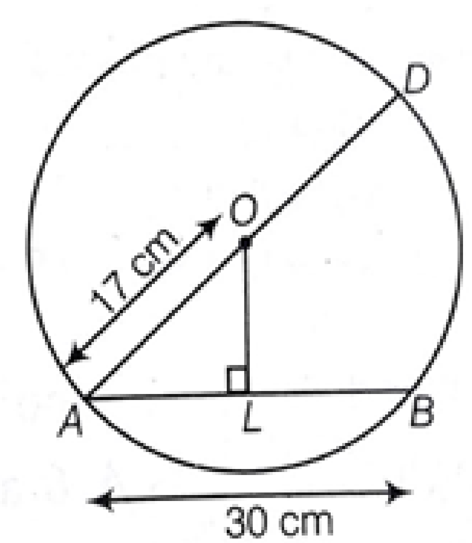
Exercise 10.1

1. AD is a diameter of a circle and AB is a chord. If AD = 34 cm, AB = 30 cm, the distance of AB from the centre of the circle is :

(A) 17 cm (B) 15 cm (C) 4 cm (D) 8 cm

Answer:



It is given that

AD = 34 cm

AB = 30 cm

Construct OL perpendicular to AB

AL = LB = 1/2 AB = 1/2 (30) = 15 cm

In triangle OLA

Using pythagoras theorem

$OA^2 = OL^2 + AL^2$

Substituting the values

$17^2 = OL^2 + 15^2$

By further calculation

$OL^2 = 289 - 225 = 64$

So we get

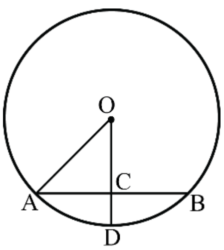
OL = 8 cm

Therefore, the distance of AB from the centre of the circle is 8 cm.

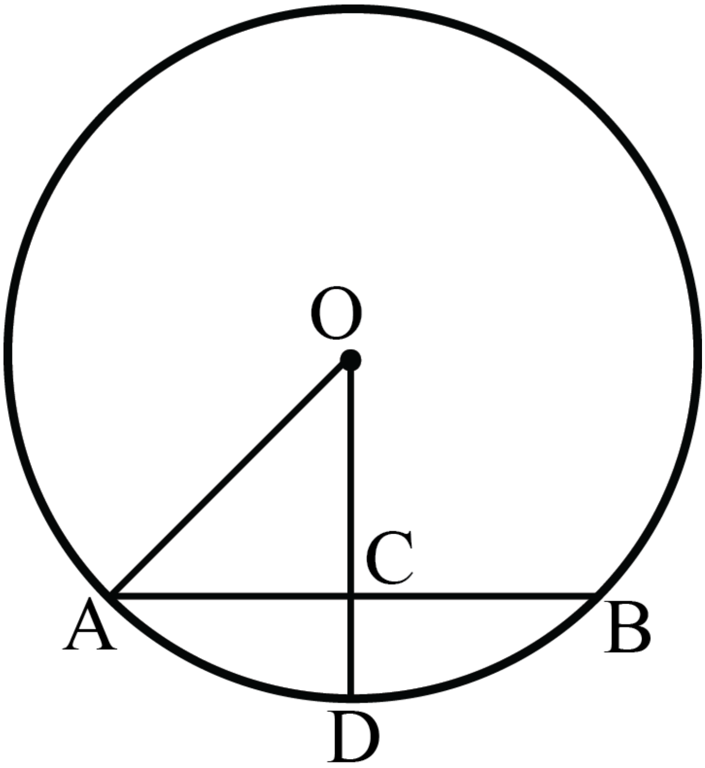
2. In Fig. 10.3, if OA = 5 cm, AB = 8 cm and OD is perpendicular to AB, then CD is equal to:

a. 2 cm **b. 3 cm** **c. 4 cm** **d. 5 cm**

Answer: a. 2 cm



Answer:



We know that Perpendicular from the centre of a circle to a chord bisects the chord

AC = CB = 1/2 AB

= 1/2 × 8

= 4 cm

It is given that

OA = 5 cm

Using the Pythagoras theorem

$AO^2 = AC^2 + OC^2$

Substituting the values

$5^2 = 4^2 + OC^2$

$OC^2 = 25 - 16$

$OC^2 = 9$

OC = 3 cm

As the radius of the circle is same OA = OD

OD = 5 cm

CD = OD - OC

= 5 - 3

= 2 cm

Therefore, CD is equal to 2 cm.

3. If AB = 12 cm, BC = 16 cm and AB is perpendicular to BC, then the radius of the circle passing through the points A, B and C is :

