Exercise 9.1

1. Identify the terms and their coefficients for each of the following expressions.
(i) $5xyz^2 - 3zy$ (ii) $1 + x + x^2$ (iii) $4x^2y^2 - 4x^2y^2z^2 + z^2$
(iv) $3 - pq + qr - rp$ (v) $x/2 + y/2 - xy$
(vi) $0.3a - 0.6ab + 0.5b$

Answer:

The numerical factor of a term is called its numerical coefficient or simply coefficient.

The terms and the respective coefficients of the given expressions are as follows.

-	Terms	Coefficients
(i)	$5xyz^2$ $-3zy$	5 -3
(ii)	1 x x^2	1 1 1
(iii)	$4x^2y^2$ $-4x^2y^2z^2$ z^2	4 -4 1
(iv)	3 -pq qr -rp	3 -1 1 -1
(v)	x/2 y/2 -xy	1/2 1/2 -1
(vi)	0.3a -0.6ab 0.5b	0.3 -0.6 0.5

2. Classify the following polynomials as monomials, binomials, and trinomials. Which polynomials do not fit in any of these three categories?
$x + y$, 1000, $x + x^2 + x^3 + x^4$, $7 + y + 5x$, $2y - 3y^2$, $2y - 3y^2 + 4y^3$, $5x - 4y + 3xy$, $4z - 15z^2$, $ab + bc + cd + da$, pqr, $p^2q + pq^2$, $2p + 2q$

Answer: An expression containing, one or more terms with a non-zero coefficient (with variables having non-negative integers as exponents) is called a polynomial. A polynomial may contain any number of terms, one or more than one.

- An expression that contains only one term in the most simplified form is called a monomial
- An expression that contains two terms in the most simplified form is called a binomial
- An expression containing three terms in the most simplified form is a trinomial and so on.

The given expressions are classified as follows,

Monomials: 1000, pqr

Binomials: $x + y$, $2y - 3y^2$, $4z - 15z^2$, $p^2q + pq^2$, $2p + 2q$

Trinomials: $7 + y + 5x$, $2y - 3y^2 + 4y^3$, $5x - 4y + 3xy$

Polynomials that do not fit in any of these categories are: $x + x^2 + x^3 + x^4$, $ab + bc + cd + da$

3. Add the following
(i) $ab - bc$, $bc - ca$, $ca - ab$
(ii) $a - b + ab$, $b - c + bc$, $c - a + ac$
(iii) $2 p^2q^2 - 3pq + 4$, $5 + 7pq - 3 p^2q^2$
(iv) $l^2 + m^2$, $m^2 + n^2$, $n^2 + l^2$, $2lm + 2mn + 2nl$

Answer: The given algebraic expressions are written by rearranging and combining like terms followed by performing the addition operation as follows:

(i) $ab - bc$, $bc - ca$, $ca – ab$

$= ab – bc + bc – ca + ca – ab$

$= ab – ab – bc + bc – ca + ca$

$= 0$

Thus, the sum of the given expressions is 0.

(ii) $(a – b + ab) + (b – c + bc) + (c – a + ac)$

➔ $a – b + ab + b – c + bc + c – a + ac$

$= a – a + b – b + c – c + ab + bc + ac$

$= 0 + 0 + 0 + ab + bc + ac$

= ab + bc + ca

Thus, the sum of the given expressions is ab + bc + ac.

(iii) 2 p^2q^2 - 3pq + 4, 5 + 7pq - 3 p^2q^2

➔ $2p^2q^2 – 3pq + 4 + 5 + 7pq – 3p^2q^2$

= $(2p^2q^2 – 3pq + 4) + (5 + 7pq – 3p^2q^2)$

= $2p^2q^2 – 3p^2q^2 – 3pq + 7pq + 4 + 5$

= $– p^2q^2 + 4pq + 9$

Thus, the sum of the given expressions is - p^2q^2 + 4pq + 9.

(iv) $l^2 + m^2$, $m^2 + n^2$, $n^2 + l^2$, 2lm + 2mn + 2nl

➔ $(l^2 + m^2) + (m^2 + n^2) + (n^2 + l^2) + (2lm + 2mn + 2nl)$

= $l^2 + l^2 + m^2 + m^2 + n^2 + n^2 + 2lm + 2mn + 2nl$

= $2l^2 + 2m^2 + 2n^2 + 2lm + 2mn + 2nl$

= $2(l^2 + m^2 + n^2 + lm + mn + nl)$

Thus, the sum of the given expressions is $2(l^2 + m^2 + n^2 + lm + mn + nl)$

4. (a) Subtract 4a - 7ab + 3b + 12 from 12a - 9ab + 5b – 3

(b) Subtract 3xy + 5yz - 7zx from 5xy - 2yz - 2zx +10xyz

(c) Subtract $4p^2q$ - 3pq + $5pq^2$ - 8p + 7q - 10 from 18 - 3p - 11q + 5 pq - $2pq^2$ + $5p^2q$

Answer: The given algebraic expressions are written by rearranging and combining like terms followed by performing the subtraction operation as follows:

(a) Subtract 4a - 7ab + 3b + 12 from 12a - 9ab + 5b – 3

➔ (12a – 9ab + 5b – 3) – (4a – 7ab + 3b + 12)

= 12a – 9ab + 5b – 3 – 4a + 7ab – 3b – 12

= 12a – 4a - 9ab + 7ab + 5b – 3b - 3 - 12

= 8a – 2ab + 2b – 15

(b) Subtract 3xy + 5yz - 7zx from 5xy - 2yz - 2zx +10xyz

➔ (5xy – 2yz – 2zx + 10xyz) – (3xy + 5yz – 7zx)

= 5xy – 2yz – 2zx + 10xyz – 3xy – 5yz + 7zx

= 5xy – 3xy – 2yz – 5yz – 2zx + 7zx + 10xyz

$= 2xy – 7yz + 5zx + 10xyz$

(c) Subtract $4p^2q - 3pq + 5pq^2 - 8p + 7q - 10$ from $18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q$

