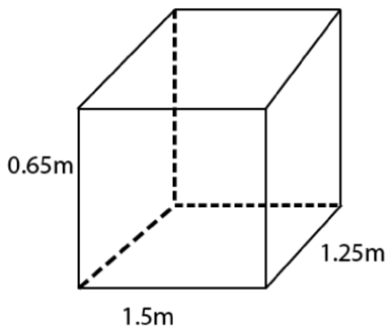
Exercise 13.1

1. A plastic box 1.5 m long, 1.25 m wide and 65 cm deep is to be made. It is to be open at the top. Ignoring the thickness of the plastic sheet, determine

(I) The area of the sheet required for making the box.

(ii) The cost of the sheet for it, if a sheet measuring 1m² costs Rs. 20.

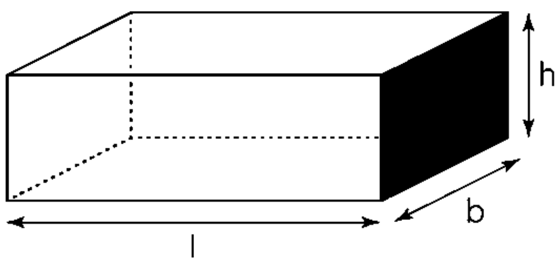


Answer: Given: A plastic box 1.5 m long, 1.25 m wide and 65 cm deep. It is opened at the top. The cost of 1 m^2 sheet is ₹20.

To find: Area of the sheet required for making the box and its cost.

Since the box is opened at the top, it has only 5 surfaces, including the 4 walls and the base. The area of the sheet required for making the cuboidal box includes the 4 walls of the box and the base.

Hence, the area of the sheet can be obtained by adding the area of the base to the lateral surface area of the cuboidal box.



Lateral surface area of cuboid = 2(l + b)h

The cost of the sheet to create the box will be equal to area of the sheet multiplied by rate of $1m^2$ sheet.

Length, l = 1.5 m

Breadth, b = 1.25 m

Height, h = 65 cm = 65/100 m = 0.65 m

The area of the sheet required to make the box = lb + 2(l + b)h

= (1.5 m × 1.25m) + 2 × (1.25 m + 1.5 m) × 0.65 m

= 1.875 m^2 + 2 × 2.75 m × 0.65 m

= 1.875 m^2 + 3.575 m^2

= 5.45 m^2

Therefore, the cost of the sheet = Rate of the sheet × Area of the sheet

= ₹ 20 / m^2 × 5.45 m^2

= ₹109

Therefore, the area of the sheet required for making the open box is 5.45 m^2 and the cost of the sheet is ₹109.

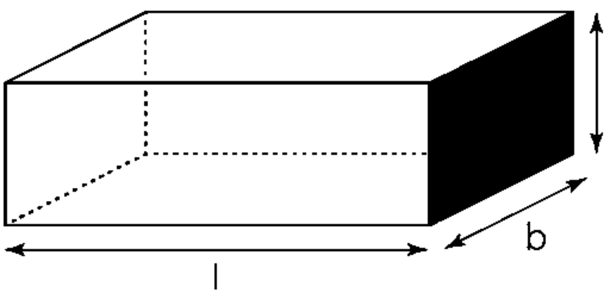
2. The length, breadth and height of a room are 5 m, 4 m and 3 m, respectively. Find the cost of whitewashing the walls of the room and ceiling at the rate of Rs 7.50 per m^2.

Answer: Given - The length, breadth, and height of a room are 5 m, 4 m, and 3 m respectively.

Since the four walls and ceiling are to be whitewashed so, it has 5 faces only, excluding the base.

Hence, the area of the room to be whitewashed can be obtained by adding the area of the ceiling to the lateral surface area of the cuboidal room.

Lateral surface area of cuboid = 2(l + b)h



The cost of whitewashing the walls of the room and ceiling will be equal to the area of the room to be whitewashed multiplied by the rate of the whitewashing per m^2.

Length, l = 5 m

Breadth, b = 4 m

Height, h = 3 m

Surface area of 5 faces = Area of the 4 walls and ceiling = lb + 2(l + b)h

lb + 2(l + b)h = (5 m × 4 m) + 2 × (5 m + 4 m) × 3 m

= 20 m^2 + 2 × 9 m × 3 m

= 20 m^2 + 54 m^2

= 74 m^2

The cost of whitewashing the walls of the room and ceiling = Rate × Area

= ₹ 7.50 / m^2 × 74 m^2

= ₹ 555

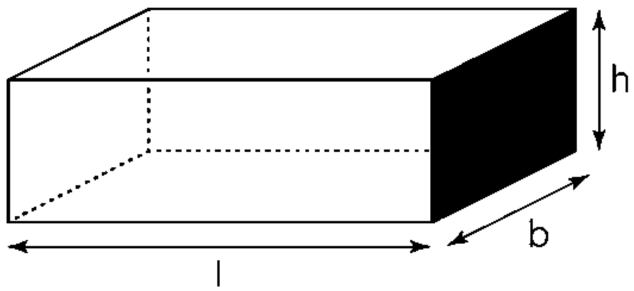
Thus, the cost of whitewashing the walls of the room and the ceiling is ₹555.

3. The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of Rs.10 per m^2 is Rs.15,000, find the height of the hall.

[Hint: Area of the four walls = Lateral surface area.]

Answer: Given - The perimeter of the floor of the rectangular hall is 250m and the cost of painting the four walls at the rate of ₹10 per m^2 is ₹15000.

The area of the four walls of the cuboidal room will be the Lateral surface area of the cuboid.



Lateral surface area of cuboid = 2(l + b)h

The area of the four walls can also be obtained by dividing the total cost of the painting by the rate of painting per m^2.

Let the length, breadth, and height of the room be l, b, and h respectively. The cost of painting the four walls is ₹15000.

The rate of painting is ₹10 / m^2

Perimeter of the floor = 250 m

Therefore, 2(l + b) = 250 m ------------ (1) [Since, perimeter of a rectangle = 2(l + b)

Now, Area of four walls = 15000/10 m^2 = 1500 m^2

2(l + b)h = 1500 m^2 [From equation(1)]

250 m × h = 1500 m^2

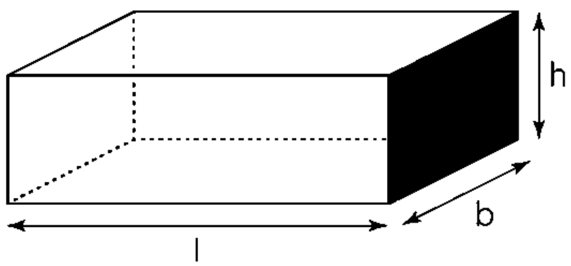
h = 1500 m^2/250 m = 6 m

Thus, the height of the hall is 6 m.

4. The paint in a certain container is sufficient to paint an area equal to 9.375 m^2. How many bricks of dimensions 22.5 cm×10 cm×7.5 cm can be painted out of this container?

Answer: Given - Dimensions of the brick 22.5cm × 10cm × 7.5cm

Since brick is cuboidal in shape, the surface area of the brick will be the total surface area of the cuboid.



Hence, the area of each brick to be painted will be the total surface area of the cuboid

Total surface area of cuboid = 2(lb + bh + hl)

The number of bricks that can be painted out of the container can be calculated by dividing the area which can be painted with paint available in the container by the area of each brick.

The area which can be painted with the paint available in the container = $9.375 m^2$.

Let the length, breadth, and height of the bricks be l, b, and h respectively.

l = 22.5 cm

b = 10 cm

h = 7.5 cm

The area of each brick to be painted = 2(lb + bh + hl)

2(lb + bh + hl) = 2 × (22.5 cm × 10 cm + 10 cm × 7.5 cm + 7.5 cm × 22.5 cm)

= 2 × (225 cm^2 + 75 cm^2 + 168.75 cm^2)

= 2 × 468.75 cm^2

= 937.5 cm^2

Number of bricks that can be painted = The area which can be painted with the paint available in the container / The area of each brick

= 9.375 m^2 / 937.5 cm^2

= (9.375 × 10000 cm^2) / 937.5 cm^2 [since 1m^2 = 10000cm^2]

= 100

Thus, the number of bricks that can be painted out of the container is 100.

5. A cubical box has each edge 10 cm, and another cuboidal box is 12.5cm long, 10 cm wide, and 8 cm high.

(i) Which box has the greater lateral surface area, and by how much?

(ii) Which box has the smaller total surface area, and by how much?

Answer: Given- The length of the edge of the cubical box is 10cm and the length, breadth, and height of the cuboidal box are 12.5 cm, 10 cm, and 8 cm respectively.

A cube is a cuboid whose length, breadth, and height and equal. A cuboid has six faces and the total surface area is the sum of the surface area of the 6 faces and the Lateral surface area is the sum of the area of the four faces.

Total surface area of cube = $6a^2$ (where, 'a' is the side of the cube)

Total surface area of cuboid = 2(lb + bh + hl)

Lateral surface area of a cube = $4a^2$

Lateral surface area of cuboid = 2(l + b)h

Edge length of the cube, a = 10 cm

Length of the cuboid, l = 12.5 cm

Breadth of the cuboid, b = 10 cm

Height of the cuboid, h = 8 cm

Lateral surface area of the cube = $4a^2$

= $4 \times (10\ cm)^2$

= $4 \times 100\ cm^2$

= $400\ cm^2$

Lateral surface area of the cuboid = (l + b)h

= 2 × (12.5 cm + 10 cm) × 8 cm

= 2 × 22.5 cm × 8 cm

= $360\ cm^2$

We see that, the cubical box has a greater lateral surface area by ($400\ cm^2$ - $360\ cm^2$) = $40\ cm^2$

Total surface area of the cube = $6a^2$

= $6 \times (10\ cm)^2$

= $6 \times 100\ cm^2$

= $600\ cm^2$

Total surface area of the cuboid = 2(lb + bh + hl)

2(lb + bh + hl) = 2 × (12.5 cm × 10 cm + 10 cm × 8 cm + 8 cm × 12.5 cm)

= $2 \times (125\ cm^2 + 80\ cm^2 + 100\ cm^2)$

= $2 \times 305\ cm^2$

= $610\ cm^2$

Cubical box has a smaller total surface area by ($610\ cm^2$ - $600\ cm^2$) = $10\ cm^2$

Thus, the cubical box has a greater lateral surface area by $40\ cm^2$ and the cubical box has a smaller total surface area by $10\ cm^2$.

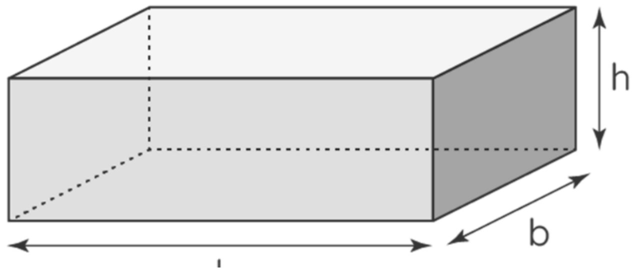
6. A small indoor greenhouse (herbarium) is made entirely of glass panes (including the base) held together with tape. It is 30cm long, 25 cm wide, and 25 cm high.

(i) What is the area of the glass?

(ii) How much tape is needed for all 12 edges?

Answer: Given - Herbarium of dimensions 30 cm × 25 cm × 25 cm.

Since the herbarium is a cuboid that is enclosed by six rectangle regions called faces having 12 edges, the area of the glass used to make the herbarium will be equal to the total surface area of the cuboid.



Total surface area of the cuboid = 2(lb + bh + hl)

Let the length, breadth and height of the herbarium are l, b and h respectively.

Length, l = 30 cm

Breadth, b = 25 cm

Height, h = 25 cm

The area of the glass = 2(lb + bh + hl)

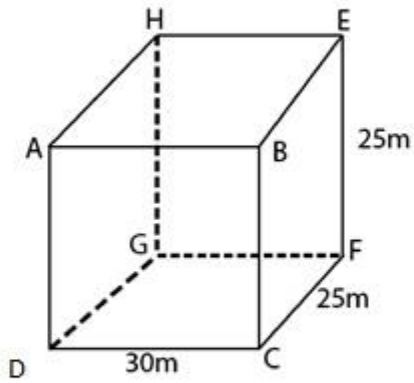
= 2 × (30 cm × 25 cm + 25 cm × 25 cm + 25 cm × 30 cm)

= 2 × (750 cm^2 + 625 cm^2 + 750 cm^2)

= 2 × 2125 cm^2

= 4250 cm^2

Since the herbarium is made entirely of glass panes (including base) held together with tape, the length of the tape which is needed for all the 12 edges will be the same as the sum of the lengths of all the 12 edges.