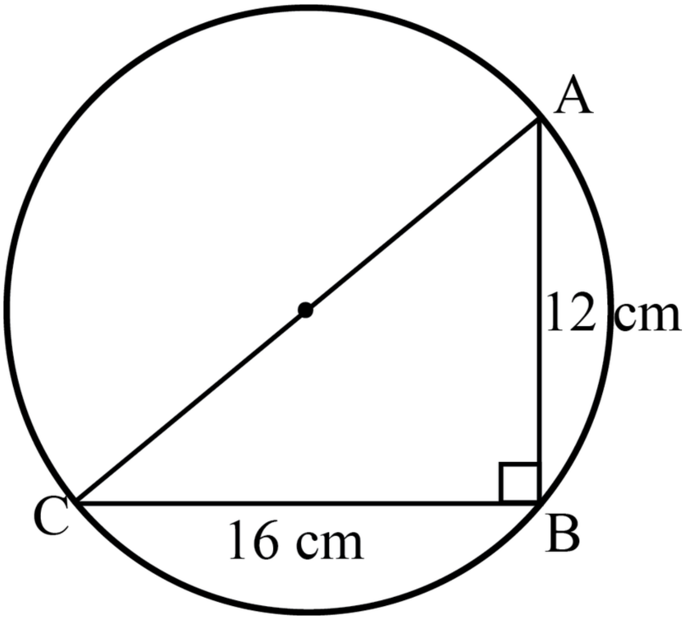
(A) 6 cm (B) 8 cm (C) 10 cm (D) 12 cm

Answer: (C) 10 cm

AB = 12 cm

BC = 16 cm

In the circle, BC is perpendicular to AB which means that AC is the diameter



In triangle ABC,

Using the Pythagoras theorem

$AC^2 = AB^2 + BC^2$

Substituting the values

$AC^2 = 12^2 + 16^2$

By further calculation

$AC^2 = 144 + 256$

$AC^2 = 400$

So we get

AC = 20 cm

We know that

Radius of circle = 1/2 AC

= 1/2 × 20

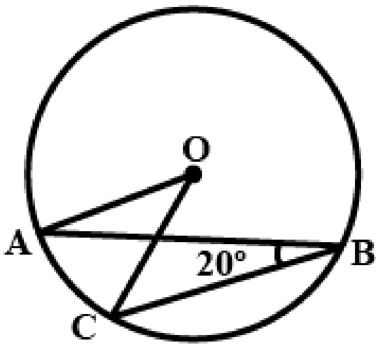
= 10 cm

Therefore, the radius of the circle is 10 cm.

4. In Fig.10.4, if ∠ABC = 20^0, then ∠AOC is equal to:

(A) 20^0 (B) 40^0 (C) 60^0 (D) 10^0

Answer: (B) 40^0



We know that

Angle at the centre of the circle is twice the angle at the circumference subtended by the same arc.

∠AOC = 2 ∠ABC

It is given that

∠ABC = 20^0

Substituting the values

∠AOC = 2 × 20^0

∠AOC = 40^0

Therefore, ∠AOC is equal to 40^0

5. In Fig.10.5, if AOB is a diameter of the circle and AC = BC, then ∠CAB is equal to:

(A) 30^0 (B) 60^0 (C) 90^0 (D) 45^0

Answer: (D) 45^0

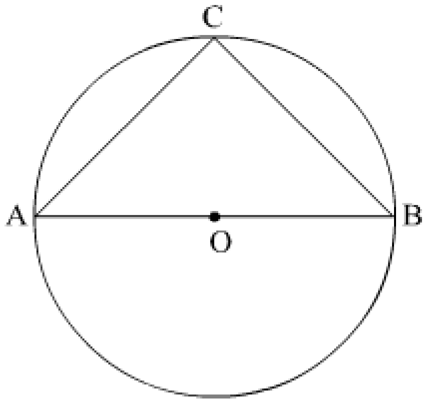


Fig. 15.201

We know that

Diameter subtends a right angle to the circle

∠BCA = 90º … (1)

It is given that

AC = BC

As the angles opposite to equal sides are equal

∠ABC = ∠CAB … (2)

In triangle ABC using the angle sum property

∠CAB + ∠ABC + ∠BCA = 180º

From equations (1) and (2)

∠CAB + ∠CAB + 90º = 180º

2∠CAB = 180 - 90

2∠CAB = 90

Dividing both sides by 2

∠CAB = 45º

Therefore, ∠CAB is equal to 45º.

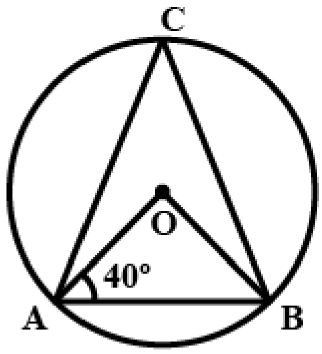
6. **In Fig. 10.6, if ∠OAB = 40^0, then ∠ACB is equal to :**

a. 50^0 **b. 40^0** **c. 60^0** **d. 70^0**

Answer: **a. 50^0**

In triangle OAB

OA = OB (radius of a circle)



∠OAB = ∠OBA

∠OBA = 40º (angles opposite to equal sides are equal)

Using the angle sum property

∠AOB + ∠OBA + ∠BAO = 180º

Substituting the values

∠AOB + 40º + 40º = 180º

By further calculation

∠AOB + 80º = 180º

∠AOB = 180º - 80º

∠AOB = 100º

As the angle subtended by an arc at the centre is twice the angle subtended by it at the remaining part of the circle

∠AOB = 2 ∠ACB

Substituting the values

100º = 2 ∠ACB

Dividing both sides by 2

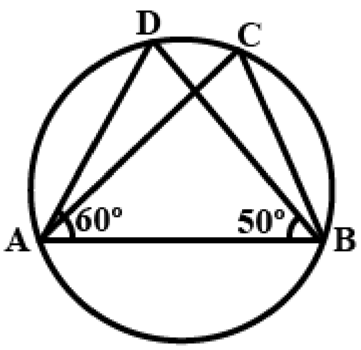
∠ACB = 50º

Therefore, ∠ACB is equal to 50º.

7. In Fig. 10.7, if ∠DAB = 60^0, ∠ABD = 50^0, then ∠ACB is equal to:

a. 60^0 **b. 50^0** **c. 70^0** **d. 80^0**

Answer: c. 70^0



It is given that

∠DAB = 60º

∠ABD = 50º

As ∠ADB = ∠ACB … (1) [angles in same segment of a circle are equal]

In triangle ABD

Using the angle sum property

∠ABD + ∠ADB + ∠DAB = 180º

Substituting the values

50 + ∠ADB + 60 = 180º

By further calculation

∠ADB + 110 = 180

So we get

∠ADB = ∠ACB = 70º

Therefore, ∠ACB is equal to 70º.