➔ $(18 – 3p – 11q + 5pq – 2pq^2 + 5p^2q) – (4p^2q – 3pq + 5pq^2 – 8p + 7q – 10)$

$= 18 – 3p – 11q + 5pq – 2pq^2 + 5p^2q – 4p^2q + 3pq – 5pq^2 + 8p – 7q + 10$

$= 18 + 10 - 3p + 8p - 11q – 7q + 5pq + 3pq - 2pq^2 – 5pq^2 + 5p^2q – 4p^2q$

$= 28 + 5p – 18q + 8pq – 7pq^2 + p^2q$

Exercise 9.2

1. Find the product of the following pairs of monomials

(i) 4,7p (ii) -4p, 7p (iii) -4p, 7pq (iv) $4p^3$, -3p (v) 4p, 0

Answer: Multiplication of two algebraic expressions or variable expressions involves multiplying two expressions that are combined with arithmetic operations such as addition, subtraction, multiplication, and division, and contain constants, variables, terms, and coefficients.

The product will be as follows.

(i) $4 \times 7 p = 4 \times 7 \times p = 28 p$

(ii) $- 4 p \times 7 p = - 4 \times p \times 7 \times p = (- 4 \times 7) \times (p \times p) = - 28 p^2$

(iii) $- 4 p \times 7 pq = - 4 \times p \times 7 \times p \times q = (- 4 \times 7) \times (p \times p \times q) = - 28 p^2q$

(iv) $4 p^3 \times (-3 p) = 4 \times (- 3) \times p \times p \times p \times p = -12 p^4$

(v) $4 p \times 0 = 4 \times p \times 0 = 0$

2. Find the areas of rectangles with the following pairs of monomials as their lengths and breadths respectively
(p, q); (10m, 5n); ($20x^2$, $5y^2$); (4x, $3x^2$); (3mn, 4np)

Answer: Given: Lengths and breadths of rectangles

We know that,

Area of a rectangle = length × breadth

Area of rectangle = Length × Breadth

Area of 1st rectangle = $p \times q = pq$

Area of 2nd rectangle = $10m \times 5n = 10 \times 5 \times m \times n = 50 mn$

Area of 3rd rectangle = $20x^2 \times 5y^2 = 20 \times 5 \times x^2 \times y^2 = 100 x^2 y^2$

Area of 4th rectangle = $4x \times 3x^2 = 4 \times 3 \times x \times x^2 = 12x^3$

Area of 5th rectangle = $3mn \times 4np = 3 \times 4 \times m \times n \times n \times p = 12mn^2 p$

3. Complete the table of products

First monomial → / ↓ Second monomial	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y$
$2x$	$4x^2$	—	—	—	—	—
$-5y$	—	—	$-15x^2y$	—	—	—
$3x^2$	—	—	—	—	—	—
$-4xy$	—	—	—	—	—	—
$7x^2y$	—	—	—	—	—	—
$-9x^2y^2$	—	—	—	—	—	—

Answer: Multiplication of two algebraic expressions or variable expressions involves multiplying two expressions that are combined with arithmetic operations such as addition, subtraction, multiplication, and division, and contain constants, variables, terms, and coefficients.

In the given table, multiply the first monomial given horizontally with the second monomial vertically to get the result.

First monomial → / Second monomial ↓	2x	-5 y	$3x^2$	-4xy	$7x^2$ y	$-9x^2$ y^2
2x	$4x^2$	-10xy	$6x^3$	$-8x^2$ y	$14x^3$ y	$-18x^3$ y^2
-5 y	-10xy	25 y^2	$-15x^2$ y	$20xy^2$	$-35x^2$ y^2	$45x^2$ y^3
$3x^2$	$6x^3$	$-15x^2$ y	$9x^4$	$-12x^3$ y	$21x^4$ y	$-27x^4$ y^2
-4xy	$-8x^2$ y	$20xy^2$	$-12x^3$ y	$16x^2$ y^2	$-28x^3$ y^2	$36x^3$ y^3
$7x^2$ y	$14x^3$ y	$-35x^2$ y^2	$21x^4$ y	$-28x^3$ y^2	$49x^4$ y^2	$-63x^4$ y^3
$-9x^2$ y^2	$-18x^3$ y^2	$45x^2$ y^3	$-27x^4$ y^2	$36x^3$ y^3	$-63x^4$ y^3	$81x^4$ y^4

4. Obtain the volume of rectangular boxes with the following length, breadth, and height respectively

(i) $5a, 3a^2, 7a^4$ (ii) 2 p, 4q, 8r (iii) xy, $2x^2$ y, $2xy^2$ (iv) a, 2b, 3c

Answer: Given: Length, breadth and height of rectangular boxes

We know that, Volume of a rectangular box = length × breadth × height

(i) Volume = $5a \times 3a^2 \times 7a^4 = 5 \times 3 \times 7 \times a \times a^2 \times a^4 = 105\ a^7$

(ii) Volume = $2\ p \times 4q \times 8r = 2 \times 4 \times 8 \times p \times q \times r = 64\ pqr$

(iii) Volume = $xy \times 2x^2y \times 2xy^2 = 2 \times 2 \times xy \times x^2y \times xy^2 = 4x^4y^4$

(iv) Volume = $a \times 2b \times 3c = 2 \times 3 \times a \times b \times c = 6abc$

5. Obtain the product of
(i) xy, yz, zx
(ii) $a, -a^2, a^3$
(iii) $2, 4y, 8y^2, 16y^3$ (iv) a, 2b, 3c, 6abc (v) m, - mn, mnp

Answer: Multiplication of two algebraic expressions or variable expressions involves multiplying two expressions that are combined with arithmetic operations such as addition, subtraction, multiplication, and division, and contain constants, variables, terms, and coefficients.