Length of the tape needed for all the 12 edges = 4(l + b + h)

= 4 × (30 cm + 25 cm + 25 cm)

= 4 × 80 cm

= 320 cm

Thus, the area of the glass is 4250 cm^2, and the tape needed for all the 12 edges is 320 cm.

7. Shanti Sweets Stall was placing an order for making cardboard boxes for packing their sweets. Two sizes of boxes were required. The bigger of dimensions 25 cm×20cm×5cm, and the smaller of dimension 15cm×12cm×5cm. For all the overlaps, 5% of the total surface area is required extra. If the cost of the cardboard is Rs. 4 for 1000 cm^2, find the cost of cardboard required for supplying 250 boxes of each kind.

Answer: Since the cardboard boxes are cuboidal in shape, the total area of the cardboard is the same as the total surface area of the cuboid added to the overlap area that is, 5% of the total surface area is required extra.

Hence, the area of each box can be obtained by adding 5% of the total surface area to the total surface area of the cuboid.

Total surface area of cuboid = 2(lb + bh + hl)

For bigger box:

Let the length, breadth and height of the bigger box be L, B and H respectively.

Length, L = 25 cm

Breadth, B = 20 cm

Height, H = 5 cm

The area of the card board = 2(LB + BH + HL)

= 2 × (25cm × 20cm + 20cm × 5cm + 5cm × 25cm)

= 2 × (500cm^2 + 100cm^2 + 125cm^2)

= 2 × 725 cm^2

= 1450 cm^2

For all the overlaps, 5% of the total surface area is required extra. Therefore,

Overlap area = 5% of 1450 cm^2

= 5/100 × 1450cm^2

= 72.5 cm^2

Net area of the card board required for each bigger box = 1450 cm^2 + 72.5 cm^2 = 1522.5 cm^2

We can now find the area of 250 such boxes and the total cost of the cardboard at ₹4 per 1000 cm^2.

Area of card board required for 250 such boxes = 250 × 1522.5 cm^2 = 380625 cm^2

The total cost of the cardboard at ₹4 per 1000 cm^2 = (4/1000) × 380625 = ₹1522.50

For smaller box:

Let the length, breadth and height of the smaller box be l, b and h respectively.

Length, l = 15 cm

Breadth, b = 12 cm

Height, h = 5 cm

The area of the cardboard = 2(lb + bh + hl)

= 2 × (15 cm × 12 cm + 12 cm × 5 cm + 5 cm × 15 cm)

= 2 × (180 cm^2 + 60 cm^2 + 75 cm^2)

= 2 × 315 cm^2

= 630 cm^2

For all the overlaps, 5% of the total surface area is required extra. Therefore,

Overlap area = 5% of 630cm^2 = (5/100) × 630 cm^2 = 31.5 cm^2

Net area of the cardboard required for smaller box = 630 cm^2 + 31.5 cm^2 = 661.5 cm^2

Area of cardboard required for 250 such boxes = 250 × 661.5 cm^2 =165375 cm^2

The total cost of the cardboard at ₹4 per 1000 cm^2 = ₹ (4/1000) × 165375 = ₹661.50

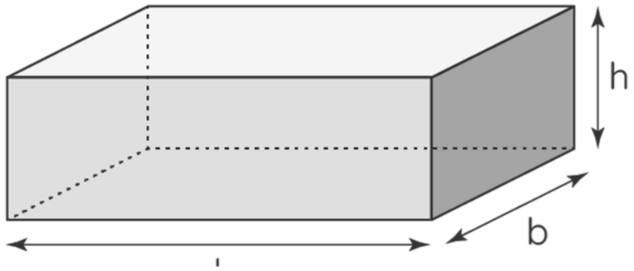
Cost of cardboard required for supplying 250 boxes of each kind = ₹1522.50 + ₹661.50 = ₹2184

Box	Dimensions (in cm)	Total surface area (in cm^2)	Extra area required for overlapping (in cm^2)	Total surface area for all overlaps (in cm^2)	Area for 250 such boxes (in cm^2)
Bigger Box	l = 25 b = 20 c = 5	1450	1450 × 5/100 = 72.5	(1450+72.5) = 1522.5	(1522.5 × 250) = 380625
Smaller Box	l = 15 b = 12 h =5	630	630 × 5 / 100 = 31.5	(630 + 31.5) = 661.5	(250 × 661.5) = 165375

8. Praveen wanted to make a temporary shelter for her car by making a box-like structure with a tarpaulin that covers all four sides and the top of the car (with the front face as a flap which can be rolled up). Assuming that the stitching margins are very small and therefore negligible, how many tarpaulins would be required to make the shelter of height 2.5m, with base dimensions 4m×3m?

Answer: Since the shelter is a box-like structure (cuboid) with a tarpaulin that covers all four sides and the top of the car, the surface area of the shelter is the sum of the lateral surface area of the cuboid and area of the top.

Lateral surface area of cuboid = 2(l + b)h



Then the area of the tarpaulin required to make the shelter = lb + 2(l + b)h

Let the length, breadth and height of the shelter be l, b and h respectively.

Length, l = 4 m

Breadth, b = 3 m

Height, h = 2.5 m

The area of the tarpaulin required to make the shelter = lb + 2(l + b)h

= (4 m × 3 m) + 2 × (4 m + 3 m) × 2.5 m

= 12 m^2 + 2 × 7 m × 2.5 m

= 12 m^2 + 35 m^2

= 47 m^2

Hence, 47 m^2 of tarpaulin will be required.

Exercise 13.2

1. The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the diameter of the base of the cylinder (Assume π =22/7).

Answer: Let the radius and height of the cylinder be 'r' and 'h' respectively.

Curved Surface Area of a right circular cylinder = 2 π r h

Diameter = 2 × radius

Height of the cylinder, h = 14 cm

CSA of the cylinder = 88 cm^2

2 π r h = 88 cm^2

2 × 22/7 × r × 14 cm = 88 cm^2

r = (88 cm^2 × 7) / (2 × 22 × 14) cm

= 1 cm

Diameter = 2 × radius

= 2 × 1 cm

= 2 cm

Thus, the diameter of the base of the cylinder is 2 cm.

2. It is required to make a closed cylindrical tank of height 1m and base diameter 140cm from a metal sheet. How many square metres of the sheet are required for the same? Assume π = 22/7)

Answer: We know that,

Diameter = 2 × radius

Let the radius and height of the cylinder be 'r' and 'h' respectively.

Height of the tank, h = 1 m

Radius of the tank, r = 140 cm / 2 = 70/100 m = 0.7 m

Total surface area of the closed cylindrical tank = 2 π r (r + h)

= 2 × 22/7 × 0.7 m × (0.7 m + 1 m)

= 4.4 m × 1.7 m

= 7.48 m^2

7.48 m^2 of the sheet is required for making the cylindrical tank.

3. A metal pipe is 77 cm long. The inner diameter of a cross-section is 4 cm, the outer diameter being 4.4cm (see fig. 13.11). Find its

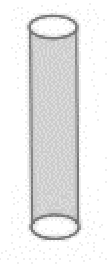


Fig. 13.11

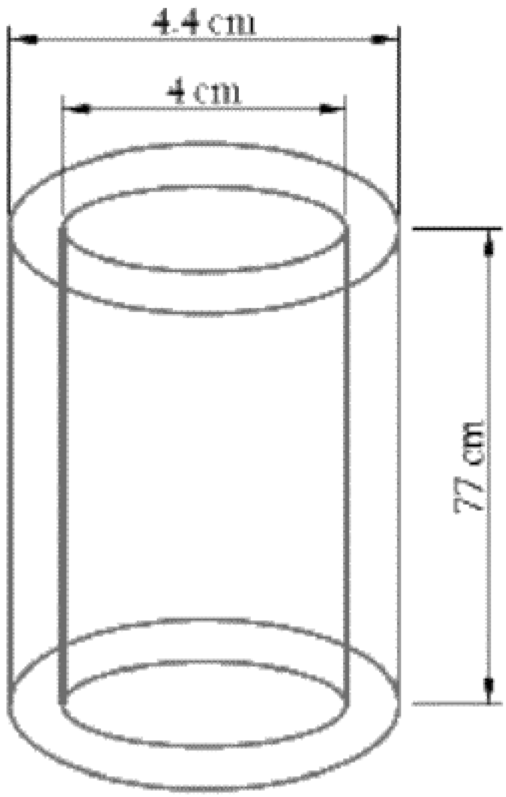
(i) inner curved surface area

(ii) outer curved surface area

(iii) total surface area

(Assume π=22/7)

Answer: Let's construct the diagram according to the given question.



The inner radius, outer radius, and height of the cylinder are r, R, and h respectively.

Inner curved surface area = $2\pi rh$

Outer curved surface area = $2\pi Rh$

Length of the pipe, h = 77 cm

Inner radius(r) of the pipe and outer radius(R) of the pipe are:

r = 4/2 cm = 2 cm

R = 4.4/2 cm = 2.2 cm

i) Inner curved surface area = $2\pi rh$

= 2 × 22/7 × 2 cm × 77 cm

= 968 cm^2

ii) Outer curved surface area = $2\pi Rh$

= 2 × 22/7 × 2.2 cm × 77 cm

= 1064.8 cm^2

iii) The total surface area of the pipe can be obtained by adding the inner and outer curved surface areas along with the area of both the circular ends.

We can find the area of circular ends by subtracting the area of the inner circle from the outer circle area.

Area of both the circular ends of the pipe = $2\pi(R^2 - r^2)$

TSA of pipe = CSA of inner surface + CSA of outer surface + Area of both the circular ends of the pipe

Hence, TSA of the pipe = $2\pi rh + 2\pi Rh + 2\pi(R^2 - r^2)$

Now, area of both the circular ends of the pipe = $2\pi(R^2 - r^2)$

= 2 × 22/7 × [$(2.2\text{ cm})^2 - (2\text{ cm})^2$]

= 2 × 22/7 × [4.84 cm^2 - 4 cm^2]

= 2 × 22/7 × 0.84 cm^2

= 5.28 cm^2

Total surface area = $2\pi rh + 2\pi Rh + 2\pi(R^2 - r^2)$

= 968 cm^2 + 1064.8 cm^2 + 5.28 cm^2 [Since, Inner curved surface area = 968 cm^2, Outer curved surface area = 1064.8 cm^2]

= 2038.08 cm^2

4. The diameter of a roller is 84 cm, and its length is 120 cm. It takes 500 complete revolutions to move once over to level a playground. Find the area of the playground in m^2 (Assume π = 22/7).

Answer: The roller is cylindrical in shape and hence it is considered as a right circular cylinder. In one revolution, the area covered will be the curved surface area of the roller.

Since it takes 500 complete revolutions to move once over to level a playground, the area of the playground will be equal to 500 times the curved surface area of the roller.

Let the radius and height of the cylinder are 'r' and 'h' respectively.

Curved Surface Area of the cylinder = 2πrh

Length of the roller, h = 120 cm

Radius of the roller, r = 84/2 cm = 42 cm

Curved Surface Area of the roller = 2πrh

= 2 × 22/7 × 42 cm × 120 cm

= 31680 cm^2

Area of the playground = Area levelled by the cylinder in 500 revolutions

= 500 × 31680 cm^2

= 15840000 cm^2

= 15840000/10000 m^2 [Since 1cm^2 = 1/10000 m^2]

= 1584 m^2

Thus, area of the playground = 1584 m^2.

5. A cylindrical pillar is 50 cm in diameter and 3.5 m in height. Find the cost of painting the curved surface of the pillar at the rate of Rs. 12.50 per m^2.

(Assume π = 22/7)

Answer: Given - The diameter of the cylindrical pillar is 50 cm and its height is 3.5 m and the rate of painting is ₹12.50 per m^2

The curved surface area of a right circular cylinder of base radius r and height h is 2πrh.

Height(h) of the pillar = 3.5 m

Radius(r) = diameter/2 = 50/2 cm = 25 cm = 0.25 m

Now, curved surface area of the pillar = 2πrh

= 2 × 22/7 × 0.25 m × 3.5 m

= 5.5 m^2

We can calculate the cost of painting by multiplying the curved surface area of the pillar and the rate of painting per meter square.

Cost of painting the curved surface area at ₹12.50 per m^2 = 12.50 × 5.5

= ₹ 68.75

Thus, cost of painting the curved surface of the pillar is ₹ 68.75.

6. Curved surface area of a right circular cylinder is 4.4 m^2. If the radius of the base of the cylinder is 0.7 m, find its height. (Assume π = 22/7).

Answer: The curved surface area of a right circular cylinder of the base radius 'r' and height 'h' is 2πrh

The curved surface area of the pillar = 4.4 m^2

Radius of the cylinder, r = 0.7 m

Height of the cylinder, h = ?

2πrh = 4.4 m^2

2 × 22/7 × 0.7 m × h = 4.4 m^2

h = 7 × 4.4 m^2 / (2 × 22 × 0.7 m)

h = 1 m

The height of the right circular cylinder is 1 m.