8. ABCD is a cyclic quadrilateral such that AB is a diameter of the circle circumscribing it and ∠ADC = 140^0, then ∠BAC is equal to:
a. 80^0
b. 50^0
c. 40^0
d. 30^0

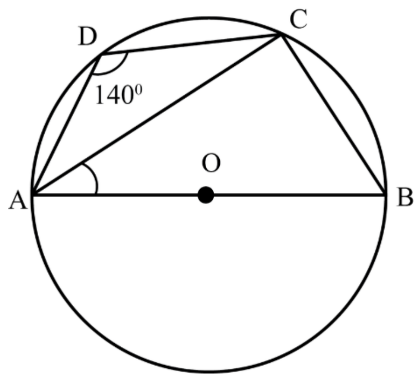
Answer: **b. 50^0**

It is given that

ABCD is a cyclic quadrilateral

∠ADC = 140^0

Sum of the opposite angles in a cyclic quadrilateral is 180^0



∠ADC + ∠ABC = 180^0

Substituting the values

140 + ∠ABC = 180^0

∠ABC = 180^0 - 140^0

∠ABC = 40^0

∠ACB is an angle in a semi circle

∠ACB = 90^0

In triangle ABC

Using the angle sum property

∠BAC + ∠ACB + ∠ABC = 180^0

Substituting the values

∠BAC + 90^0 + 40^0 = 180^0

By further calculation

∠BAC + 130^0 = 180^0

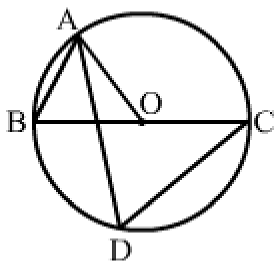
∠BAC = 180 - 130

∠BAC = 50^0

Therefore, ∠BAC is equal to 50^0

9. In Fig. 10.8, BC is a diameter of the circle and ∠BAO = 60^0. Then ∠ADC is equal to :

a. 30^0 b. 45^0 c. 60^0 d. 120^0



Answer: c. 60^0

In triangle AOB

∠OBA = ∠BAO (angles opposite to equal sides are equal)

It is given that

∠BAO = 60^0

So ∠OBA = 60^0

As the angles in the same segment AC are equal

∠ABC = ∠ADC

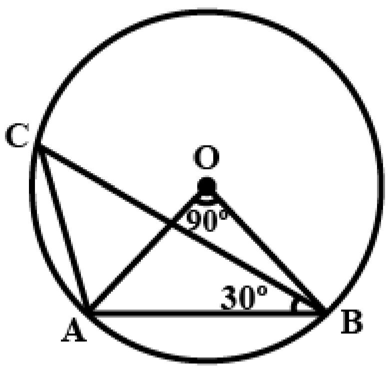
We get

∠ADC = 60^0

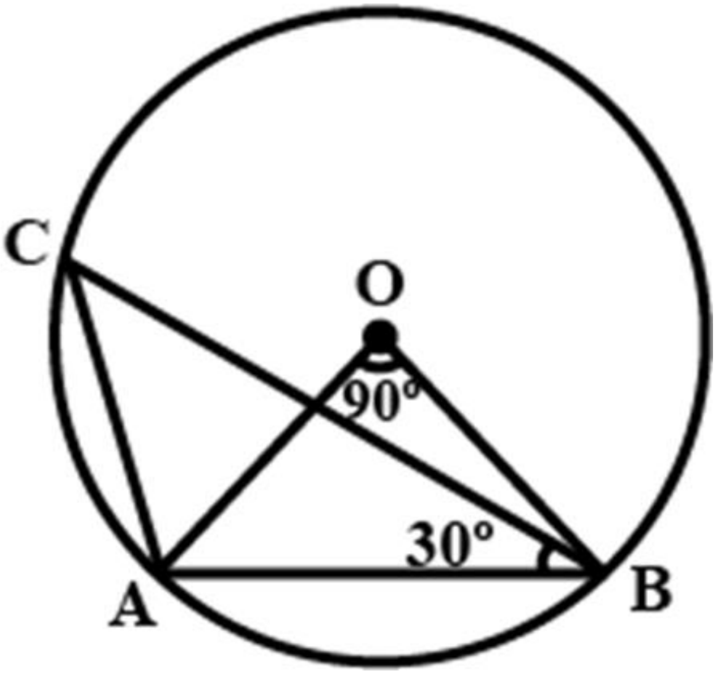
Therefore, ∠ADC is equal to 60^0

10. In Fig. 10.9, ∠AOB = 90^0 and ∠ABC = 30^0, then ∠CAO is equal to:

a. 30^0 b. 45^0 c. 90^0 d. 60^0



Answer: d. 60^0



In triangle OAB

∠OAB + ∠ABO + ∠BOA = 180º

As the angles opposite to equal sides are equal

∠OAB + ∠OAB + 90º = 180º

2∠OAB + 90º = 180º

By further calculation

2∠OAB = 180º - 90º

2∠OAB = 90º

Dividing both sides by 2

∠OAB = 45º …. (1)

In triangle ACB

∠ACB + ∠CBA + ∠CAB = 180º

Substituting the values

45º + 30º + ∠CAB = 180º

By further calculation

∠CAB = 180º - 75º

∠CAB = 105º

We know that

∠CAO + ∠OAB = 105º

Substituting the values

∠CAO + 45º = 105º

∠CAO = 105º - 45º

∠CAO = 60º

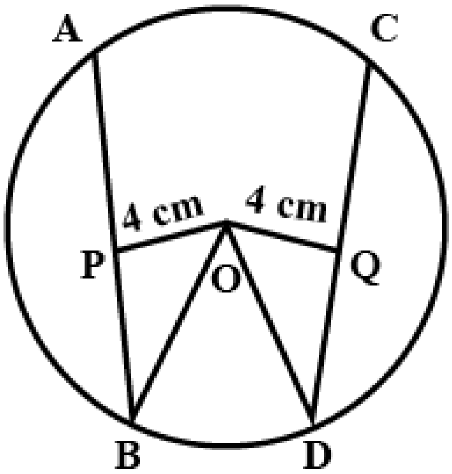
Therefore, ∠CAO is equal to 60º.