(i) $xy \times yz \times zx = x^2y^2z^2$

(ii) $a \times (-a^2) \times a^3 = -a^6$

(iii) $2 \times 4y \times 8y^2 \times 16y^3 = 2 \times 4 \times 8 \times 16 \times y \times y^2 \times y^3 = 1024y^6$

(iv) $a \times 2b \times 3c \times 6abc = 2 \times 3 \times 6 \times a \times b \times c \times abc = 36a^2b^2c^2$

(v) $m \times (-mn) \times mnp = -m^3n^2p$

Exercise 9.3

1. Carry out the multiplication of the expressions in each of the following pairs
(i) 4p, q + r (ii) ab, a - b (iii) $a + b, 7a^2b^2$
(iv) $a^2 - 9, 4a$ (v) pq + qr + rp, 0

Answer: Multiplication of two algebraic expressions or variable expressions involves multiplying two expressions that are combined with arithmetic operations such as addition, subtraction, multiplication, and division, and contain constants, variables, terms, and coefficients.

(i) $(4p) \times (q + r) = (4p \times q) + (4p \times r) = 4pq + 4pr$

(ii) $(ab) \times (a - b) = (ab \times a) + [ab \times (-b)] = a^2b - ab^2$

(iii) $(a + b) \times (7a^2b^2) = (a \times 7a^2b^2) + (b \times 7a^2b^2) = 7a^3b^2 + 7a^2b^3$

(iv) $(a^2 - 9) \times (4a) = (a^2 \times 4a) + [(-9) \times (4a)] = 4a^3 - 36a$

(v) $(pq + qr + rp) \times 0 = (pq \times 0) + (qr \times 0) + (rp \times 0) = 0$

2. Complete the table

	First expression	Second expression	Product
(i)	a	b + c + d	-
(ii)	x + y -5	5xy	-
(iii)	p	$6p^2 - 7p + 5$	-
(iv)	$4p^2q^2$	$p^2 - q^2$	-
(v)	a + b + c	abc	-

Answer:

(i) a (b + c + d) = ab + ac + ad

(ii) (x + y - 5) (5xy) = $5x^2y + 5xy^2 - 25xy$

(iii) $p(6p^2 - 7p + 5) = 6p^3 - 7p^2 + 5p$

(iv) $4p^2q^2(p^2 - q^2) = 4p^4q^2 - 4p^2q^4$

(v) (a + b + c) (abc) = $a^2bc + ab^2c + abc^2$

	First expression	Second expression	Product
(i)	a	b + c + d	ab + ac + ad
(ii)	x + y -5	5xy	$5x^2y + 5xy^2 - 25xy$
(iii)	p	$6p^2 - 7p + 5$	$6p^3 - 7p^2 + 5p$
(iv)	$4p^2q^2$	$p^2 - q^2$	$4p^4q^2 - 4p^2q^4$
(v)	a + b + c	abc	$a^2bc + ab^2c + abc^2$

3. Find the product

(i) $(a^2) \times (2a^{22}) \times (4a^{26})$ **(ii) $(2/3\ xy) \times (-9/10\ x^2 y^2)$**

(iii) $(-10/3\ pq^3) \times (6/5\ p^3q)$ **(iv) $x \times x^2 \times x^3 \times x^4$**

Answer:

.(i) $(a^2) \times (2a^{22}) \times (4a^{26}) = 2 \times 4 \times a^2 \times a^{22} \times a^{26} = 8a^{50}$

[According to exponent rules, $a^m \times a^n = a^{m+n}$]

(ii) $\frac{2}{3} xy \times (\frac{-9}{10} x^2 y^2) = \frac{2}{3} \times (\frac{-9}{10}) \times x \times y \times x^2 \times y^2 = \frac{-3}{5} x^3 y^3$

[Since, $a^m \times a^n = a^{m+n}$]

(iii) $\left(\frac{-10}{3}pq^3\right) \times \left(\frac{6}{5}p^3q\right) = \frac{-10}{3} \times \frac{6}{5} \times p \times q^3 \times p^3 \times q = -4\,p^4q^4$

[Since, $a^m \times a^n = a^{m+n}$]

(iv) $x \times x^2 \times x^3 \times x^4 = x^{10}$

[Since, $a^m \times a^n = a^{m+n}$]

4(a) Simplify 3x(4x - 5) + 3 and find its values for (i) x = 3 (ii) x = 1/2

(b) Simplify a (a^2 + a + 1)+ 5 and find its values for (i) a = 0, (ii) a =1, (iii) a = - 1

Answer:

(a) $3x(4x - 5) + 3$

$= 12x^2 - 15x + 3$

(i) For x = 3,

$= 12x^2 - 15x + 3$

$= 12(3)^2 - 15(3) + 3$

$= 108 - 45 + 3$

$= 66$

(ii) For x = 1/2

$= 12x^2 - 15x + 3$

$= 12\left(\frac{1}{2}\right)^2 - 15\left(\frac{1}{2}\right) + 3$

$= 3 - \frac{15}{2} + 3$

$= \frac{-3}{2}$

(b) $a(a^2 + a + 1) + 5 = a^3 + a^2 + a + 5$

(i) For a = 0,

➔ $a^3 + a^2 + a + 5$

$= 0 + 0 + 0 + 5 = 5$

(ii) For a = 1,

$a^3 + a^2 + a + 5$

$= (1)^3 + (1)^2 + 1 + 5$

$= 1 + 1 + 1 + 5 = 8$

(iii) For a = -1,

$a^3 + a^2 + a + 5$

$= (-1)^3 + (-1)^2 + (-1) + 5$

$= - 1 + 1 -1 + 5 = 4$

5. (a) Add: p(p - q), q(q - r) and r (r - p)

(b) Add: 2x (z - x - y) and 2y(z- y - x)

(c) Subtract: 3l (l - 4m + 5n) from 4l (10n - 3m + 2l)

(d) Subtract: 3a(a + b + c) - 2b(a - b + c) from 4c (-a + b + c)

Answer: We know that addition and subtraction take place between like terms.

(a) First expression = $p (p - q) = p^2 - pq$

Second expression = $q (q - r) = q^2 - qr$

Third expression = $r (r - p) = r^2 - pr$

Adding these expressions, we get

$$\begin{array}{r} p^2 - pq \qquad\qquad \\ + \; q^2 - qr \qquad \\ + \; r^2 - pr \\ \hline p^2 - pq + q^2 - qr + r^2 - pq \end{array}$$

Therefore, the sum of the given expressions is $p^2 + q^2 + r^2 - pq - qr - rp$.