7. The inner diameter of a circular well is 3.5m. It is 10m deep. Find

(i) its inner curved surface area.

(ii) the cost of plastering this curved surface at the rate of Rs. 40 per m^2.

(Assume π = 22/7)

Answer: Since the well is cylindrical its curved surface area with base radius 'r' and height 'h' is 2πrh.

Diameter of the well, d = 3.5 m

Radius of the well, r = d/2 = 3.5/2 m = 1.75 m

Depth of the well, h = 10 m

i) The inner curved surface area of the well = 2πrh

= 2 × 22/7 × 1.75 m × 10 m

= 110 m^2

ii) We can calculate the cost of plastering by multiplying the curved surface area of the well and the rate of plastering per square meter.

Cost of plastering the curved surface area at ₹ 40 per m^2 = 110 × 40 = ₹ 4400

Thus, the inner curved surface area is 110 m^2 and the cost of plastering the circular well is ₹ 4400.

8. In a hot water heating system, there is a cylindrical pipe of length 28 m and diameter 5 cm. Find the total radiating surface in the system. (Assume π = 22/7).

Answer: Since the well is cylindrical its curved surface area with base radius 'r' and height 'h' is 2πrh.

Diameter of the pipe, d = 5 cm

Radius of the pipe, r = d/2 = 5/2 cm = 2.5 cm = 2.5/100 m = 0.025 m

Length of the pipe, h = 28 m

The total radiating surface area of the pipe = 2πrh

= 2 × 22/7 × 0.025 m × 28 m

= 4.4 m^2

Thus, the total radiating surface is 4.4 m^2.

9. Find

(i) the lateral or curved surface area of a closed cylindrical petrol storage tank that is 4.2 m in diameter and 4.5m high.

(ii) How much steel was actually used, if 1/12 of the steel actually used was wasted in making the tank? (Assume π = 22/7)

Answer: The curved surface area of a right circular cylinder of base radius 'r' and height 'h' is 2πrh and its total surface area is 2πr(r + h).

Diameter of the tank, d = 4.2 m

Radius of the tank, r = d/2 = 4.2/2 m = 2.1 m

Height of the tank, h = 4.5 m

i) Lateral or curved surface area of the tank = 2πrh

= 2 × 22/7 × 2.1 m × 4.5 m

= 59.4 m²

ii) Total surface area of the tank = 2πr (h + r)

= 2 × 22/7 × 2.1 m × (4.5 m + 2.1 m)

= 2 × 22/7 × 2.1 m × 6.6 m

= 87.12 m²

Let the amount of steel required to make the tank be 'x'.

Amount of steel required - Amount of steel wasted = Total surface area of the tank

x - x/12 = 87.12 m² [Since, 1/12 of the steel was wasted]

11x / 12 = 87.12 m²

x = 12/11 × 87.12 m²

x = 95.04 m²

Thus, Curve surface area = 59.4m², Steel actually used to make the tank = 95.04m²

10. In fig. 13.12, you see the frame of a lampshade. It is to be covered with a decorative cloth.

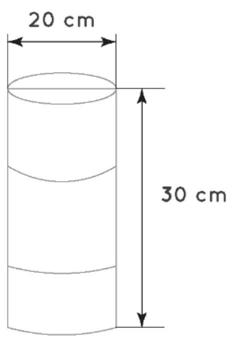


Fig. 13.12

The frame has a base diameter of 20 cm and a height of 30 cm. A margin of 2.5 cm is to be given for folding it over the top and bottom of the frame. Find how much cloth is required to cover the lampshade. (Assume π = 22/7)

Answer: Given: The frame has a base diameter of 20 cm and height of 30 cm.

Since the frame is cylindrical in shape, the cloth required to cover the lampshade will be equal to the curved surface area of cylinder.

The amount of cloth required to cover the cylinder will be equal to the curved surface area of the cylinder.

The curved surface area of a right circular cylinder = 2πrh

Diameter of the frame, d = 20 cm

Radius of the frame, r = d/2 = 20/2 cm = 10 cm

Height of the cylinder frame = 30 cm

Here, height will be equal to the height of cylinder plus 2.5 cm margin on both the sides which is used as an extra part for folding.

Height of the lampshade, h = 30 cm + 2.5 cm + 2.5 cm = 35 cm

Thus, cloth required for covering the lampshade = 2πrh

$= 2 \times 22/7 \times 10$ cm $\times 35$ cm

$= 2200$ cm²

Thus, the area of the cloth required is 2200 cm².

11. The students of Vidyalaya were asked to participate in a competition for making and decorating penholders in the shape of a cylinder with a base using cardboard. Each penholder was to be of radius 3 cm and height 10.5 cm. The Vidyalaya was to supply the competitors with cardboard. If there were 35 competitors, how much cardboard was required to be bought for the competition? (Assume π =22/7).

Answer: The curved surface area of a right circular cylinder of base radius 'r' and height 'h' is $2\pi rh$ and total surface area $=2\pi r(r + h)$.

So, the total amount of cardboard required will be the product of the surface area of pen holders and the total number of participating students.

Area of cardboard required for each penholder $= 2\pi rh + \pi r^2 = \pi r(2h + r)$

Area of cardboard required for 35 penholders $= 35 \times \pi r (2h + r)$

Radius of the penholder, $r = 3$ cm

Height of the penholder, $h = 10.5$ cm

Area of cardboard required for 35 penholders

$= 35 \times \pi r(2h + r)$

$= 35 \times 22/7 \times 3$ cm $\times (2 \times 10.5$ cm $+ 3$ cm$)$

$= 330$ cm $\times 24$ cm

$= 7920$ cm²

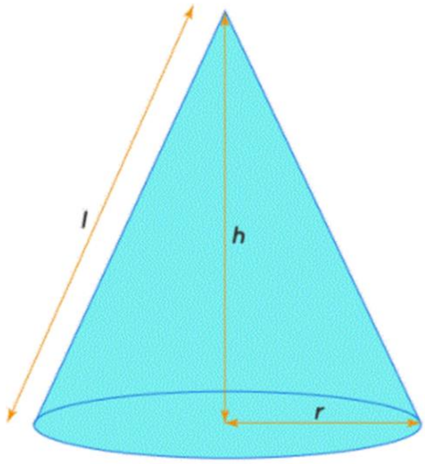
7920 cm² of cardboard will be required for the competition.

Exercise 13.3

1. Diameter of the base of a cone is 10.5 cm, and its slant height is 10 cm. Find its curved surface area. (Assume π=22/7).

Answer: Diameter of the base of the cone is 10.5 cm and slant height is 10 cm.

Curved surface area of a right circular cone of base radius, 'r' and slant height, 'l' is πrl.



Diameter, d = 10.5cm

Radius, r = 10.5/2 cm = 5.25 cm

Slant height, l = 10 cm

Curved surface area = πrl

= 22/7 × 5.25 × 10

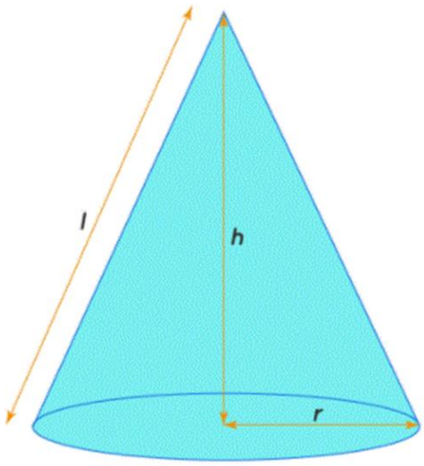
= 165 cm^2

Thus, curved surface area of the cone = 165 cm^2.

2. Find the total surface area of a cone, if its slant height is 21 m and the diameter of its base is 24 m. (Assume π = 22/7).

Answer: Diameter is 24m and the slant height of the cone is 21m.

The total surface area of the cone is the sum of the curved surface area and area of the base which is a circle.



Curved surface area of a right circular cone of base radius 'r' and slant height 'l' is πrl.

Total surface area of the cone = $\pi rl + \pi r^2 = \pi r(l + r)$

Diameter, d = 24m

Radius, r = 24/2m = 12m

Slant height, l = 21 m

Total surface area of the cone = $\pi r(l + r)$

= 22/7 × 12 m × (12 m + 21 m)

= 22/7 × 12 m × 33 m

= 8712/7 m^2

= 1244.57 m^2

Thus, total surface area of the cone = 1244.57 m^2

3. Curved surface area of a cone is 308 cm^{2}, and its slant height is 14 cm. Find

(i) radius of the base and (ii) total surface area of the cone.

(Assume π = 22/7)

Answer: Curved surface area of the cone is 308 cm^2 and its slant height is 14 cm.

The total surface area of the cone is the sum of the curved surface area and area of the base which is a circle.

Curved surface area of a right circular cone of base radius, r and slant height, l is πrl.

Total surface area of the cone = $\pi rl + \pi r^2 = \pi r(l + r)$

Let the radius be r

Slant height, l = 14 cm

Curved surface area = 308 cm^2

πrl = 308

r = 308 cm^2/πl

r = 308 cm^2/14 cm × 7/22

= 7 cm

Total surface area = πr (l + r)

= 22/7 × 7 cm × (7 cm + 14 cm)

= 22 cm × 21 cm

= 462 cm^2

Thus, radius of the cone is 7 cm and total surface area of the cone is 462 cm^2.

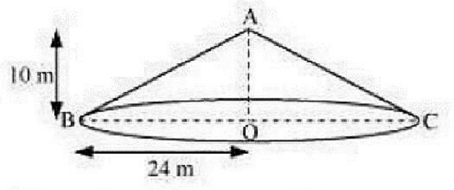
4. A conical tent is 10 m high, and the radius of its base is 24 m. Find

(i) slant height of the tent.

(ii) cost of the canvas required to make the tent, if the cost of 1 m^2 canvas is Rs 70.

(Assume π=22/7)

Answer: The total surface area of the cone is the sum of the curved surface area and area of the base which is a circle.



Curved surface area of a right circular cone of base radius, 'r' and slant height, 'l' is πrl

Slant height, l = $\sqrt{r^2 + h^2}$ where h is the height of the cone and r is the radius of the base.

i) Radius, r = 24 m

Height, h = 10 m

Slant height, l = $\sqrt{r^2 + h^2}$

= $\sqrt{(24)^2 + (10)^2}$

= $\sqrt{576 + 100}$

= $\sqrt{676}$

= 26 m

ii) Curved surface area of the cone = πrl

= 22/7 × 24 m × 26 m

= 13728/7 m^2

The cost of the canvas required to make the tent, at ₹ 70 per m^2 = 70 × Curved surface area of the cone

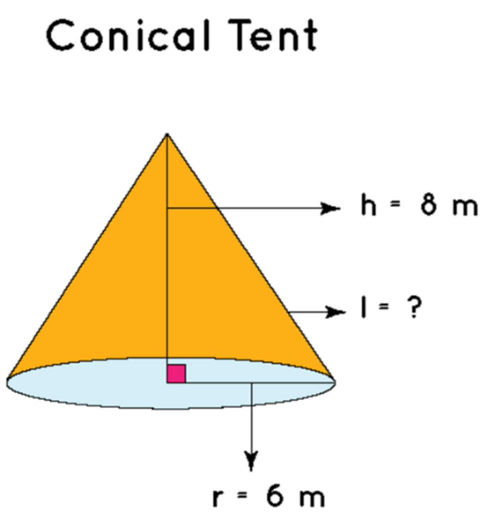
= 13728/7 × 70

= ₹ 137280

Thus, slant height of the tent is 26 m and the cost of the canvas is ₹ 137280.

5. What length of tarpaulin 3 m wide will be required to make a conical tent of height 8 m and base radius 6m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm. [Use π=3.14].

Answer: Since the tent is in a conical shape, the area of tarpaulin = the curved surface area of the cone.



The curved surface area of a right circular cone with base radius(r) and slant height(l) is πrl

where, Slant height, l = $\sqrt{(r^2 + h^2)}$, h is the height of the cone.

The length of the tarpaulin can be calculated by dividing its area by its breadth.

Since the extra length of material = 20 cm, the actual length of the tarpaulin will be obtained by adding 20 cm to the length of the tarpaulin.

Radius, r = 6 m

Height, h = 8 m

Slant height, $l = \sqrt{r^2 + h^2}$

$= \sqrt{(6)^2 + (8)^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

= 10 m

Therefore, the curved surface area = πrl

= 3.14 × 6m × 10m

= 188.4 m^2

Now, width of the tarpaulin = 3m

Area of the tarpaulin = 188.4 m^2

So, Area of the tarpaulin = width of the tarpaulin × length of the tarpaulin

188.4 m^2 = 3 × length of the tarpaulin

⇒ Length of the tarpaulin = 188.4 m^2/3

= 62.8 m

Extra length of the material = 20cm = 20/100m = 0.2m

Actual length required = 62.8m + 0.2m = 63m

Thus, the required length of the tarpaulin is 63 m.