Exercise 10.2

Write True or False and justify your answer in each of the following:

1. Two chords AB and CD of a circle are each at distances 4 cm from the centre. Then AB = CD.

Answer: True



It is given that

AB and CD are the chords of a circle at a distance of 4 cm from the centre

We know that

OM and ON are perpendiculars of 4 cm from the centre O to AB and CD

It bisects AB and CD

Here,

AM = 1/2 AB

DN = 1/2 DC

∠OMA = ∠OND = 90º

In between the triangles AOM and DON

OM = ON = 4 cm [Given]

∠OMA = ∠OND = 90º

OA = OD (radii of the same circle)

From the RHS rule

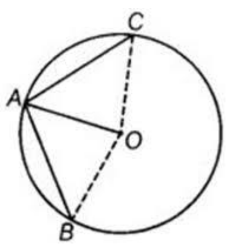
Δ AOM ≅ Δ DON

AM = DN and AB = DC

Therefore, the statement is true.

2. Two chords AB and AC, of a circle with centre O are on the opposite sides of OA. Then ∠OAB = ∠OAC.

Answer: False



AB and AC are the two chords

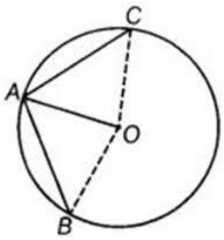
Let us join OB and OC

In triangle OAB and triangle OAC

OA = OA (common side)

OC = OB (radius of circle)

It is not possible to prove that either a third side or any angle is equal and triangle OAB is not congruent to triangle OAC.

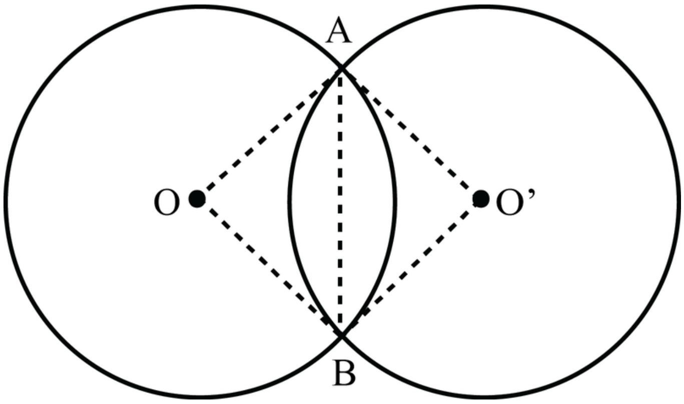


∠OAB ≠ ∠OAC

Therefore, the statement is false.

3. Two congruent circles with centres O and O' intersect at two points A and B. Then ∠AOB = ∠AO'B.

Answer: True



Let us join AB, OA, OB, O'A and OB'

In triangle AOB and triangle AO'B

OA = AO' (circles have same radius)

OB = BO' (circles have same radius)

AB = AB (common chord)

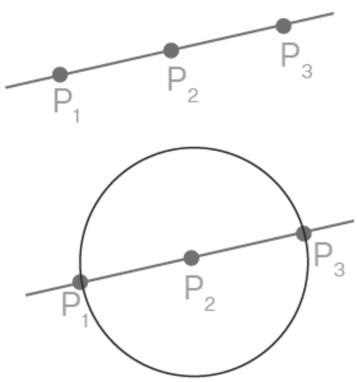
From the SSS congruence criterion

Δ AOB ≅ Δ AO'B

∠AOB = ∠AO'B (cpctc)

Therefore, the statement is true.

4. Through three collinear points a circle can be drawn.



Answer: False

A circle can be drawn only through two collinear points.

5. A circle of radius 3 cm can be drawn through two points A, B such that AB = 6 cm.

Answer: True

Radius of a circle = 3 cm

We know that

Diameter of a circle = 2 × radius of the circle

A circle is drawn through two points AB of 6 cm

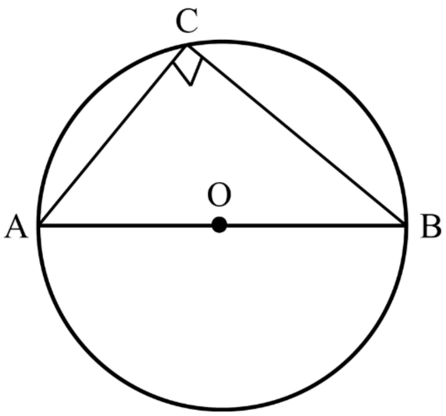
It means that the diameter is 6 cm

Radius = diameter/2

Radius = 6/2 = 3 cm

6. If AOB is a diameter of a circle and C is a point on the circle, then $AC^2 + BC^2 = AB^2$.

Answer: True



A diameter of a circle subtends a right angle to any point on the circle

If AOB is the diameter and C is a point on the circle

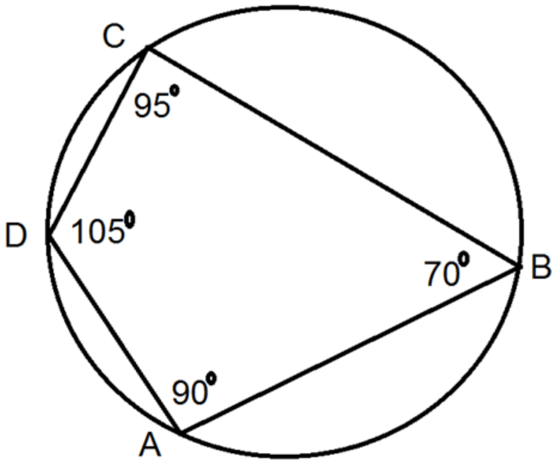
Triangle ACB is right angled at C

In triangle ACB

Using the Pythagoras theorem

$AC^2 + BC^2 = AB^2$

7. ABCD is a cyclic quadrilateral such that ∠A = 90°, ∠B = 70°, ∠C = 95° and ∠D = 105°. Is the given statement true or false and justify your answer.