Alternative method:

$p (p – q) + q (q – r) + r (r – p)$

$= (p^2 – pq) + (q^2 – qr) + (r^2 – pr)$

$= p^2 + q^2 + r^2 – pq – qr – pr$

(b) First expression = $2x (z - x - y) = 2xz - 2x^2 - 2xy$

Second expression = $2 y (z - y - x) = 2 yz - 2 y^2 - 2 yx$

Adding these expressions, we get

$$
\begin{array}{r}
2xz - 2x^2 - 2xy \\
+ \qquad - 2xy + 2 yz – 2y^2 \\
\hline
2xz – 2x^2 – 4xy + 2yz – 2y^2
\end{array}
$$

Alternative method:

$2x (z – x – y) + 2y (z – y – x)$

$= (2xz – 2x^2 – 2xy) + (2yz – 2y^2 – 2xy)$

$= 2xz – 4xy + 2yz – 2x^2 – 2y^2$

Therefore, the sum of the given expressions is $-2x^2 - 2 y^2 - 4xy + 2 yz + 2zx$.

(c) First expression = $3l (l - 4m + 5n) = 3l^2 -12lm + 15ln$

Second expression = $4l (10n - 3m + 2l) = 40ln -12lm + 8l^2$

Subtracting these expressions, we get

$$
\begin{array}{l}
\quad 40ln -12lm + 8l^2 \\
- \; 15ln -12lm + 3l^2 \\
(-) \quad (+) \quad (-) \\
\hline
+ 25ln \qquad 5l^2
\end{array}
$$

Alternative method:

$4l (10 n – 3 m + 2 l) – 3l (l – 4 m + 5 n)$

$= (40ln – 12lm + 8l^2) – (3l^2 – 12lm + 15ln)$

$= 40ln – 12lm + 8l^2 – 3l^2 +12lm -15 ln$

$= 25 ln + 5l^2$

(d) First expression = $3a (a + b + c) - 2b (a - b + c)$

$= 3a^2 + 3ab + 3ac - 2ba + 2b^2 - 2bc$

$= 3a^2 + 2b^2 + ab + 3ac - 2bc$

Second expression = $4c (- a + b + c) = -4ac + 4bc + 4c^2$

Subtracting these expressions, we obtain

$$\begin{array}{lllll}
-4ac + 4bc + 4c^2 & & & & \\
\;\;3ac - 2bc & + 3a^2 & + 2b^2 & + ab & \\
(-)\quad(+) & (-) & (-) & (-) & \\
\hline
-7ac + 6bc + 4c^2 & - 3a^2 & - 2b^2 & - ab &
\end{array}$$

Alternative method:

d) $4c (– a + b + c) – (3a (a + b + c) – 2 b (a – b + c))$

$= (-4ac + 4bc + 4c^2) – (3a^2 + 3ab + 3ac – (2ab – 2b^2 + 2bc))$

$=-4ac + 4bc + 4c^2 – (3a^2 + 3ab + 3ac – 2ab + 2b^2 – 2bc)$

$= -4ac + 4bc + 4c^2 – 3a^2 – 3ab – 3ac +2ab – 2b^2 + 2bc$

$= -7ac + 6bc + 4c^2 – 3a^2 – ab – 2b^2$

Exercise 9.4

1. Multiply the binomials

(i) $(2x + 5)$ and $(4x - 3)$ (ii) $(y - 8)$ and $(3y - 4)$

(iii) $(2.5l - 0.5m)$ and $(2.5l + 0.5m)$

(iv) $(a + 3b)$ and $(x + 5)$ (v) $(2pq + 3q^2)$ and $(3pq - 2q^2)$

(vi) $(3a^2/4 + 3b^2)$ and $4[a^2 - (2b^2/3)]$

Answer:

(i) $(2x + 5)$ and $(4x - 3)$

$(2x + 5) \times (4x - 3)$

$= 2x \times (4x - 3) + 5 \times (4x - 3)$

$= 8x^2 - 6x + 20x - 15$

$= 8x^2 + 14x - 15$ (By adding like terms)

(ii) (y - 8) and (3y - 4)

$(y - 8) \times (3y - 4) = y \times (3y - 4) - 8 \times (3y - 4)$

$= 3y^2 - 4y - 24y + 32$

$= 3y^2 - 28y + 32$ (By adding like terms)

(iii) (2.5l - 0.5m) and (2.5l + 0.5m)

$(2.5l - 0.5m) \times (2.5l + 0.5m)$

$= 2.5l \times (2.5l + 0.5m) - 0.5m\,(2.5l + 0.5m)$

$= 6.25l^2 + 1.25lm - 1.25lm - 0.25m^2$

$= 6.25l^2 - 0.25m^2$

(iv) (a + 3b) and (x + 5)

$(a + 3b) \times (x + 5)$

$= a \times (x + 5) + 3b \times (x + 5)$

$= ax + 5a + 3bx + 15b$

(v) $(2pq + 3q^2)$ and $(3pq - 2q^2)$

$(2pq + 3q^2) \times (3pq - 2q^2)$

$= 2pq \times (3pq - 2q^2) + 3q^2 \times (3pq - 2q^2)$

$= 6p^2q^2 - 4pq^3 + 9pq^3 - 6q^4$

$= 6p^2q^2 + 5pq^3 - 6q^4$

(vi) $(3a^2/4 + 3b^2)$ and $4[a^2 - (2b^2/3)]$

$\Rightarrow \frac{3a^2}{4} + 3b^2 \ \text{x} \ 4\left[a^2 - \left(\frac{2b^2}{3}\right)\right]$

$\Rightarrow \frac{3a^2}{4} + 3b^2 \ \text{x} \ \left[4a^2 - \left(\frac{8b^2}{3}\right)\right]$

$\Rightarrow \frac{3a^2}{4}\left[4a^2 - \left(\frac{8b^2}{3}\right)\right] + 3b^2\left[4a^2 - \left(\frac{8b^2}{3}\right)\right]$