6. The slant height and base diameter of the conical tomb are 25m and 14 m, respectively. Find the cost of whitewashing its curved surface at the rate of Rs. 210 per 100 m^2. (Assume π = 22/7).

Answer: The slant height of the conical tomb is 25 m and the base diameter is 14 m.

The curved surface area of a right circular cone of base radius(r) and slant height(l) is πrl

Slant height, $l = \sqrt{r^2 + h^2}$, where h is the height of the cone.

Diameter, d = 14 m

Radius, r = 14/2 m = 7 m

Slant height, l = 25 m

Curved surface area = πrl

= 22/7 × 7 m × 25 m

= 550m^2

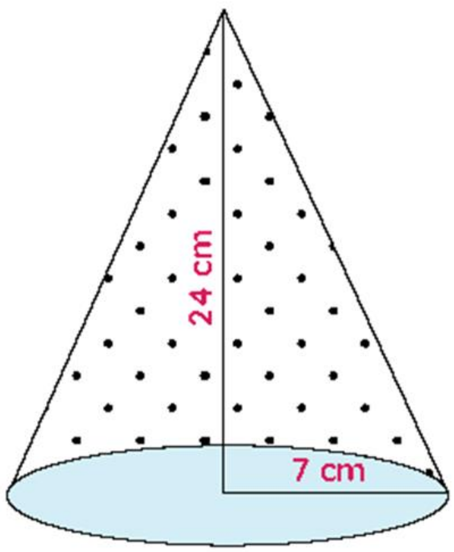
Cost of the whitewashing at ₹ 210 per 100 m^2

= (210/100) × 550 = ₹ 1155

Thus, the cost of whitewashing the conical tomb is ₹1155.

7. A joker's cap is in the form of a right circular cone of base radius 7 cm and height 24cm. Find the area of the sheet required to make 10 such caps. (Assume π =22/7).

Answer: Given: A right circular cone (joker's cap) of base radius 7 cm and height 24 cm.



Since the cap is conical in shape, the area of the sheet required to make each cap will be equal to the curved surface area of the cone.

The curved surface area of a right circular cone of base radius(r) and slant height(l) is πrl

Slant height, l = $\sqrt{r^2 + h^2}$ where h is the height of the cone.

Radius, r = 7 cm Height, h = 24 cm

Slant height,

$l = \sqrt{r^2 + h^2}$

$= \sqrt{(7)^2 + (24)^2}$

$= \sqrt{49 + 576}$

$= \sqrt{625}$

= 25 cm

Area of the sheet required to make each cap = πrl

= 22/7 × 7 cm × 25 cm

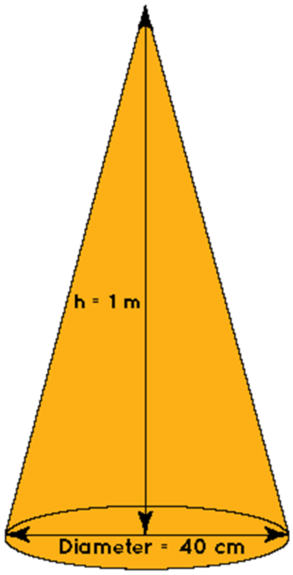
= 550 cm^2

Area of the sheet required to make 10 such caps = 10 × 550 cm^2 = 5500 cm^2

Thus, the area of the sheet required to make 10 such caps is 5500 cm2.

8. A bus stop is barricaded from the remaining part of the road by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is Rs. 12 per m^2, what will be the cost of painting all these cones? (Use π = 3.14 and take √(1.04) =1.02).

Answer: Given: The base diameter of the cone is 40 cm and its height is 1m.



Since the outer side of each cone is to be painted, the area to be painted will be equal to the curved surface area of the cone.

The curved surface area of a right circular cone with base radius(r) and slant height(l) is πrl

Slant height, $l = \sqrt{(r^2 + h^2)}$ where h is the height of the cone.

Diameter, d = 40cm = 40/100 m = 0.4m

Radius, r = 0.4/2m = 0.2m

Height, h = 1 m

Slant height, $l = \sqrt{(0.2)^2 + (1)^2}$

$= \sqrt{0.04m^2 + 1m^2}$

$= \sqrt{1.04} = 1.02m$ (given)

The curved surface area = πrl

= 3.14 × 0.2m × 1.02m

= 0.64056 m^2

Curved surface area of 50 cones = 50 × 0.64056 m^2 = 32.028 m^2

Cost of painting of 50 cones at ₹ 12 per m^2 = 32.028 × 12

= ₹ 384.34 (approx.)

Thus, the cost of painting all the cones is ₹ 384.34 (approx.)

Exercise 13.4

1. Find the surface area of a sphere of radius

(i) 10.5cm (ii) 5.6cm (iii) 14cm

(Assume π=22/7)

Answer: i) Radius, r = 10.5cm

Surface area of the sphere = $4\pi r^2$

= 4 × 22/7 × $(10.5cm)^2$

= 1386 cm^2

ii) Radius, r = 5.6cm

Surface area of the sphere = $4\pi r^2$

= 4 × 22/7 × $(5.6cm)^2$

= 394.24 cm^2

iii) Radius, r = 14cm

Surface area of the sphere = $4\pi r^2$

= 4 × 22/7 × $(14cm)^2$

= 2464 cm^2

Thus, the surface areas of the sphere of radii 10.5 cm, 5.6 cm, and 14 cm are 1386 cm^2, 394.24 cm^2, and 2464 cm^2 respectively.

2. Find the surface area of a sphere of diameter

(i) 14cm (ii) 21cm (iii) 3.5cm

(Assume π = 22/7)

Answer: i) Diameter, d = 14 cm

Radius, r = 14cm/2 = 7cm

Surface area of a sphere = $4\pi r^2$

= 4 × 22/7 × 7cm × 7cm

= 616 cm^2

ii) Diameter, d = 21 cm

Radius, r = 21cm/2

Surface area of a sphere = $4\pi r^2$

= 4/2 × 22/7 × 21/2 × 21/2

= 1386 cm^2

iii) Diameter, d = 3.5 m

Radius, r = 3.5/2 = 1.75m

Surface area of a sphere = $4\pi r^2$

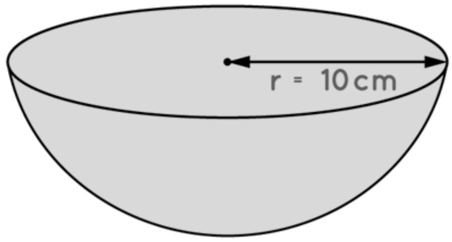
= 4 × 22/7 × 1.75m x 1.75m

= 38.5 m^2

The surface area of a sphere with diameters 14 cm, 21 cm, and 3.5 m are 616 cm^2, 1386 cm^2, and 38.5 m^2 respectively.

3. Find the total surface area of a hemisphere of radius 10 cm. [Use π=3.14].

Answer: A hemisphere is exactly half of a sphere having one circular surface at the top.



The total surface area of a hemisphere is half of the surface area of a sphere along with the top circular area added to it.

TSA of hemisphere = $(4\pi r^2)/2 + \pi r^2 = 3\pi r^2$

The radius of the hemisphere, r = 10cm

Total Surface area = $3\pi r^2$

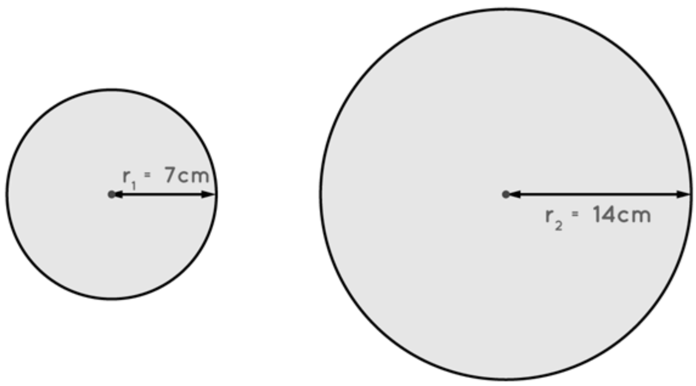
= 3 × 3.14 × 10cm × 10cm

= 942 cm^2

The total surface area of the hemisphere is 942 cm^2..

4. The radius of a spherical balloon increases from 7cm to 14cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.

Answer: The radius of the spherical balloon before and after filling air has radii of 7 cm and 14 cm respectively as shown below.



The surface area of a sphere = $4\pi r^2$

The radius of the balloon before pumping air, r_1 = 7cm

The radius of the balloon after pumping air, r_2 = 14cm

The surface area of the balloon before pumping air, $SA_1 = 4\pi(r_1)^2$

The surface area of the balloon after pumping air, $SA_2 = 4\pi(r_2)^2$

The ratio of the surface areas of the balloon,

= SA_1/CSA_2

= $4\pi(r_1)^2/4\pi(r_2)^2$

= $(r_1)^2/(r_2)^2$

= $(r_1/r_2)^2$

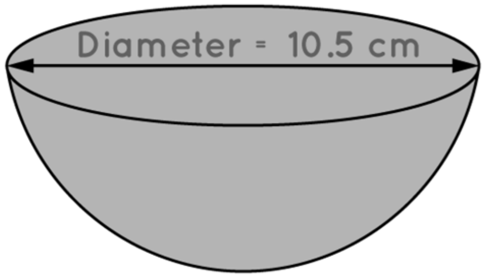
= $(7/14)^2$

= $(1/2)^2$

= (1/4)

The ratio of the surface areas of the balloons = 1: 4

5. A hemispherical bowl made of brass has an inner diameter 10.5cm. Find the cost of tin-plating it on the inside at the rate of Rs 16 per 100 cm^2. (Assume π = 22/7).



Answer: Hemisphere is exactly half of a sphere, so the curved surface area is half of the surface area of the sphere. CSA of the hemisphere with radius r, CSA = $2\pi r^2$.

Since the bowl is to be tin-plated from inside, the area to be tin-plated will be equal to the CSA of the hemisphere.

Inner diameter, d = 10.5cm

Inner radius, r = 10.5/2cm = 5.25cm

CSA of hemispherical bowl = $2\pi r^2$

= 2 × 22/7 × 5.25cm × 5.25cm

= 173.25 cm^2

The cost of tin-plating 100 cm^2 of the bowl = ₹16

The cost of tin-plating 1 cm^2 of the bowl = ₹16/100

The cost of tin-plating 173.25 cm^2 area of the bowl = (₹16/100) ×173.25 = 27.72

Thus, the cost of tin-plating is ₹ 27.72.

6. Find the radius of a sphere whose surface area is 154 cm^2. (Assume π = 22/7).

Answer: Given: The surface area of a sphere is 154 cm^2.

The surface area of the sphere of radius r, SA = $4\pi r^2$.

The surface area of the sphere = 154 cm^2

$4\pi r^2$ = 154 cm^2

r^2 = 154/4π

r^2 = (154/4) (7/22)

r^2 = (154 × 7) / (4 × 22)

r^2 = (49/4)

r = (7/2) = 3.5 cm

Thus, the radius of the sphere whose surface area is 154 cm^2 is 3.5 cm.

7. The diameter of the moon is approximately one-fourth of the diameter of the earth.

Find the ratio of their surface areas.

Answer: Given - The diameter of the moon is approximately one-fourth of the diameter of the earth.

Since the moon and earth are spherical in shape, so the surface area of a sphere of radius r, SA = $4\pi r^2$

Let the radius of the earth be R and the radius of the moon be r.

Diameter of the moon = 1/4 × diameter of the earth

Thus, the radius of the moon = 1/4 × radius of the earth [Since, radius = 2 × Diameter]

r = 1/4 × R

r/R = 1/4 ------------ (1)

Now, the surface area of earth = $4\pi R^2$

The surface area of moon = $4\pi r^2$

The ratio of their surface areas = $4\pi r^2/4\pi R^2$

= r^2/R^2

= $(r/R)^2$

= $(1/4)^2$ [From equation(1)]

= 1/16

The ratio of their surface areas = 1:16

8. A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5cm. Find the outer curved surface of the bowl. (Assume π =22/7).

Answer: Given - Inner radius of the bowl is 5 cm and the thickness of the steel is 0.25cm.

Since the hemispherical bowl is made up of 0.25cm thick steel, we can find the outer radius of the bowl by adding thickness to the inner radius.

The curved surface area of a hemisphere of radius r = $2\pi r^2$

The inner radius of the bowl, r = 5cm

Thickness of steel = 0.25cm

Outer radius of the bowl, R = 5cm + 0.25cm = 5.25cm

Outer CSA of the hemisphere = $2\pi R^2$

= 2 × 22/7 × 5.25cm × 5.25cm

= 173.25 cm^2

Thus, the outer curved surface area of the hemisphere is 173.25 cm^2.