Answer: False

It is given that

ABCD is a cyclic quadrilateral

∠A = 90°

∠B = 70°

∠C = 95°

∠D = 105°

We know that

In a cyclic quadrilateral, the sum of opposite angles is 180°

It can be written as

∠A + ∠C = 90° + 95° = 185° ≠ 180°

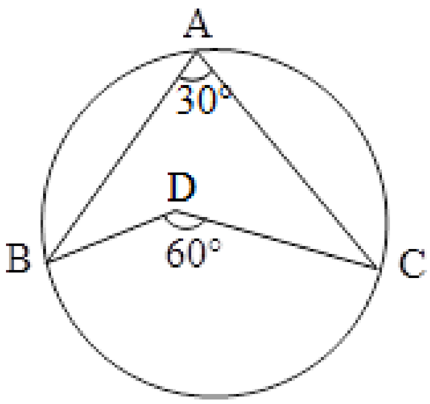
∠B + ∠D = 70° + 105° = 175° ≠ 180°

Here the sum of opposite angles is not equal to 180°

So it is not a cyclic quadrilateral

8. If A, B, C, D are four points such that ∠BAC = 30° and ∠BDC = 60°, then D is the centre of the circle through A, B and C. Is the given statement true or false and justify your answer.

Answer: False



A, B, C, D are four points

∠BAC = 30°

∠BDC = 60°

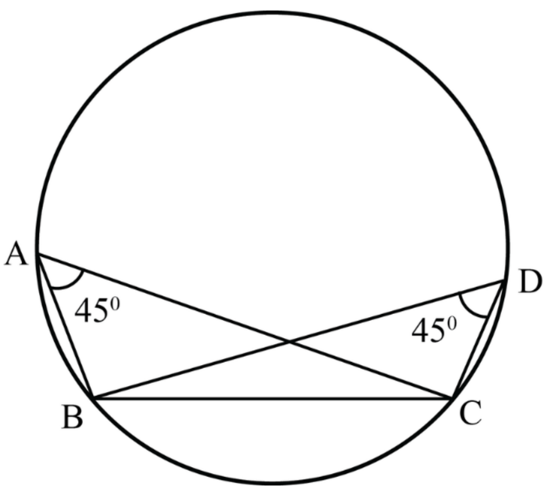
D is the centre of the circle through A, B and C

We know that

There can be many points D such that ∠BDC = 60° and each point cannot be the centre of the circle through A,B,C.

9. If A, B, C and D are four points such that ∠BAC = 45° and ∠BDC = 45°, then A, B, C and D are concyclic.

Answer: True



A, B, C and D are four points

∠BAC = 45°

∠BDC = 45°

We have to find if the points are concyclic

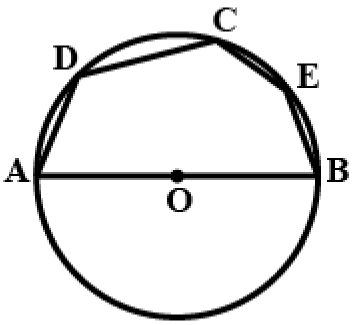
Consider the figure

We know that

Angles in the same segment of a circle are equal

So A, B, C and D are concyclic.

10. In Fig. 10.10, if AOB is a diameter and ∠ADC = 120°, then ∠CAB = 30°. Is the given statement true or false and justify your answer.



Answer: True

Let us join CA and CB

ADCB is a cyclic quadrilateral

We know that, the sum of opposite angles will be 180°

∠ADC + ∠CBA = 180°

It is given that

∠ADC = 120°

∠CAB = 30°

Substituting the values

120° + ∠CBA = 180°

∠CBA = 180° - 120° = 60°

In triangle ACB

∠ACB = 90° (Angle subtended by the diameter of a circle to its centre is 90°)

∠CAB + ∠CBA + ∠ACB = 180°

Using the angle sum property of a triangle

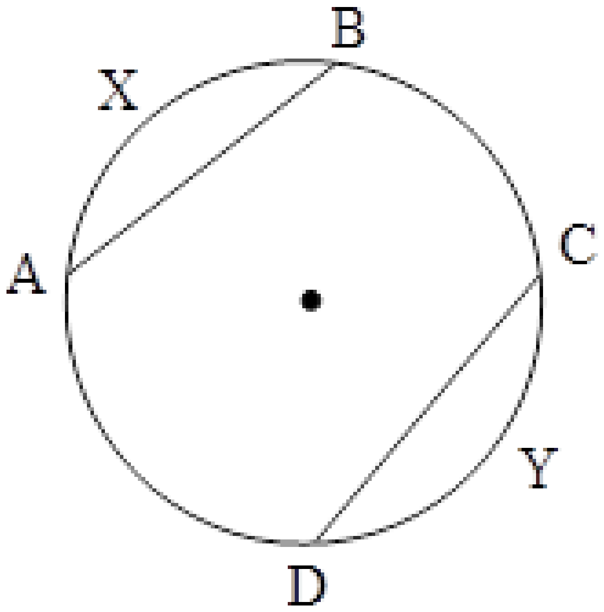
∠CAB + 60° + 90° = 180°

By further calculation

∠CAB = 180° - 150° = 30°

Exercise 10.3

1. If arcs AXB and CYD of a circle are congruent, find the ratio of AB and CD.



Answer: Given, AXB and CYD are arc of a circle which are congruent.

We have to find the ratio of AB and CD

Join AB and CD

We know that if two arcs of a circle are congruent, then their corresponding chords are equal.