$\Rightarrow \frac{3a^2}{4} \text{ x } 4a^2 - \frac{3a^2}{4} \text{ x } \frac{8b^2}{3} + 3b^2 \text{ x } 4a^2 - 3b^2 \text{ x } \frac{8b^2}{3}$

$= 3a^4 - 2b^2a^2 + 12b^2a^2 - 8b^4$

$= 3a^4 + 10a^2b^2 - 8b^4$

2. Find the product

(i) $(5 - 2x)(3 + x)$

(ii) $(x + 7y)(7x - y)$

(iii) $(a^2 + b)(a + b^2)$

(iv) $(p^2 - q^2)(2p + q)$

Answer:

(i) $(5 - 2x)(3 + x)$

$(5 - 2x)(3 + x)$

$= 5(3 + x) - 2x(3 + x)$

$= 15 + 5x - 6x - 2x^2$

$= 15 - x - 2x^2$

(ii) $(x + 7y)(7x - y)$

$(x + 7y)(7x - y)$

$= x(7x - y) + 7y(7x - y)$

$= 7x^2 - xy + 49xy - 7y^2$

$= 7x^2 + 48xy - 7y^2$

(iii) $(a^2 + b)(a + b^2)$

$(a^2 + b)(a + b^2)$

$= a^2(a + b^2) + b(a + b^2)$

$= a^3 + a^2b^2 + ab + b^3$

(iv) $(p^2 - q^2)(2p + q)$

$(p^2 - q^2)(2p + q)$

$= p^2(2p + q) - q^2(2p + q)$

$= 2p^3 + p^2q - 2pq^2 - q^3$

3. Simplify

(i) $(x^2 - 5)(x + 5) + 25$

(ii) $(a^2 + 5)(b^3 + 3) + 5$

(iii) $(t + s^2)(t^2 - s)$

(iv) $(a + b)(c - d) + (a - b)(c + d) + 2(ac + bd)$

(v) $(x + y)(2x + y) + (x + 2y)(x - y)$

(vi) $(x + y)(x^2 - xy + y^2)$

(vii) $(1.5x - 4y)(1.5x + 4y + 3) - 4.5x + 12y$

(viii) $(a + b + c)(a + b - c)$

Answer:

(i) $(x^2 - 5)(x + 5) + 25$

$= x^2(x + 5) - 5(x + 5) + 25$

$= x^3 + 5x^2 - 5x - 25 + 25$

$= x^3 + 5x^2 - 5x$

(ii) $(a^2 + 5)(b^3 + 3) + 5$

$= a^2(b^3 + 3) + 5(b^3 + 3) + 5$

$= a^2b^3 + 3a^2 + 5b^3 + 15 + 5$

$= a^2b^3 + 3a^2 + 5b^3 + 20$

(iii) $(t + s^2)(t^2 - s)$

$= t(t^2 - s) + s^2(t^2 - s)$

$= t^3 - st + s^2t^2 - s^3$

(iv) $(a + b)(c - d) + (a - b)(c + d) + 2(ac + bd)$

$= a(c - d) + b(c - d) + a(c + d) - b(c + d) + 2(ac + bd)$

$= ac - ad + bc - bd + ac + ad - bc - bd + 2ac + 2bd$

$= (ac + ac + 2ac) + (ad - ad) + (bc - bc) + (2bd - bd - bd)$

$= 4ac$

(v) $(x + y)(2x + y) + (x + 2y)(x - y)$

$= x(2x + y) + y(2x + y) + x(x - y) + 2y(x - y)$

$= 2x^2 + xy + 2xy + y^2 + x^2 - xy + 2xy - 2y^2$

$= (2x^2 + x^2) + (y^2 - 2y^2) + (xy + 2xy - xy + 2xy)$

$= 3x^2 - y^2 + 4xy$

(vi) $(x + y)(x^2 - xy + y^2)$

$= x(x^2 - xy + y^2) + y(x^2 - xy + y^2)$

$= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3$

$= x^3 + y^3 + (xy^2 - xy^2) + (x^2y - x^2y)$

$= x^3 + y^3$

(vii) $(1.5x - 4y)(1.5x + 4y + 3) - 4.5x + 12y$

$= 1.5x(1.5x + 4y + 3) - 4y(1.5x + 4y + 3) - 4.5x + 12y$

$= 2.25x^2 + 6xy + 4.5x - 6xy - 16y^2 - 12y - 4.5x + 12y$

$= 2.25x^2 + (6xy - 6xy) + (4.5x - 4.5x) - 16y^2 + (12y - 12y)$

$= 2.25x^2 - 16y^2$

(viii) $(a + b + c)(a + b - c)$

$= a(a + b - c) + b(a + b - c) + c(a + b - c)$

$= a^2 + ab - ac + ab + b^2 - bc + ac + bc - c^2$

$= a^2 + b^2 - c^2 + (ab + ab) + (bc - bc) + (ac - ac)$

$= a^2 + b^2 - c^2 + 2ab$

Exercise 9.5

1. Use a suitable identity to get each of the following products

(i) $(x + 3)(x + 3)$

(ii) $(2y + 5)(2y + 5)$

(iii) $(2a - 7)(2a - 7)$

(iv) $(3a - (1/2))(3a - (1/2))$

(v) $(1.1 m - 0.4)(1.1 m + 0.4)$

(vi) $(a^2 + b^2)(-a^2 + b^2)$

(vii) $(6x - 7)(6x + 7)$

(viii) $(-a + c)(-a + c)$

(ix) $(x/2 + 3y/4)(x/2 + 3y/4)$

(x) $(7a - 9b)(7a - 9b)$

Answer:

(i) $(x + 3) (x + 3)$

$= (x + 3)^2$

$= (x)^2 + 2(x) (3) + (3)^2$ [Using the algebraic identity $(a + b)^2 = a^2 + 2ab + b^2$]

$= x^2 + 6x + 9$

(ii) $(2y + 5)(2y + 5)$

$= (2y + 5)^2$

$= (2y)^2 + 2(2y) (5) + (5)^2$ [Since, $(a + b)^2 = a^2 + 2ab + b^2$]