9. A right circular cylinder just encloses a sphere of radius r (see fig. 13.22). Find

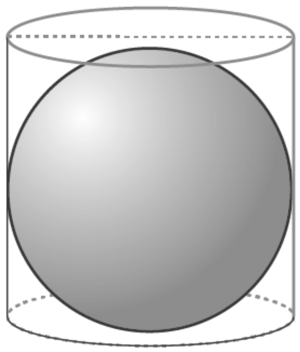


Fig. 13.22

(i) surface area of the sphere,

(ii) curved surface area of the cylinder,

(iii) ratio of the areas obtained in(i) and (ii).

Answer: Since the cylinder encloses the sphere as we can see in the figure, the radius of the cylinder will be equal to the radius of the sphere and the height of the cylinder will be equal to the diameter of the sphere.

The radius of the sphere = Radius of the cylinder = r

Height of the cylinder, h = diameter of the sphere = 2r

Thus, h = 2r

The surface area of a sphere with radius r = $4\pi r^2$

Curved surface area of a cylinder = $2\pi rh$

(i) Surface area of the sphere = $4\pi r^2$

(ii) Curved surface area of the cylinder = $2\pi rh$

= $2\pi r \times 2r$ [Since, h = 2r]

= $4\pi r^2$

(iii) The ratio of the areas obtained in (i) and (ii) is:

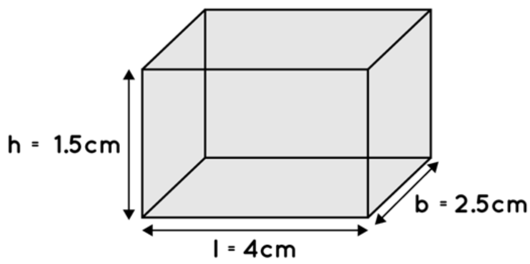
$4\pi r^2/4\pi r^2 = 1/1$

Thus, the surface area of the sphere and the curved surface area of the cylinder is $4\pi r^2$ and the ratio between these areas is 1:1.

Exercise 13.5

1. A matchbox measures 4 cm × 2.5cm × 1.5cm. What will be the volume of a packet containing 12 such boxes?

Answer: Dimensions of a matchbox are 4 cm × 2.5 cm × 1.5 cm



Since the matchbox is cuboidal in shape, the volume of each matchbox will be equal to the volume of the cuboid.

The volume of the cuboid of length l, breadth b, and height h, V = l × b × h

Length of the matchbox, l = 4 cm, Breadth of the matchbox, b = 2.5 cm, Height of the matchbox, h = 1.5 cm

The volume of each matchbox = l × b × h

= 4 cm × 2.5 cm × 1.5 cm

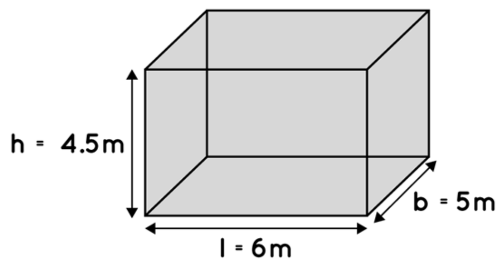
= 15 cm^3

Volume of 12 matchboxes = 12 × 15cm^3 = 180cm^3

Thus, the volume of the packet containing 12 such matchboxes is 180 cm^3.

2. A cuboidal water tank is 6m long, 5m wide and 4.5m deep. How many litres of water can it hold? (1 m^3= 1000 l).

Answer: Length, breadth, and depth of the cuboidal water tank are 6 m, 5 m, and 4.5 m respectively.



Since the water tank is cuboidal in shape, the volume of water in the tank will be equal to the volume of the cuboid.

The volume of the cuboid of length l, breadth b, and height h, is V = l × b × h

Length of the cuboidal tank, l = 6 m, Breadth of the cuboidal tank, b = 5 m, Height of the cuboidal tank, h = 4.5 m

Volume of the cuboidal tank = l × b × h

V = 6m × 5m × 4.5m

V = 135 m^3

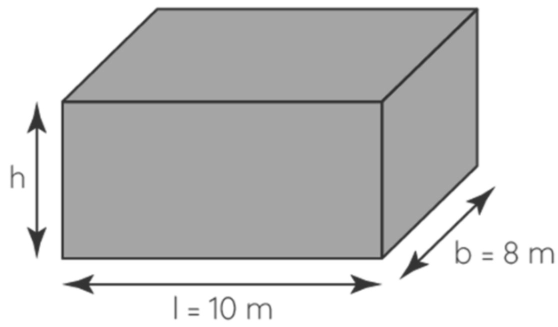
V = 135 × 1000L

V = 135000L (1m^3 = 1000L)

Thus, the cuboidal water tank can hold 135000 litres of water.

3. A cuboidal vessel is 10m long and 8m wide. How high must it be made to hold 380 cubic metres of a liquid?

Answer: Given: Length and breadth of the cuboidal vessel are 10 m and 8 m respectively. It must hold 380 m^3 of a liquid.



Since the vessel is cuboidal in shape, the volume of liquid in the vessel will be equal to the volume of the cuboid.

The volume of the cuboid of length l, breadth b, and height h, is V = l × b × h

Let the height of the cuboidal vessel be h.

Length of the cuboidal vessel, l = 10 m

The breadth of the cuboidal vessel, b = 8 m

The capacity of the cuboidal vessel (V) = 380 m^3

Volume of the liquid in the cuboidal vessel = l × b × h

l × b × h = 380m^3

10 m × 8 m × h = 380m^3

h = 380 / (10 × 8)

h = 4.75 m

Thus, the cuboidal vessel must be made 4.75 m high

4. Find the cost of digging a cuboidal pit 8m long, 6m broad and 3m deep at the rate of Rs 30 per m^3.

Answer: The length, breadth, and depth of the cuboidal pit are 8 m, 6 m, and 3 m respectively. The rate of digging the pit is ₹30 per m^3.

We have to find the cost of digging the cuboidal pit.

Since the pit is cuboidal in shape, the volume of the pit will be equal to the volume of the cuboid.

The volume of the cuboid of length l, breadth b, and height h, is l × b × h

We can find the cost of digging the pit by multiplying the volume of the pit and the rate of digging.

Length of the cuboidal pit, l = 8 m

Breadth of the cuboidal pit, b = 6 m

Height of the cuboidal pit, h = 3 m

Volume of the cuboidal pit = l × b × h

= 8m × 6m × 3m

= 144 m^3

Thus, the cost of digging the pit at ₹ 30 per m^3 = ₹ 30 × 144 = ₹ 4320

5. The capacity of a cuboidal tank is 50000 litres of water. Find the breadth of the tank, if its length and depth are respectively 2.5 m and 10 m.

Answer: Given: the length and depth of the cuboidal tank are 2.5 m and 10 m respectively. The capacity of the tank is 50000 litres.

We have to find the breadth of the cuboidal tank.

Since the tank is cuboidal in shape, the volume of the tank will be equal to the volume of the cuboid.

The volume of cuboid of length l, breadth b, and height h, is l × b × h

First, we will change volume in cubic meters because all the measurements are in meters.

The capacity of the tank = 50000L

= 50000/1000 m^3 (∵ $1m^3$ = 1000L)

= 50 m^3

Length of the cuboidal tank, l = 2.5 m

Height of the cuboidal tank, h = 10 m

Let the breadth of the cuboidal tank be b

Volume of the cuboidal tank = l × b × h

l × b × h = 50 m^3

b = 50 m^3/l × h

b = 50 m^3/(2.5m × 10m)

= 2 m

6. A village, having a population of 4000, requires 150 litres of water per head per day.

It has a tank measuring 20 m×15 m×6 m. For how many days will the water in this tank last?

Answer: Given - 150 litres of water requires per head per day in a village of 4000 population. The dimensions of the tank are 20 m × 15 m × 6 m

We have to find the number of days, the water of this tank will last.

Since the tank is cuboidal in shape, the volume of water in the tank will be equal to the volume of the cuboid.

The volume of cuboid of length l, breadth b, and height h, is l × b × h

First, we will find the requirement of water per day for a total of 4000 population in cubic metres and the volume of the water in the tank.

The number of days for which the water of the tank will last can be obtained by dividing the volume of the water in the tank by the requirement of water per day for the total population.

Requirement of water per head per day is 150 litres.

Requirement of water per day for 4000 population = 4000 × 150L

= 600000L

= 600000/1000 m^3 (∵ 1000L = 1 m^3)

= 600 m^3

Length of the tank, l = 20 m, Breadth of the tank, b = 15 m, and Height of the tank, h = 6 m

The volume of the water in the tank = l × b × h

= 20m × 15m × 6m

= 1800 m^3

Number of days for which the water of the tank will last = 1800m^3/600m^3 = 3 days

7. A godown measures 40 m×25m×15 m. Find the maximum number of wooden crates, each measuring 1.5m×1.25 m×0.5 m, that can be stored in the godown.

Answer: Given: Dimensions of the godown are 40m × 25m × 15m and that of crate is 1.5m × 1.25m × 0.5m

We have to find the number of crates that can be stored in the godown.

Since the godown and crates are cuboidal in shape, their volume will be equal to the volume of the cuboid.

The volume of cuboid of length l, breadth b, and height h, is l × b × h

The maximum number of wooden crates that can be stored in the godown will be the ratio of the volume of the godown to the volume of a wooden crate.

Length of the godown, breadth of the godown and height of the godown are L = 40 m, B = 25 m and H = 15 m respectively.

Capacity of the godown = L × B × H

= 40m × 25m × 15m

= 15000 m^3

Length of the crate, l = 1.5 m

Breadth of the crate, b =1.25 m

Height of the crate, h = 0.5 m

Volume of each crate = l × b × h

= 1.5m × 1.25m × 0.5m

= 0.9375 m^3

Number of crates = 15000m^3/0.9375m^3 = 16000

The maximum number of wooden crates that can be stored in the godown is 16000.

8. A solid cube of side 12 cm is cut into eight cubes of equal volume. What will be the side of the new cube? Also, find the ratio between their surface areas.

Answer: Given: a solid cube of side 12 cm is cut into eight cubes of equal volume.

We have to find the side of the new cube and the ratio between their surface areas.

Since the solid cube is cut into eight cubes of equal volume, each smaller cube so obtained will have the one-eighth volume of the solid cube of side 12 cm.

For the ratio of their surface areas, we will find the surface area of the two types of cubes individually.

Volume of the cube of edge length 'a' = a^3

The surface area of the cube of edge length 'a' = $6a^2$

Edge of the solid cube, a = 12 cm

The volume of the solid cube = a^3

= $(12\text{ cm})^3$

= 1728 cm^3

The cube is cut into 8 equal cubes of the same volume.

Volume of each small cube = (1/8) × 1728 cm^3 = 216 cm^3

Let 'x' be the side of each small cube.

Volume of each small cube = x^3 = 216 cm^3

$x^3 = (6\text{ cm})^3$ [Since $6^3 = 216$]

x = 6 cm

Surface area of the solid cube = $6a^2$

Surface area of the small cube = $6x^2$

Ratio between their surface areas = $6a^2/6x^2$

= a^2/x^2

= $(a/x)^2$

= $(12/6)^2$

= 4/1

The side of the new cube is 6 cm and the ratio between the surface areas is 4 : 1.

9. A river 3m deep and 40m wide is flowing at the rate of 2km per hour. How much water will fall into the sea in a minute?

Answer: Given: the depth and width of the river are 3 m and 40 m respectively and water is flowing at the rate of 2 km per hour.

We have to find the amount of water that will fall into the sea in a minute.

Since the water in the river flowing in a cuboidal shape and the volume of the water that falls into the sea is nothing but the volume of the cuboid.

The volume of the cuboid of length l, breadth b, and height h, = lbh

Water is flowing at the rate of 2 km per hour, but we need to change this into meters per minute so that we can obtain the length of the flowing water in a minute.

Hence, we can easily find the volume of water that falls into the sea by calculating the volume of the cuboid.