Since arcs AXB and CYD are congruent

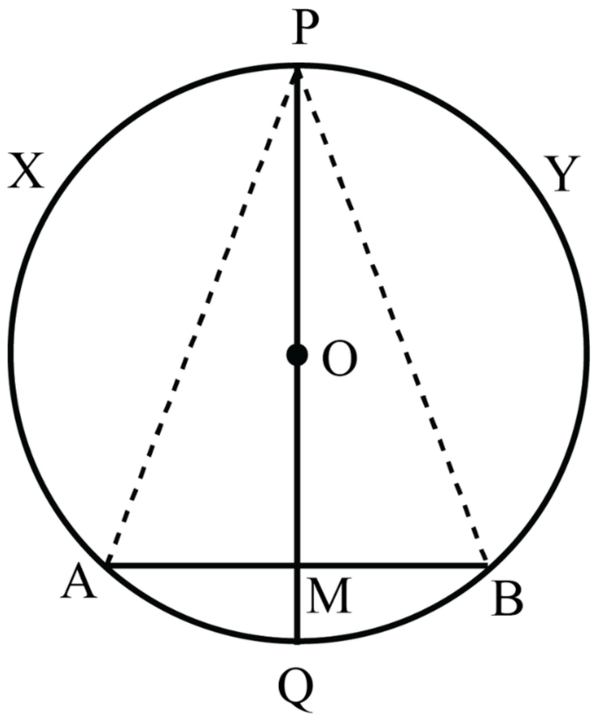
So, AB = CD

Now, AB : CD = AB : AB = 1 : 1

Therefore, the ratio of AB and CD is 1:1

2. If the perpendicular bisector of a chord AB of a circle PXAQBY intersects the circle at P and Q, prove that arc PXA ∠ Arc PYB.

Answer: Given, the perpendicular bisector of a chord AB of a circle PXAQBY intersects the circle at P and Q.



We have to prove that arc PXA ≅ Arc PYB.

PQ is the perpendicular bisector of AB

So, AM = B M

Considering triangle APM and BPM,

Since PQ is the perpendicular bisector, M is the midpoint of AB

AM = BM

∠AMP = ∠BMP = 90°

Common side = MP

SAS criterion states that if two sides of one triangle are proportional to two sides of another triangle and their included angles are congruent, then the triangles are similar.

By SAS criterion, the triangles APM and BPM are similar.

The Corresponding Parts of Congruent Triangles are Congruent (CPCTC) theorem states that when two triangles are similar, then their corresponding sides and angles are also congruent or equal in measurements.

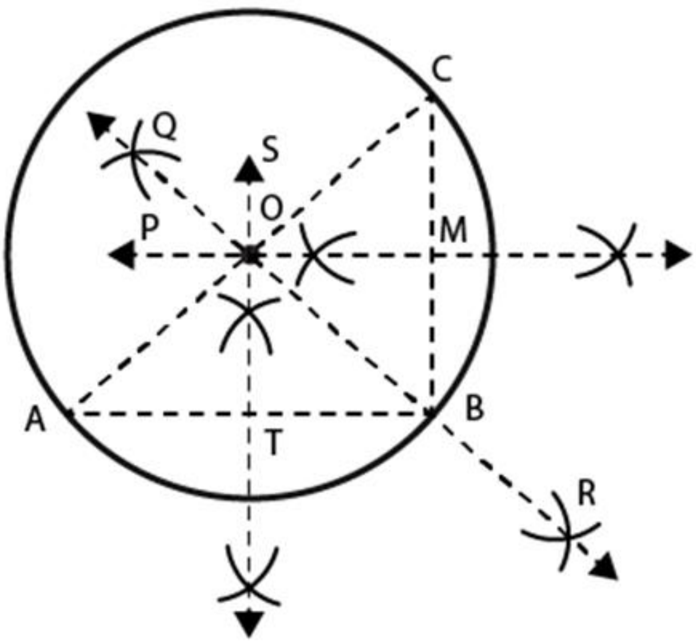
By CPCTC,

AP = BP

We know that if two chords of a circle are congruent, then their corresponding arcs are equal.

Therefore, arc PXA = arc PYB

3. A, B and C are three points on a circle. Prove that the perpendicular bisectors of AB, BC and CA are concurrent.



Answer: Given, A, B and C are three points on a circle.

We have to prove that the perpendicular bisectors of AB, BC and CA are concurrent.

Draw perpendicular bisectors of AB and AC which meet at a point O.

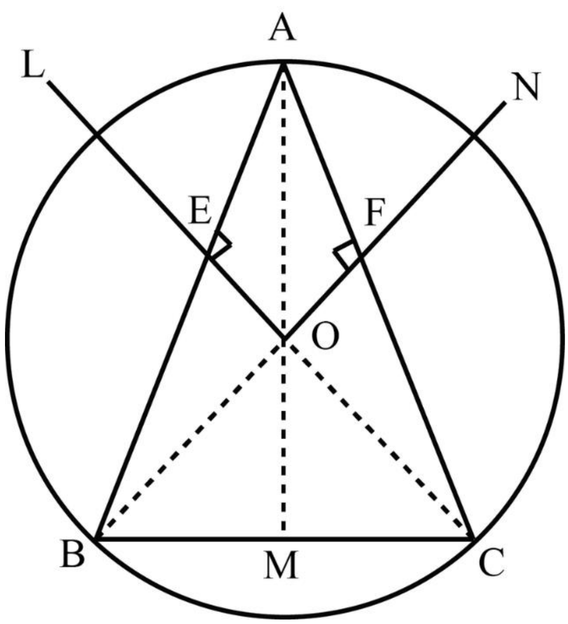
Join OA, OB and OC.

Perpendicular bisector of BC also passes through O

So, OL = ON = OM

Considering triangles OEA and OEB,

Since OL is the perpendicular bisector of AB



AE = BE

∠AEO = ∠BEO = 90°

Common side = OE

SAS criterion states that if two sides of one triangle are proportional to two sides of another triangle and their included angles are congruent, then the triangles are similar.