$= 4y^2 + 20y + 25$

(iii) $(2a - 7)(2a - 7)$

$= (2a - 7)^2$

$= (2a)^2 - 2(2a) (7) + (7)^2$ [Since, $(a - b)^2 = a^2 - 2ab + b^2$]

$= 4a^2 - 28a + 49$

(iv) (3a - (1/2)) (3a - (1/2))

$= \left(3a - \frac{1}{2}\right)^2$ [Since, $(a - b)^2 = a^2 - 2ab + b^2$]

$= (3a)^2 – 2 \times 3a \times \frac{1}{2} + \left(\frac{1}{2}\right)^2$

$= 9a^2 - 3a + \frac{1}{4}$

(v) (1.1 m - 0.4)(1.1 m + 0.4)

$= (1.1m)^2 - (0.4)^2$ [Since, $(a + b)(a - b) = a^2 - b^2$]

$= 1.21m^2 - 0.16$

(vi) ($a^2 + b^2$)($-a^2 + b^2$)

$= (b^2 + a^2)(b^2 - a^2)$

$= (b^2)^2 - (a^2)^2$ [Since, $(a + b)(a - b) = a^2 - b^2$]

$= b^4 - a^4$

(vii) (6x - 7)(6x + 7)

$= (6x)^2 - (7)^2$ [Since, $(a + b)(a - b) = a^2 - b^2$]

$= 36x^2 - 49$

(viii) (-a + c)(-a + c)

$= (-a + c)^2$

$= (-a)^2 + 2(-a) (c) + (c)^2$ [Since, $(a + b)^2 = a^2 + 2ab + b^2$]

$= a^2 - 2ac + c^2$

(ix) (x/2 + 3y/4)(x/2 + 3y/4)

$= \left(\frac{x}{2} + \frac{3y}{4}\right)^2$ [Since, $(a + b)^2 = a^2 + 2ab + b^2$]

$= \left(\frac{x}{2}\right)^2 + 2 \times \frac{x}{2} \times \frac{3y}{4} + \left(\frac{3y}{4}\right)^2$

$= \frac{x^2}{4} + \frac{3xy}{4} + \frac{9y^2}{16}$

(x) (7a - 9b) (7a - 9b)

$= (7a - 9b)^2$

$= (7a)^2 - 2(7a)(9b) + (9b)^2$ [Since, $(a - b)^2 = a^2 - 2ab + b^2$]

$= 49a^2 - 126ab + 81b^2$

2. Use the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$ to find the following products
(i) (x + 3)(x + 7)

(ii) (4x + 5)(4x + 1)

(iii) (4x - 5)(4x -1)

(iv) (4x + 5)(4x -1)

(v) (2x + 5y)(2x + 3y)

(vi) $(2a^2 + 9)(2a^2 + 5)$

(vii) (xyz - 4)(xyz - 2)

Answer: We will be using the algebraic identity $(x + a)(x + b) = x^2 + (a + b)x + ab$ to solve the given questions.

(i) (x + 3) (x + 7)

$= (x)^2 + (3 + 7)x + 3 \times 7$

$= x^2 + 10x + 21$

(ii) (4x + 5) (4x + 1)

$= (4x)^2 + (5 + 1)4x + 5 \times 1$

$= 16x^2 + 24x + 5$

(iii) (4x - 5) (4x - 1)

= $(4x)^2$ - (-5 - 1)4x + -5 x -1

= $16x^2$ - 24x + 5

(iv) (4x + 5)(4x -1)

= $(4x)^2$ + ((+5) + (- 1) 4x + 5 x -1

= $16x^2$ +16x – 5

(v) (2x + 5y) (2x + 3y)

= $(2x)^2$ + (5y + 3y) 2x + 5y x 3y

= $4x^2$ + 16 xy + $15y^2$

(vi) ($2a^2$ + 9) ($2a^2$ + 5)

= $(4a)^2$ + (9 + 5) $2a^2$ + 9 x 5

= $16a^2$ + $28a^2$ + 45

(vii) (xyz - 4) (xyz - 2)

= $(xyz)^2$ + ((-4) + (-2)) xyz + -4 x -2

= $x^2y^2z^2$ – 6xyz + 8

3. Find the following squares by using the identities

(i) $(b - 7)^2$

(ii) $(xy + 3z)^2$

(iii) $(6x^2 - 5y)^2$

(iv) $(2m/3 + 3n/2)^2$

(v) $(0.4p - 0.5q)^2$

(vi) $(2xy + 5y)^2$

Answer: We will be using the following algebraic identities

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

(i) $(b - 7)^2$

$= (b)^2 - 2(b)(7) + (7)^2$ [Since, $(a - b)^2 = a^2 - 2ab + b^2$]

$= b^2 - 14b + 49$

(ii) $(xy + 3z)^2$

$= (xy)^2 + 2(xy)(3z) + (3z)^2$ [Since, $(a + b)^2 = a^2 + 2ab + b^2$]

$= x^2y^2 + 6xyz + 9z^2$

(iii) $(6x^2 - 5y)^2$

$= (6x^2)^2 - 2(6x^2)(5y) + (5y)^2$ [Since, $(a - b)^2 = a^2 - 2ab + b^2$]

$= 36x^4 - 60x^2y + 25y^2$

(iv) $\left(\frac{2m}{3} + \frac{3n}{2}\right)^2$

$= \left(\frac{2m}{3}\right)^2 + 2\left(\frac{2m}{3}\right)\left(\frac{3n}{2}\right) + \left(\frac{3n}{2}\right)^2$ [Since, $(a + b)^2 = a^2 + 2ab + b^2$]

$= \frac{4m^2}{9} + \frac{6mn}{3} + \frac{9n^2}{4}$

$= \frac{4m^2}{9} + 2mn + \frac{9n^2}{4}$

(v) $(0.4p - 0.5q)^2$

$= (0.4p)^2 - 2(0.4p)(0.5q) + (0.5q)^2$ [Since, $(a - b)^2 = a^2 - 2ab + b^2$]