Width of the river, b = 40 m

Depth of the river, h = 3 m

Flowing rate of water = 2 km / h

= 2000 m / 60 min

= 100/3 m/min

Length of the water flowing in 1 minute, l = 100/3 m

The volume of the water that falls into the sea in 1 minute = l × b × h

= 100/3 m × 40 m × 3 m

= 4000 m^3

Thus, 4000 m^3 of water will fall into the sea in a minute

Exercise 13.6

1. The circumference of the base of the cylindrical vessel is 132cm, and its height is 25cm.

How many litres of water can it hold? ($1000\ cm^3 = 1L$) (Assume π = 22/7)

Answer: Since the base of a cylindrical vessel is a circle, so its radius can be easily obtained using the circumference = 2πr

The volume of a cylinder of base radius, r and height, h = $\pi r^2 h$

Let the radius of the base be 'r'

Height of the cylinder, 'h' = 25 cm

Circumference of the base = 132 cm

2πr = 132 cm

r = 132 / 2π

= (132 / 2) × (7 / 22)

= 21 cm

The capacity of the cylindrical vessel = $\pi r^2 h$

= 22/7 × 21 cm × 21 cm × 25 cm

= $34650\ cm^3$

= 34650/1000 (Since $1000\ cm^3 = 1\ L$)

= 34.65 L

2. The inner diameter of a cylindrical wooden pipe is 24cm, and its outer diameter is 28 cm. The length of the pipe is 35cm. Find the mass of the pipe, if $1cm^3$ of wood has a mass of 0.6g. (Assume π = 22/7).

Answer: Since the cylindrical wooden pipe is made up of two concentric circles at the top and bottom, we will find the volume of both cylinders.

The volume of a cylinder of base radius, r, and height, h = $\pi r^2 h$

The volume of wood can be obtained by finding the difference between the volumes of both the outer and inner cylinders.

Outer diameter of the pipe = 28 cm

Outer radius of the pipe, R = 28/2 = 14 cm

Inner diameter of the pipe = 24 cm

Inner radius of the pipe, r = 24/2 = 12cm

Length of the pipe, h = 35 cm

Volume of the outer cylinder, $V_1 = \pi R^2 h$

V_1 = 22/7 × 14cm × 14cm × 35cm

= 21560 cm^3

Volume of the inner cylinder, $V_2 = \pi r^2 h$

V_2 = 22/7 × 12cm × 12cm × 35cm

= 15840 cm^3

The volume of the wood used = Volume of the outer cylinder – Volume of the inner cylinder

= 21560 cm^3 - 15840 cm^3

= 5720 cm^3

Mass of 1 cm^3 wood is 0.6 g

Mass of 5720 cm^3 wood = 5720 × 0.6g

= 3432 g

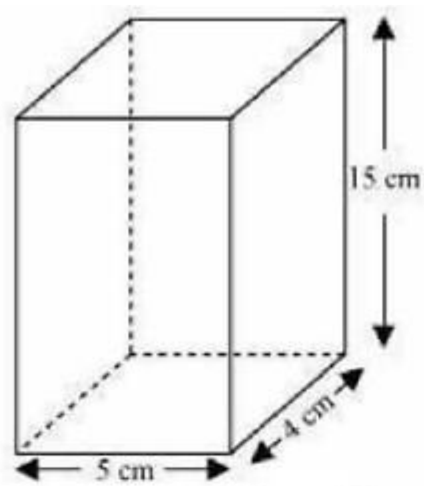
= (3432/1000) kg

= 3.432 kg

Thus, the mass of the wooden pipe is 3.432 kg

3. A soft drink is available in two packs – (i) a tin can with a rectangular base of length 5cm and width 4cm, having a height of 15 cm and (ii) a plastic cylinder with a circular base of diameter 7cm and height 10cm. Which container has greater capacity, and by how much? (Assume π=22/7).

Answer: Since the tin can is cuboidal in shape while the other is cylindrical, we will find the volume of both containers.



The volume of a cylinder of base radius, r, and height, h = $\pi r^2 h$

The volume of a cuboid of length ' l ', breadth ' b', and height ' h ' = l × b × h

Dimensions of tin can with a rectangular base are:

Length of the cuboidal tin can, l = 5cm

The breadth of the cuboidal tin can, b = 4cm

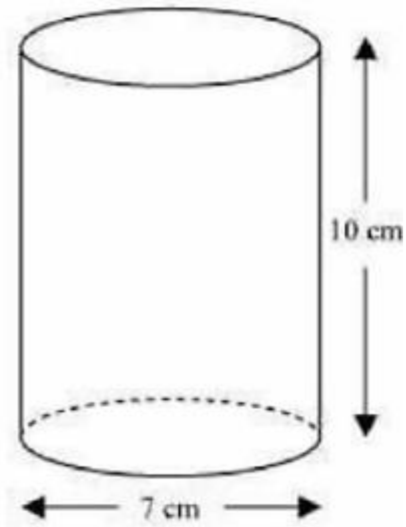
Height of the cuboidal tin can, h = 15cm

The volume of the cuboidal tin can = l × b × h

= 5 cm × 4 cm × 15 cm

= 300 cm^3

Dimensions of the plastic cylinder with a circular base are:



The diameter of the cylindrical plastic can = 7 cm

The radius of the cylindrical plastic can, r = 7/2 cm

Height of the cylindrical plastic can, h = 10cm

The volume of the cylindrical plastic can = $\pi r^2 h$

= 22/7 × 7/2 cm × 7/2 cm × 10cm

= 385 cm^3

Clearly, the plastic cylinder with a circular base has greater capacity than the tin container.

Difference = 385 cm^3 - 300 cm^3 = 85 cm^3

The plastic cylindrical has more capacity than the tin can by 85 cm^3.

4. If the lateral surface of a cylinder is 94.2cm^2 and its height is 5cm, then find

(i) radius of its base (ii) its volume. [Use π= 3.14]

Answer: Since the lateral surface area and height are known, we can easily obtain the radius and its volume.

Lateral surface area (CSA) of a cylinder of base radius r, and height h = $2\pi rh$

The volume of a cylinder of base radius r, and height h = $\pi r^2 h$

(i) Let the radius of the cylinder be r.

Height of the cylinder, h = 5 cm

Lateral surface area = 92.4 cm^2

$2\pi rh = 92.4$

r = 92.4 / 2hπ

r = 92.4 / (2 × 5 × 3.14)

r = 3 cm (approx.)

(ii) Volume of cylinder = $\pi r^2 h$

= 3.14 × 3 cm × 3 cm × 5 cm

= 141.3 cm^3

Thus, the radius of the base is 3 cm and the volume is 141.3 cm^3.

5. It costs Rs 2200 to paint the inner curved surface of a cylindrical vessel 10m deep. If the cost of painting is at the rate of Rs 20 per m^2, find

(i) inner curved surface area of the vessel

(ii) radius of the base

(iii) capacity of the vessel

(Assume π = 22/7)

Answer: Since the cost to paint the inner curved surface and its rate is known, we can obtain the inner CSA.

The ratio between the total cost and the rate per m^2 will give the inner CSA in m^2.

CSA of a cylinder of base radius r, and height h = 2πrh

The volume of a cylinder of base radius r, and height h = πr^2h

Total cost to paint inner CSA = ₹ 2200

Rate of painting = ₹ 20 per m^2

Inner CSA of the cylindrical vessel = 2200/20 = 110 m^2

Height of the vessel, h = 10m

Inner CSA of the vessel = 110 m^2

2πrh = 110 m^2

r = 110 / 2πh

= 110 /(2 × 10) × 7/22

= 7/4 m

= 1.75 m

Volume of the vessel = πr^2h

= 22/7 × 1.75 m × 1.75 m × 10 m

= 96.25 m^3

Thus, inner curved surface area is 110 m^2, radius of the base is 1.75 m and capacity of the vessel is 96.25 m^3.

6. The capacity of a closed cylindrical vessel of height 1m is 15.4 litres. How many square metres of the metal sheet would be needed to make it? (Assume π = 22/7).

Answer: Since the cylinder is a closed vessel, a metal sheet would be needed for the curved surface area and area of the two bases, top and bottom, that is TSA of the cylinder.

Hence, area of the metal sheet will be equal to TSA of the cylinder.

TSA of a cylinder of base radius r, and height h = $2\pi r(r + h)$

Volume of a cylinder of base radius r, and height h = $\pi r^2 h$

Capacity of the vessel = 15.4 litres

= 15.4 / 1000 m^3 (Since, 1000 l = $1m^3$)

= 0.0154 m^3

Let the radius of the vessel be r

Height of the vessel, h = 1 m

Volume of the vessel = 0.0154 m^3

$\pi r^2 h = 0.0154\ m^3$

$r^2 = 0.0154 / \pi h$

$r^2 = 0.0154 /1 \times 7/22$

$r^2 = 0.0049\ m^2$

Thus, r = 0.07 m

TSA of the cylinder = $2\pi r(r + h)$

= $2 \times 22/7 \times 0.07m \times (0.07m + 1m)$

= $0.44\ m \times 1.07\ m$

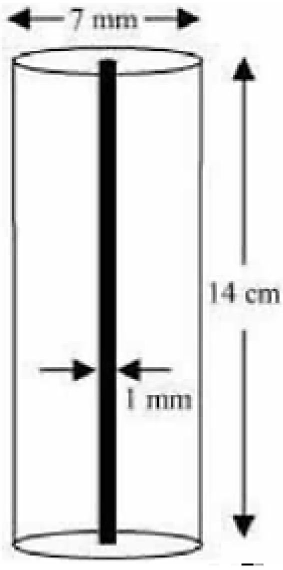
= 0.4708 m^2

0.4708 m^2 of metal sheet would be needed to make the vessel.

7. A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled in the interior. The diameter of the pencil is 7 mm, and the diameter of the graphite is 1 mm. If the length of the pencil is 14 cm, find the volume of the wood and that of the graphite. (Assume π = 22/7).

Answer: Since the lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled in the interior, the height of the graphite and wood will be the same as the height of the pencil.

Volume of the wood can be calculated by subtracting volume of graphite from volume of the pencil.



Volume of a cylinder of base radius, r and height, h = $\pi r^2 h$

For graphite:

Diameter of the graphite = 1 mm

Radius (r) = 1 mm = 0.5/10 cm = 0.05 cm

h = 14 cm

Volume of the graphite = πr^2 h

= 22/7 × 0.05 cm × 0.05 cm × 14 cm

= 0.11 cm^3

For pencil:

Diameter of the pencil = 7 mm

Radius (R) = 7 / 2 mm = 3.5/10 cm = 0.35 cm

h = 14 cm

Volume of the pencil = πR^2 h

= 22/7 × 0.35 cm × 0.35 cm × 14 cm

= 5.39 cm^3

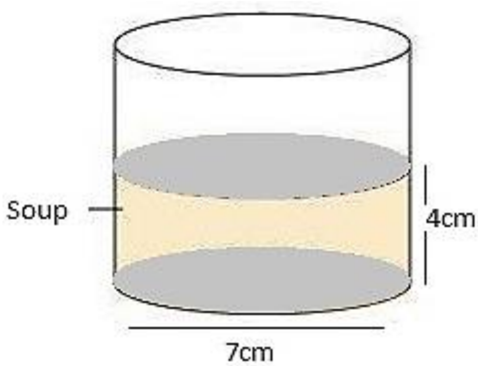
Volume of wood = Volume of the pencil – Volume of the graphite

= 5.39 cm^3 - 0.11cm^3

= 5.28 cm^3

The volume of the wood is 5.28 cm^3 and the volume of graphite is 0.11 cm^3.

8. A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7cm. If the bowl is filled with soup to a height of 4cm, how much soup the hospital has to prepare daily to serve 250 patients? (Assume π = 22/7).



Answer: Since the cylindrical bowl is filled with soup, the volume of the soup will be equal to the volume of the cylindrical bowl.

The volume of a cylinder of base radius r, and height h = $\pi r^2 h$

The amount of soup to be prepared will be the product of the volume of soup in each bowl and the total number of patients.

Diameter of the bowl = 7 cm

Radius of the bowl, r = 7/2 cm = 3.5cm

Height of the bowl, h = 4 cm

The volume of soup in each bowl = $\pi r^2 h$

= 22/7 × 3.5 cm × 3.5 cm × 4 cm

= 154 cm^3

The volume of soup for 1 patient = 154 cm^3

Thus, the volume of soup for 250 patients

= 250 × 154 cm^3

= 38500 cm^3

= 38500 / 1000 (Since, 1000 cm^3 = 1l)

= 38.5 l

The hospital has to prepare 38.5 litres of soup daily to serve 250 patients.

Exercise 13.7

1. Find the volume of the right circular cone with

(i) radius 6cm, height 7 cm (ii) radius 3.5 cm, height 12 cm (Assume π = 22/7)

Answer: Volume of a cone of base radius r, and height h, = $1/3\pi r^2 h$

i) Radius of the cone, r = 6 cm

Height of the cone, h = 7 cm

Volume of the cone = $1/3\pi r^2 h$

= 1/3 × 22/7 × 6 cm × 6 cm × 7 cm

= 264 cm^3

ii) Radius of the cone, r = 3.5 cm

Height of the cone, h = 12 cm

Volume of the cone = $1/3\pi r^2 h$

= 1/3 × 22/7 × 3.5 cm × 3.5 cm × 12 cm

= 154 cm^3

2. Find the capacity in litres of a conical vessel with

(i) radius 7cm, slant height 25 cm (ii) height 12 cm, slant height 13 cm

(Assume $\pi = 22/7$)

Answer: Capacity of a conical vessel is nothing but the volume of the cone.