By SAS criterion, the triangles OEA and OEB are similar.

So, OA = OB

Similarly, by SAS criterion, the triangles OFA and OFC are similar.

By CPCTC,

OA = OC

Let OA = OB = OC = r

Draw a perpendicular from O to BC and join them.

Considering triangles OMB and OMC,

OB = OC

Common side = OM

∠OMB = ∠OMC = 90°

RHS criterion states that if the hypotenuse and side of one right-angled triangle are equal to the hypotenuse and the corresponding side of another right-angled triangle, the two triangles are congruent.

By RHS criterion, the triangles OMB and OMC are similar.

The Corresponding Parts of Congruent Triangles are Congruent (CPCTC) theorem states that when two triangles are similar, then their corresponding sides and angles are also congruent or equal in measurements.

By CPCTC,

BM = MC

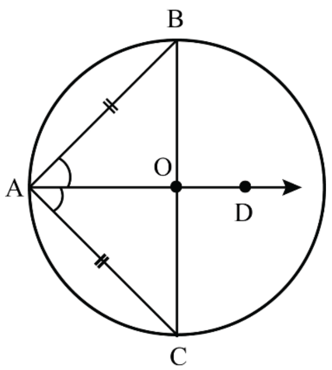
This implies OM is the perpendicular bisector of BC

Therefore, OL, OM and ON are concurrent.

4. AB and AC are two equal chords of a circle. Prove that the bisector of the angle BAC passes through the centre of the circle.

Answer: Given, AB and AC are two equal chords of a circle

We have to prove that the bisector of the angle BAC passes through the centre of the circle.



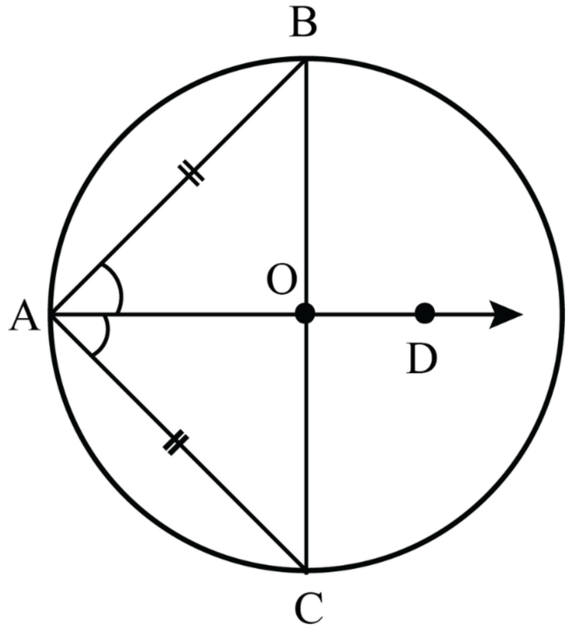
Considering triangles APB and APC,

Given, AB= AC

∠BAP = ∠CAP

Common side = AP

SAS criterion states that if two sides of one triangle are proportional to two sides of another triangle and their included angles are congruent, then the triangles are similar.



By SAS criterion, the triangles APB and APC are similar.

The Corresponding Parts of Congruent Triangles are Congruent (CPCTC) theorem states that when two triangles are similar, then their corresponding sides and angles are also congruent or equal in measurements.

By CPCTC,

BP = CP

∠APB = ∠APC ---------- (1)

We know that the linear pair of angles is always equal to 180 degrees.

∠APB + ∠APC = 180°

From (1),

∠APB + ∠APB = 180°

2∠APB = 180°

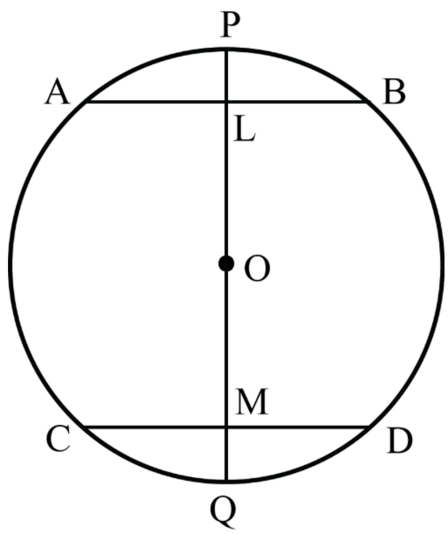
∠APB = 180°/2

∠APB = 90°

This implies AP is the perpendicular bisector of chord BC.

Therefore, AP passes through the center of the circle.

5. If a line segment joining mid-points of two chords of a circle passes through the centre of the circle, prove that the two chords are parallel.



Answer: Consider a circle with centre O

AB and CD are two chords of a circle

PQ is the diameter which bisects the chord AB and CD at L and M.

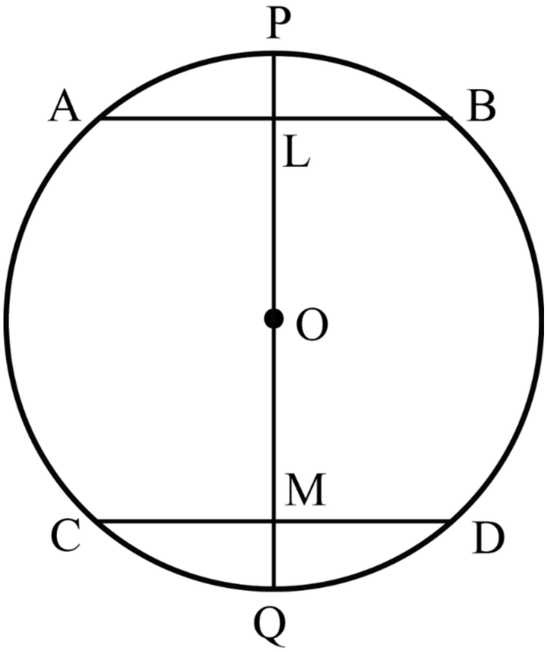
The diameter PQ passes through the centre O of the circle.