$= 0.16p^2 - 0.4pq + 0.25q^2$

(vi) $(2xy + 5y)^2$

$= (2xy)^2 + 2(2xy)(5y) + (5y)^2$ [Since, $(a + b)^2 = a^2 + 2ab + b^2$]

$= 4x^2y^2 + 20xy^2 + 25y^2$

4. Simplify

(i) $(a^2 - b^2)^2$

(ii) $(2x + 5)^2 - (2x - 5)^2$

(iii) $(7m - 8n)^2 + (7m + 8n)^2$

(iv) $(4m + 5n)^2 + (5m + 4n)^2$

(v) $(2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$

(vi) $(ab + bc)^2 - 2ab^2c$

(vii) $(m^2 - n^2m)^2 + 2m^3n^2$

Answer:

By using the following Algebraic identities

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

(i) $(a^2 - b^2)^2$

$= (a^2)^2 - 2(a^2)(b^2) + (b^2)^2$ [Since, $(a - b)^2 = a^2 - 2ab + b^2$]

$= a^4 - 2a^2b^2 + b^4$

(ii) $(2x + 5)^2 - (2x - 5)^2$

$= (2x)^2 + 2(2x)(5) + (5)^2 - [(2x)^2 - 2(2x)(5) + (5)^2]$

[Since, $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$]

$= 4x^2 + 20x + 25 - [4x^2 - 20x + 25]$

$= 4x^2 + 20x + 25 - 4x^2 + 20x - 25$

$= 40x$

(iii) $(7m - 8n)^2 + (7m + 8n)^2$

$= (7m)^2 - 2(7m)(8n) + (8n)^2 + (7m)^2 + 2(7m)(8n) + (8n)^2$

[Since, $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$]

$= 49m^2 - 112mn + 64n^2 + 49m^2 + 112mn + 64n^2$

$= 98m^2 + 128n^2$

(iv) $(4m + 5n)^2 + (5m + 4n)^2$

$= (4m)^2 + 2(4m)(5n) + (5n)^2 + (5m)^2 + 2(5m)(4n) + (4n)^2$ [Since, $(a + b)^2 = a^2 + 2ab + b^2$]

$= 16m^2 + 40mn + 25n^2 + 25m^2 + 40mn + 16n^2$

$= 41m^2 + 80mn + 41n^2$

(v) $(2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$

$= (2.5p)^2 - 2(2.5p)(1.5q) + (1.5q)^2 - [(1.5p)^2 - 2(1.5p)(2.5q) + (2.5q)^2]$

[Since, $(a - b)^2 = a^2 - 2ab + b^2$]

$= 6.25p^2 - 7.5pq + 2.25q^2 - [2.25p^2 - 7.5pq + 6.25q^2]$

$= 6.25p^2 - 7.5pq + 2.25q^2 - 2.25p^2 + 7.5pq - 6.25q^2$

$= 4p^2 - 4q^2$

(vi) $(ab + bc)^2 - 2ab^2c$

$= (ab)^2 + 2(ab)(bc) + (bc)^2 - 2ab^2c$ [Since, $(a + b)^2 = a^2 + 2ab + b^2$]

$= a^2b^2 + 2ab^2c + b^2c^2 - 2ab^2c$

$= a^2b^2 + b^2c^2$

(vii) $(m^2 - n^2m)^2 + 2m^3n^2$

$= (m^2)^2 - 2(m^2)(n^2m) + (n^2m)^2 + 2m^3n^2$ [Since, $(a - b)^2 = a^2 - 2ab + b^2$]

$= m^4 - 2m^3n^2 + n^4m^2 + 2m^3n^2$

$= m^4 + n^4m^2$

5. Show that

(i) $(3x + 7)^2 - 84x = (3x - 7)^2$

(ii) $(9p - 5q)^2 + 180pq = (9p + 5q)^2$

(iii) $(4m/3 - 3n/4)^2 + 2mn = 16m^2/9 + 9n^2/16$

(iv) $(4pq + 3q)^2 - (4pq - 3q)^2 = 48pq^2$

(v) $(a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) = 0$

Answer:

(i) $(3x + 7)^2 - 84x = (3x - 7)^2$

L.H.S $= (3x + 7)^2 - 84x$

$= (3x)^2 + 2(3x)(7) + (7)^2 - 84x$ [Using algebraic identity, $(a + b)^2 = a^2 + 2ab + b^2$]

$= 9x^2 + 42x + 49 - 84x$

$= 9x^2 - 42x + 49$

R.H.S $= (3x - 7)^2$

$= (3x)^2 - 2(3x)(7) + (7)^2$ [Since, $(a - b)^2 = a^2 - 2ab + b^2$]

$= 9x^2 - 42x + 49$

L.H .S = R.H .S

(ii) $(9p - 5q)^2 + 180pq = (9p + 5q)^2$

L.H .S = $(9p - 5q)^2 + 180pq$

$= (9p)^2 - 2(9p)(5q) + (5q)^2 + 180pq$ [Since, $(a - b)^2 = a^2 - 2ab + b^2$]

$= 81p^2 - 90pq + 25q^2 + 180pq$

$= 81p^2 + 90pq + 25q^2$

R.H.S = $(9p + 5q)^2$

$= (9p)^2 + 2(9p)(5q) + (5q)^2$ [Since, $(a + b)^2 = a^2 + 2ab + b^2$]

$= 81p^2 + 90pq + 25q^2$

L.H .S = R.H .S

(iii) $(4m/3 - 3n/4)^2 + 2mn = 16m^2/9 + 9n^2/16$

$\Rightarrow \left(\frac{4m}{3} - \frac{3n}{4}\right)^2 + 2mn = \frac{16m^2}{9} + \frac{9n^2}{16}$

L.H.S = $\left(\frac{4m}{3} - \frac{3n}{4}\right)^2 + 2mn$

$= \left(\frac{4m}{3}\right)^2 - 2 \times \frac{4m}{3} \times \frac{3n}{4} + \left(\frac{3n}{4}\right)^2 + 2mn$