Volume of a cone of base radius r, and height h = $1/3\pi r^2 h$

Slant height of the cone, $l = \sqrt{r^2 + h^2}$

i) Radius of the conical vessel, r = 7cm

Slant height of the conical vessel, l = 25cm

Height of the conical vessel, $h = \sqrt{l^2 - r^2}$

$= \sqrt{(25)^2 - (7)^2}$

$= \sqrt{625 - 49}$

$= \sqrt{576}$

h = 24 cm

Capacity of the conical vessel = $1/3\ \pi r^2 h$

= 1/3 × 22/7 × 7 cm × 7 cm × 24 cm

= 1232 cm^3

= 1232 × (1/1000L) [∵ 1000 cm^3 = 1litre]

= 1.232 litres

ii) Height of the conical vessel, h = 7cm

Slant height of the conical vessel, l = 13cm

Radius of the conical vessel, $r = \sqrt{l^2 - h^2}$

$= \sqrt{(13)^2 - (12)^2}$

$= \sqrt{169 - 144}$

$= \sqrt{25}$

r = 5 cm

Capacity of the conical vessel = $1/3\pi r^2 h$

= 1/3 × 22/7 × 5 cm × 5 cm × 12 cm

= 2200/7 cm^3

= 2200/7 × 1/1000 l [∵ 1000 cm^3 = 1litre]

= 11/35 litres

3. The height of a cone is 15cm. If its volume is 1570cm^3, find the diameter of its base. (Use π = 3.14)

Answer: Volume of a cone of base radius 'r', and height 'h' = $1/3\pi r^2h$

Volume of the cone = 1570 cm^3

Height of the cone, 'h' = 15cm

Radius of the cone, 'r' = ?

$1/3\pi r^2h$ = 1570 cm^3

r^2 = (1570 cm^3 × 3) / πh

r^2 = (1570 cm^3 × 3) / (3.14 × 15 cm) = 100 cm^2

r = √100 cm^2

r = 10 cm

Radius of the base = 10 cm

4. If the volume of a right circular cone of height 9cm is 48πcm^3, find the diameter of its base.

Answer: The volume of a cone of base radius 'r', and height 'h', = $1/3\pi r^2h$

Volume of the cone = 48π cm^3

Height of the cone, 'h' = 9 cm

Radius of the cone, 'r' = ?

$1/3\pi r^2h$ = 48π cm^3

r^2 = 48 cm^3 × 3 / h

r^2 = 48 cm^3 × 3 / 9 cm

r^2 = 16 cm^2

r = √16 cm^2

r = 4 cm

Base diameter, 'd' = 2 × radius(r)

= 2 × 4 cm

= 8 cm

The diameter of the box of the right circular cone is 8 cm.

5. A conical pit of a top diameter 3.5m is 12m deep. What is its capacity in kilolitres?

(Assume $\pi = 22/7$)

Answer: Volume of a cone having radius 'r', and height 'h' = $1/3\pi r^2h$

Diameter of the conical pit, 'd' = 3.5 m

Radius of the conical pit, 'r' = 3.5/2 m = 1.75 m

Depth of the conical pit, 'h' = 12m

Volume of conical pit = $1/3\pi r^2h$

= 1/3 × 22/7 × 1.75 m × 1.75 m × 12 m

= 38.5 m^3

= 38.5 × 1 kilolitres (1m^3 = 1000 Litres = 1 kilolitres)

= 38.5 kl

Capacity of the conical pit is 38.5 kilolitres.

6. The volume of a right circular cone is 9856cm^3. If the diameter of the base is 28cm, find

(i) height of the cone

(ii) slant height of the cone

(iii) curved surface area of the cone

(Assume $\pi = 22/7$)

Answer: Volume of a cone having radius 'r', and height 'h' = $1/3\pi r^2h$

Curved surface area of the cone having radius, 'r' and slant height, 'l' = πrl

Slant height of the cone, $l = \sqrt{r^2 + h^2}$ --- Equation(3)

Diameter of the cone, d = 28 cm

Radius of the cone, r = 28/2 cm = 14 cm

i) Height of the cone, h = ?

Volume of the cone = 9856 cm^3 and radius(r) = 14 cm

$1/3\pi r^2h$ = 9856 cm^3

h = 9856 cm^3 × $3/\pi r^2$

= 9856 cm^3 × 3/(14 cm × 14 cm) × 7/22

= 48 cm

ii) Slant height of the cone, l'= ?

radius(r) = 14 cm and height(h) = 48 cm

$l = \sqrt{r^2 + h^2}$

$= \sqrt{(14)^2 + (48)^2}$

$= \sqrt{196 + 2304}$

$= \sqrt{2500}$

= 50 cm

iii) Curved surface area of the cone = ?

radius(r) = 14 cm and slant height(l) = 50 cm

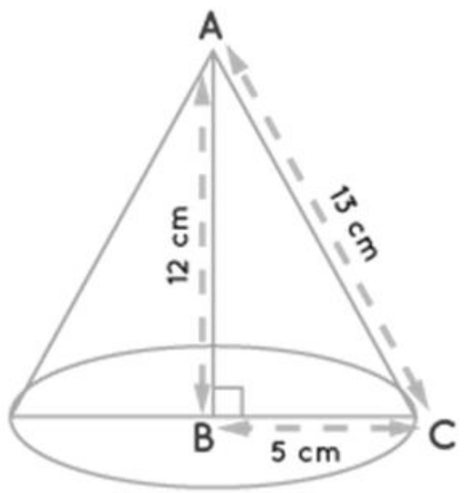
Curved surface area = πrl

= 22/7 × 14 cm × 50 cm

= 2200 cm^2

7. A right triangle ABC with sides 5cm, 12cm and 13cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

Answer: Since the triangle is revolved about the side 12 cm, a solid cone is formed with a height of 12 cm and radius of the base of 5 cm as shown below.



Volume of a cone having radius 'r', and height 'h', = $1/3\pi r^2h$

Radius of the cone, 'r' = 5 cm

Height of the cone, 'h' = 12 cm

Volume of the cone = $1/3\pi r^2h$

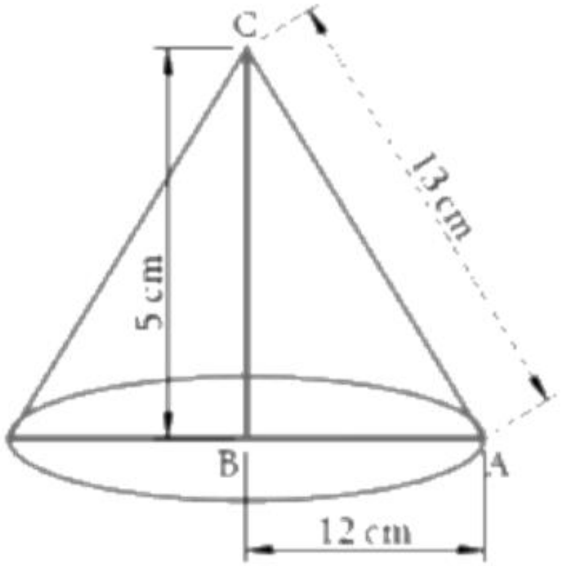
= 1/3 × π × 5 cm × 5 cm × 12 cm

= 100π cm^3

Volume of the cone is 100π cm^3

8. If the triangle ABC in Question 7 is revolved about the side 5cm, then find the volume of the solids so obtained. Find also the ratio of the volumes of the two solids obtained in Questions 7 and 8.

Answer: Since the triangle is revolved about the side 5 cm, a solid cone if formed with a height of 5 cm and radius of the base of 12 cm.



Volume of a cone having radius 'r' and height 'h' = $1/3\pi r^2h$

Radius of the cone, r =12cm

Height of the cone, h = 5cm

Volume of the cone = $1/3\pi r^2h$

= (1/3) × π × 12cm × 12cm × 5cm

= 240π cm^3

Volume of the cone in question 7 = 100π cm

Ratio = Volume of the cone in question 7 : Volume of the cone in question 8

= 100π : 240π

= 5 :12

The volume of the cone is 240π cm^3 and the required ratio is 5 :12 .

9. A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas.

(Assume π = 22/7).

Answer: Since the heap of wheat is in the form of a cone and the canvas required to cover the heap will be equal to the curved surface area of the cone.

Volume of a cone of base radius, 'r' and height, 'h' = $1/3\pi r^2h$

Curved surface area of the cone having a base radius, 'r' and slant height, 'l' = πrl

Slant height of the cone, $l = \sqrt{r^2 + h^2}$

Diameter of the conical heap, d = 10.5 m

Radius of the conical heap, r = 10.5/2 m = 5.25 m

Height of the conical heap, h = 3 m

Volume of the conical heap = $1/3\pi r^2h$

= 1/3 × 22/7 × 5.25 m × 5.25 m × 3 m

= 86.625 m^3

Slant height, l = $\sqrt{r^2 + h^2}$

= $\sqrt{(5.25)^2 + (3)^2}$

= √27.5625 + 9

= √36.5625

= 6.046 m (approx.)

The area of the canvas required to cover the heap of wheat = πrl

= 22/7 × 5.25 m × 6.046 m

= 99.759 m^2

The volume of the conical heap is 86.625 m^3 and the area of the canvas required is 99.759 m^2.

Exercise 13.8

1. Find the volume of a sphere whose radius is

(i) 7 cm (ii) 0.63 m

(Assume π =22/7)

Answer: We will find the volume of a sphere by using the volume of the spheres formula = $4/3\pi r^3$

i) Radius of the sphere, r = 7 cm

Volume of the sphere = $4/3\pi r^3$

= 4/3 × 22/7 × 7 cm × 7 cm × 7 cm

= 4312/3 cm^3

= 1437.33 cm^3

ii) Radius of the sphere, r = 0.63m

Volume of the sphere = $4/3\pi r^3$

= 4/3 × 22/7 × 0.63 m × 0.63 m × 0.63 m

= 1.047816 m^3

= 1.05 m^3 (approx.)

2. Find the amount of water displaced by a solid spherical ball of diameter

(i) 28 cm (ii) 0.21 m

(Assume π =22/7)

Answer: The amount of water displaced by a solid spherical ball is nothing but its volume.

The volume of a sphere = $4/3\pi r^3$

i) Diameter of the spherical ball, d = 28cm

Radius of the spherical ball, r = 28/2cm = 14cm

Therefore, the amount of water displaced = volume of the sphere = $4/3\pi r^3$

= 4/3 × 22/7 × 14cm × 14cm × 14cm

= 34496/3 cm^3

= 11498.66 cm^3

ii) Diameter of the spherical ball, d = 0.21 m

Radius of the spherical ball, r = 0.21/2 m = 0.105 m

Therefore, the amount of water displaced = volume of a sphere = $4/3\pi r^3$

= 4/3 × 22/7 × 0.105m × 0.105m × 0.105m

= 0.004851 m^3

3. The diameter of a metallic ball is 4.2cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm^3? (Assume π=22/7).

Answer: The diameter of the metallic ball = 4.2 cm

The density of the metal = 8.9 g per cm^3.

We know that,

Density = Mass / Volume

Volume of a sphere = $4/3\pi r^3$

Diameter of the metallic ball, d = 4.2cm

Radius of the metallic ball, r = 4.2/2cm = 2.1cm

The volume of the metallic ball = $4/3\pi r^3$

= 4/3 × 22/7 × 2.1cm × 2.1cm × 2.1cm

= 38.808 cm^3

Now, Mass = Volume × Density

Mass of the metallic ball = 38.808 cm^3 × 8.9g / cm^3

= 345.3912 g

= 345.39 g (approx.)

Thus, the mass of the ball is 345.39 g.

4. The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

Answer: The volume of a sphere of base radius r is $4/3\pi\ r^3$

Let the radius of the earth be R and the radius of the moon be r

Diameter of the moon = 1/4 × diameter of the earth

The radius of the moon = 1/4 × radius of the earth [Since, diameter = 2 × Radius]

r = 1/4 × R

r = R/4

The volume of the earth = $4/3\pi\ R^3$

The volume of the moon = $4/3\pi\ r^3$

= $4/3\pi\ (R/4)^3$ [replacing r with R/4]

= $1/64 \times 4/3\pi\ R^3$

The volume of the moon = 1/64 × Volume of the earth

Hence the volume of the moon is 1/64 times the volume of the earth.

5. How many litres of milk can a hemispherical bowl of diameter 10.5cm hold? (Assume π = 22/7).

Answer: The quantity of milk that the hemispherical bowl can hold is nothing but the volume of the hemispherical bowl.

