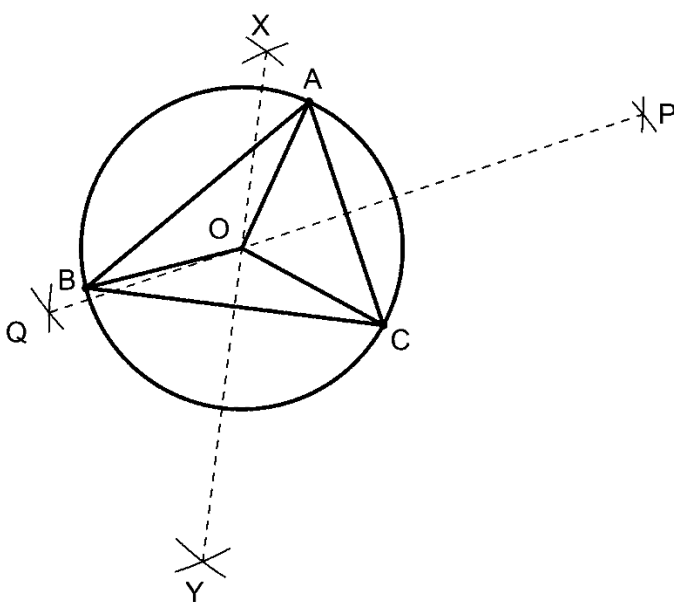
Where should an ice cream parlor be set up so that a maximum number of persons can approach it?

(Hint: The parlor should be equidistant from A, B, and C)

Answer: The maximum number of persons can approach the ice cream parlor if it is equidistant from A, B, and C. In a triangle, the circumcentre is the only point that is equidistant from its vertices.

So, the ice cream parlor should be set up at the circumcentre O of $\triangle ABC$ as shown below.

We can find circumcentre O of this triangle by drawing perpendicular bisectors of the sides of this triangle.



In this situation, the maximum number of people can approach it.