$= \frac{16m^2}{9} - \frac{12mn}{6} + \frac{9n^2}{16} + 2mn$

$= \frac{16m^2}{9} - 2mn + \frac{9n^2}{16} + 2mn$

$= \frac{16m^2}{9} + \frac{9n^2}{16}$

L.H.S = R.H.S

(iv) $(4pq + 3q)^2 - (4pq - 3q)^2 = 48pq^2$

L.H.S = $(4pq + 3q)^2 - (4pq - 3q)^2$

= $(4pq)^2 + 2 (4pq) (3q) + (3q)^2 - [(4pq)^2 - 2(4pq) (3q) + (3q)^2]$

[Since, $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$]

= $16p^2q^2 + 24pq^2 + 9q^2 - [16p^2q^2 - 24pq^2 + 9q^2]$

= $16p^2q^2 + 24pq^2 + 9q^2 - 16p^2q^2 + 24 pq^2 - 9q^2$

= $48pq^2$

L.H.S = R.H.S

(v) (a - b) (a + b) + (b - c) (b + c) + (c - a) (c + a) = 0

L.H.S = (a - b) (a + b) + (b - c) (b + c) + (c - a) (c + a)

= $(a^2 - b^2) + (b^2 - c^2) + (c^2 - a^2)$ [Since, $a^2 - b^2 = (a - b) (a + b)$]

= $a^2 - b^2 + b^2 - c^2 + c^2 - a^2$

= 0

L.H.S = R.H.S

6. Using identities, evaluate

i) 71^2 ii) 99^2 iii) 102^2 iv) 998^2 v) 5.2^2

vi) 297 × 303 vii) 78 × 82 viii) 8.9^2 ix) 10.5 × 9.5

Answer: The three basic algebraic identities, which we will be using to evaluate the expressions are as follows.

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

$(a + b) (a - b) = a^2 - b^2$

(i) 71^2

$= (70 + 1)^2$

$= (70)^2 + 2(70)(1) + (1)^2$ [Since, $(a + b)^2 = a^2 + 2ab + b^2$]

$= 4900 + 140 + 1 = 5041$

(ii) 99^2

$= (100 - 1)^2$

$= (100)^2 - 2(100)(1) + (1)^2$ [Since, $(a - b)^2 = a^2 - 2ab + b^2$]

$= 10000 - 200 + 1 = 9801$

(iii) 102^2

$= (100 + 2)^2$

$= (100)^2 + 2(100)(2) + (2)^2$ [Since, $(a + b)^2 = a^2 + 2ab + b^2$]

$= 10000 + 400 + 4 = 10404$

(iv) 998^2

$= (1000 - 2)^2$

$= (1000)^2 - 2(1000)(2) + (2)^2$ [Since, $(a - b)^2 = a^2 - 2ab + b^2$]

$= 1000000 - 4000 + 4 = 996004$

(v) $(5.2)^2$

$= (5.0 + 0.2)^2$

$= (5.0)^2 + 2(5.0)(0.2) + (0.2)^2$ [Since, $(a + b)^2 = a^2 + 2ab + b^2$]

$= 25 + 2 + 0.04 = 27.04$

(vi) 297 × 303

$= (300 - 3) \times (300 + 3)$ [Since, $(a + b)(a - b) = a^2 - b^2$]

$= (300)^2 - (3)^2$

$= 90000 – 9 = 89991$

(vii) 78 × 82

$= (80 - 2) \times (80 + 2)$ [Since, $(a + b)(a - b) = a^2 - b^2$]

$= (80)^2 - (2)^2$

$= 6400 – 4 = 6396$

(viii) 8.9^2

$= (9.0 - 0.1)^2$

$= (9.0)^2 - 2(9.0)(0.1) + (0.1)^2$ [Since, $(a - b)^2 = a^2 - 2ab + b^2$]

$= 81 - 1.8 + 0.01 = 79.21$

(ix) 10.5 × 9.5

$= (10 + 0.5) \times (10 - 0.5)$

$= 10^2 - 0.5^2$ [Since, $(a + b)(a - b) = a^2 - b^2$]

$= 100 - 0.25 = 99.75$

7. Using $a^2 - b^2 = (a + b)(a - b)$, find

(i) $51^2 – 49^2$ **(ii) $(1.02)^2 - (0.98)^2$**

(iii) $153^2 - 147^2$ **(iv) $12.1^2 - 7.9^2$**

Answer: We will be solving the questions using the algebraic identity $a^2 - b^2 = (a + b)(a - b)$

(i) $51^2 – 49^2$

$= (51 + 49)(51 - 49)$

$= (100)(2) = 200$

(ii) $(1.02)^2 - (0.98)^2$

$= (1.02 + 0.98)(1.02 - 0.98)$

$= (2)(0.04) = 0.08$

(iii) $153^2 - 147^2$

= (153 + 147) (153 - 147)

= (300) (6) = 1800

(iv) $12.1^2 - 7.9^2$

= (12.1 + 7.9) (12.1 - 7.9)

= (20.0)(4.2) = 84

8. Using $(x + a)(x + b) = x^2 + (a + b)x + ab$

(i) 103 × 104 **(ii) 5.1 × 5.2**

(iii) 103 × 98 **(iv) 9.7 × 9.8**

Answer: We will be using the algebraic Identity $(x + a)(x + b) = x^2 + (a + b)x + ab$

(i) 103 × 104

= (100 + 3)(100 + 4)

= $(100)^2$ + (3 + 4) (100) + (3)(4)

= 10000 + 700 + 12 = 10712

(ii) 5.1 × 5.2

= (5 + 0.1) (5 + 0.2)

= $(5)^2$ + (0.1 + 0.2) (5) + (0.1) (0.2)

= 25 + 1.5 + 0.02 = 26.52

(iii) 103 × 98

= (100 + 3) (100 - 2)

= $(100)^2$ + [3 + (-2)] (100) + (3) (-2)

= 10000 + 100 – 6 = 10094

(iv) 9.7 × 9.8

= (10 - 0.3)(10 - 0.2)

= $(10)^2$ + [(-0.3) + (-0.2)] (10) + (-0.3) (-0.2)

= 100 + (- 0.5)10 + 0.06

= 100 - 5 + 0.06 = 95.06