The volume of a hemisphere of base radius, r = $2/3\pi\ r^3$

Diameter of the hemispherical ball, d = 10.5 cm

Radius of the hemispherical ball, r = 10.5/2 cm = 5.25 cm

Volume of the hemispherical ball = $2/3\pi\ r^3$

= 2/3 × 22/7 × 5.25 cm × 5.25 cm × 5.25 cm

= 303.1875 cm^3

= 303.1875 /1000 ($1000cm^3 = 1L$)

= 0.3031875 litres

= 0.303 litres (approx.)

The hemispherical bowl can hold 0.303 litres of milk.

6. A hemispherical tank is made up of an iron sheet 1cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank. (Assume π = 22/7).

Answer: Since the hemispherical tank is made of 1 cm thick iron, we can find the outer radius of the tank by adding thickness to the inner radius.

The Volume of a hemisphere of base radius, r = $2/3\pi r^3$

The inner radius of the tank, r = 1m

Thickness of iron = 1cm = 1/100 m = 0.01 m

Outer radius of the tank, R = 1 m + 0.01m = 1.01m

The volume of the iron used to make the tank can be calculated by subtracting the volume of the tank with inner radius from the volume of the tank with outer radius.

Volume of the iron used to make the tank = $2/3\pi R^3 - 2/3\pi r^3$

= $2/3\pi (R^3 - r^3)$

= $2/3 \times 22/7 \times [(1.01m)^3 - (1m)^3]$

= $2/3 \times 22/7 \times [1.030301\ m^3 - 1\ m^3]$

= $2/3 \times 22/7 \times 0.030301\ m^3$

= $0.06348\ m^3$ (approx.)

$0.06348\ m^3$ of iron used to make the tank.

7. Find the volume of a sphere whose surface area is 154 cm^2. (Assume π = 22/7).

Answer: Since the surface area of the sphere is given, we can obtain the radius easily using the formula of the surface area of the sphere.

Let the radius of the sphere be r.

Surface area of the sphere = $4\pi r^2$

Volume of a sphere = $4/3\pi r^3$

Now, Surface area of the sphere = $4\pi r^2 = 154cm^2$

$r^2 = 154cm^2 / 4\pi$

$r^2 = (154\ cm^2) \div (4 \times 22/7)$

r = $\sqrt{49/4}$ cm²

r = 7/2 cm

Now, radius of the sphere = 7/2 cm

Therefore, volume of the sphere = $4/3\pi r^3$

= 4/3 × 22/7 × 7/2 cm × 7/2 cm × 7/2 cm

= 539/3 cm^3

Therefore, volume of the sphere is 539/3 cm^3.

8. A dome of a building is in the form of a hemisphere. From inside, it was whitewashed at the cost of Rs. 4989.60. If the cost of white-washing is 20 per square metre, find the

(i) inside surface area of the dome (ii) volume of the air inside the dome

(Assume π = 22/7)

Answer: Surface area of a hemisphere = $2\pi r^2$

Volume of a hemisphere = $2/3\pi r^3$

(i) Inside surface area of the dome = Total cost for whitewashing the dome inside / Rate of whitewashing

⇒ 4989.60/20 = 249.48 m^2

(ii) Let 'r' be the radius of a hemispherical dome.

Inner surface area of the hemispherical dome = $2\pi r^2$

$2\pi r^2 = 249.48\ m^2$

⇒ $r^2 = 249.48/2\pi\ m^2$

⇒ $r^2 = 249.48 \div (2 \times 22/7)$

⇒ $r^2 = 39.69$

⇒ r = √39.69

⇒ r = 6.3 m

The volume of the air inside the dome will be the same as the volume of the hemisphere.

Now the volume of the air inside the dome = $2/3\pi r^3$

= 2/3 × 22/7 × 6.3 m × 6.3 m × 6.3 m

= 523.9 m^3 (approx.)

Therefore, the inner surface area of the dome is 249.48 m2 and he volume of the air inside the dome is 523.9 m^3.

9. Twenty-seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S'. Find the

(i) radius r' of the new sphere,

(ii) ratio of Sand S'.

Answer: Since 27 solid iron spheres are melted to form a single solid sphere, the volume of the newly formed sphere will be equal to the volume of 27 solid iron spheres together.

The surface area of a sphere = $4\pi r^2$

The volume of a sphere = $4/3\pi r^3$

Therefore, The volume of 27 solid spheres with radius r = 27 × [$4/3\pi r^3$] = $36\pi r^3$ ---------------- (1)

Also, the volume of the new sphere with radius r' = $4/3\pi r'^3$----------------- (2)

(i) Volume of the new sphere = Volume of 27 solid spheres

$(4/3)\pi r'^3 = 36\pi r^3$ [From equation (1) and (2)]

$\Rightarrow r'^3 = 36\pi r^3 \times 3/4\pi$

$\Rightarrow r'^3 = 27r^3$

$\Rightarrow r' = \sqrt[3]{27r^3}$

r' = 3r

Radius of the new sphere, r' = 3r

(ii) Ratio of S and S'

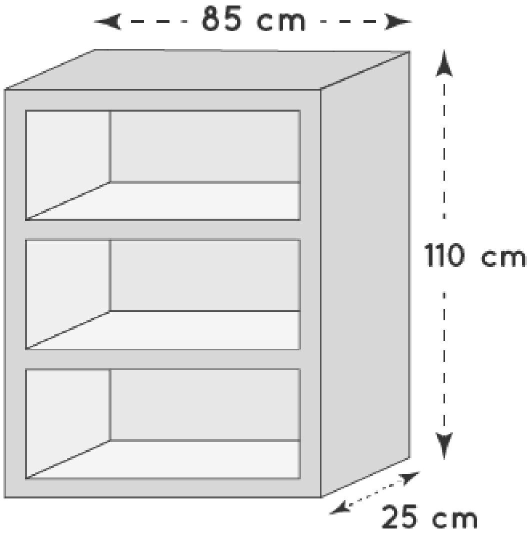
Now, surface area of each iron sphere, S = $4\pi r^2$

Surface area of the new sphere, S' = $4\pi r'^2 = 4\pi(3r)^2 = 36\pi r^2$

The ratio of the S and S' = $4\pi r^2/36\pi r^2 = 1/9$

Hence, the radius of new sphere is 3r and the ratio of S and S' is 1:9.

Exercise 13.9



1. A wooden bookshelf has external dimensions as follows: Height = 110cm, Depth = 25cm,

Breadth = 85cm (see fig. 13.31). The thickness of the plank is 5cm everywhere. The external faces are to be polished, and the inner faces are to be painted. If the rate of polishing is 20 paise per cm^2 and the rate of painting is 10 paise per cm^2, find the total expenses required for polishing and painting the surface of the bookshelf.

Answer: Since the bookshelf is cuboidal in shape and opened at the front with three shelves of the same dimensions, the area to be polished will be 5 surfaces of cuboidal bookshelf and the front border with plank's thickness.

The area to be painted will be 3 shelves of the bookshelf with internal dimensions and the area of each shelf will be 5 surfaces of the cuboidal shelf.

We can calculate the total cost of polishing and painting by multiplying the rate and their respective area.

The Volume of a cuboid of length l, breadth b, and height h, is = l × b × h

External measures of the bookshelf is,

Breadth, B = 85 cm

Depth, D = 25 cm

Height, H = 110 cm

The thickness of the plank, t = 5 cm

Internal measures of the bookshelf is,

The breadth of each shelf, b = B - 2t

⇒ b = 85 cm - 2 × 5 cm = 75 cm

Depth of each shelf, d = D - t

⇒ d = 25 cm - 5 cm = 20 cm

Height of each shelf, h = H - 4t

= (110 cm - 4 × 5 cm) ÷ 3

= 90 cm / 3 = 30 cm

Now, Surface area to be polished = External 5 surfaces of the bookshelf + border of the shelf

= 2(B + H) D + BH + 2Ht + 4bt

= [2 × (85 cm + 110 cm) × 25 cm] + (85 cm × 110 cm) + (2 × 110 cm × 5 cm) + (4 × 75 cm × 5 cm)

= 9750 cm^2 + 9350 cm^2 + 1100 cm^2 + 1500 cm^2

= 21700 cm^2

Cost of polishing at the rate of 20 paise per cm^2 = 21700 cm^2 × (₹ 20/100) / cm^2 = ₹ 4340

Surface area to be painted = Internal 5 surfaces of 3 shelves

= 3 [2(b + h)d + bh]

= 3 [2 × (75 cm + 30 cm) × 20 cm + (75 cm × 30 cm)]

= 3 [4200cm^2 + 2250 cm^2]

= 3 × 6450 cm^2

= 19350 cm^2

Cost of painting at the rate of 10 paise per cm^2 = 19350 cm^2 × (₹ 10/100) / cm^2 = ₹ 1935

Total expense required for polishing and painting = ₹ 4340 + ₹ 1935 = ₹ 6275

Thus, the total expense required for polishing and painting the surface of the bookshelf is ₹ 6275.

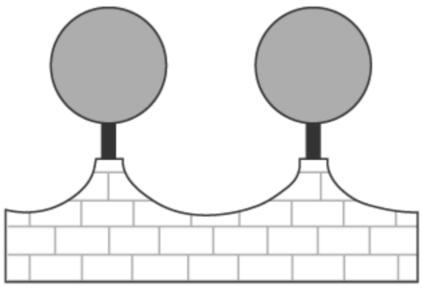


Fig 13.32

2. The front compound wall of a house is decorated by wooden spheres of diameter 21 cm, placed on small supports as shown in fig. 13.32. Eight such spheres are used forth is the purpose and are to be painted silver. Each support is a cylinder of radius 1.5cm and height 7cm and is to be painted black. Find the cost of paint required if silver paint costs 25 paise per cm^2, and black paint costs 5 paise per cm^2.

Answer: Since each sphere is placed on the cylinder, the area which is to be painted silver will be calculated by subtracting the top circular area of the cylinder from the surface area of the sphere.

The area of the cylinder which is to be painted black is the CSA of the cylinder.

The surface area of a sphere = $4\pi r^2$

CSA of the cylinder = $2\pi rh$

Diameter of the wooden sphere = 21cm

The radius (R) of the wooden sphere = 21/2 cm

Surface area for wooden sphere = $4\pi R^2$

= 4 × 22/7 × 21/2 cm × 21/2 cm

= 1386 cm^2

Since the support is a cylinder of radius, r = 1.5 cm

Area of the circular end of the cylinder = πr^2

= 22/7 × 1.5 cm × 1.5 cm

= 7.07 cm^2

So, the area of each wooden sphere to be painted = 1386 cm^2 - 7.07 cm^2 = 1378.93 cm^2

Total area of the 8 spheres to be painted = 8 × 1378.93 cm^2 = 11031.44 cm^2

Cost of silver painting the wooden spheres at the rate of 25 paise per cm^2

= 11031.44 × ₹ (25/100)

= ₹ 2757.86

Now,

Radius of the cylinder, r = 1.5 cm

Height of the cylinder, h = 7 cm

Curve surface area of the cylinder = $2\pi rh$

= 2 × 22/7 × 1.5 cm × 7 cm

= 66 cm^2

CSA of 8 cylindrical support to be painted = 8 × 66 cm^2 = 528 cm^2

Cost of black painting the cylindrical support at 5 paise per cm^2

= 528 × ₹ (5/100)

= ₹ 26.40

Hence the cost of paint required = ₹ 2757.86 + ₹ 26.40

= ₹ 2784.26

Thus, the cost of paint required is ₹ 2784.26 (approx.)

3. The diameter of a sphere is decreased by 25%. By what per cent does its curved surface area decrease?

Answer: Given: diameter of a sphere is decreased by 25%.

We have to find the percentage by which its Curved Surface Area decreases.

Let the radius of the sphere be r.

Then its diameter is 2r.

The Surface area of a sphere = $4\pi r^2$

The Curved surface area of the sphere = $4\pi r^2$

Now it is given in the question that the diameter of the sphere is decreased by 25% hence a new sphere is formed.

Therefore, the diameter of the new sphere can be written as:

= 2r - (25%) of (2r)

= 2r - (25/100) × (2r)

= 2r - (r/2)

= 3r/2

Radius of the new sphere = 1/2 × 3r/2 = 3r/4

Hence, curved surface area of the new sphere = $4\pi (3r/4)^2$

= $4\pi (9r^2/16)$

= $(9\pi r^2)/4$

Now, decrease in the original curved surface area = $4\pi r - (9\pi r^2)/4$

= $(16\pi r^2 - 9\pi r^2)/4$

= $(7\pi r^2)/4$

So, the percentage decrease in the curved surface area is,

= $[(7\pi r^2)/4 \times 1/(4\pi r^2)] \times 100\%$

= $[7/16] \times 100\%$

= 43.75%