We have to prove that the chords are parallel.

Since L is the midpoint of AB

OL ⊥ AB

We know that the line joining the centre of a circle to the midpoint of a chord is perpendicular to the chord.



∠ALO = 90° -------------------------- (1)

Similarly, OM ⊥ CD

∠OMD = 90° ---------------------- (2)

From (1) and (2),

∠ALO = ∠OMD = 90°

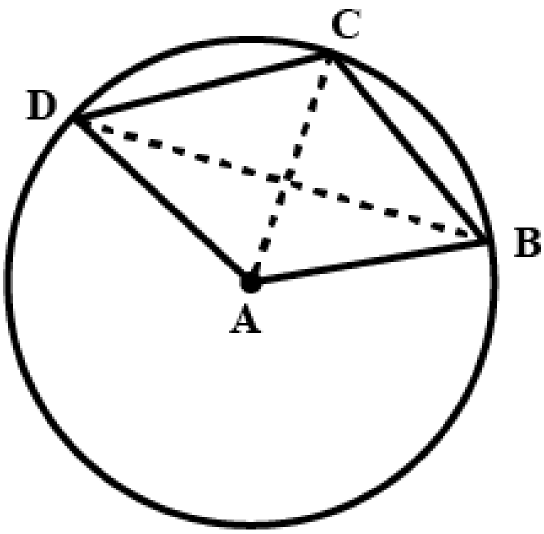
We know that the alternate angles are equal.

From the figure,

∠ALO and ∠OMD are alternate angles.

Therefore, AB || CD

6. ABCD is such a quadrilateral that A is the centre of the circle passing through B, C and D. Prove that ∠CBD + ∠CDB = ½ ∠BAD.



Answer: Given, ABCD is a quadrilateral.

A is the centre of the circle passing through B, C and D.

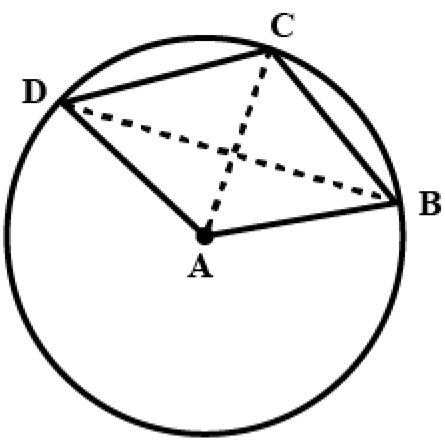
We have to prove that ∠CBD + ∠CDB = 1/2 ∠BAD

Join the diagonals AC and BD of the quadrilateral

Now, arc DC subtends an angle ∠DAC at the centre and ∠CBD at a point B in the remaining part of the circle.

We know that in a circle the angle subtended by an arc at the centre is twice the angle subtended by it at the remaining part of the circle.

So, ∠DAC = 2∠CBD ------------------ (1)



Similarly, arc BC subtends an angle ∠BAC at the centre and ∠CDB at a point D in the remaining part of the circle.

So, ∠BAC = 2∠CDB -------------------- (2)

Adding (1) and (2),

∠DAC + ∠BAC = 2∠CBD + 2∠CDB

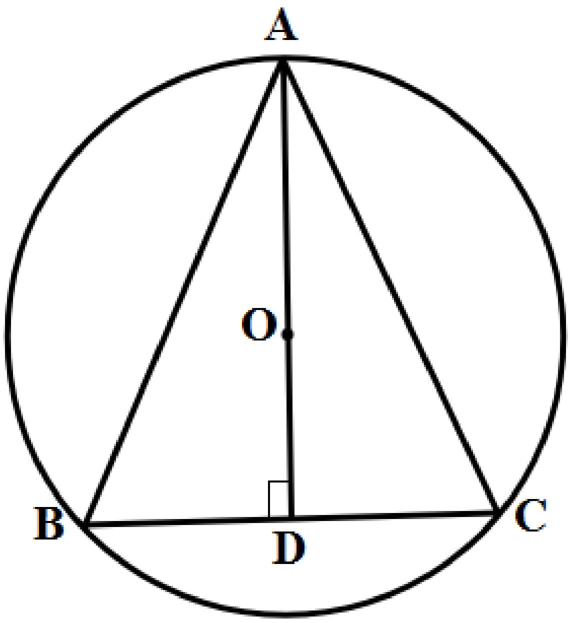
From the figure,

∠DAC + ∠BAC = ∠DAB

So, ∠DAB = 2(∠CBD + ∠CDB)

Therefore, ∠CBD + ∠CDB = 1/2 ∠DAB

7. O is the circumcentre of the triangle ABC, and D is the mid-point of the base BC. Prove that ∠BOD = ∠A.



Answer: Given, ABC is a triangle

O is the circumcentre of the triangle

D is the midpoint of the base BC

We have to prove that ∠BOD = ∠A.

Join OB, OC and OD

Considering triangles BOD and COD,

OB = OC = Radius of the circle

Since D is the midpoint of BC

BD = DC

Common side = OD

The Side-Side-Side congruence rule states that, if all the three sides of a triangle are equal to the three sides of another triangle then the triangles are congruent.

By SSS criterion, the triangles BOD and COD are similar.

The Corresponding Parts of Congruent Triangles are Congruent (CPCTC) theorem states that when two triangles are similar, then their corresponding sides and angles are also congruent or equal in measurements.

By CPCTC,

∠BOD = ∠COD

So, ∠BOC = 2 ∠BOD ---------------------- (1)

We know that in a circle the angle subtended by an arc at the centre is twice the angle subtended by it at the remaining part of the circle.

∠BOC = 2 ∠BAC

From (1),

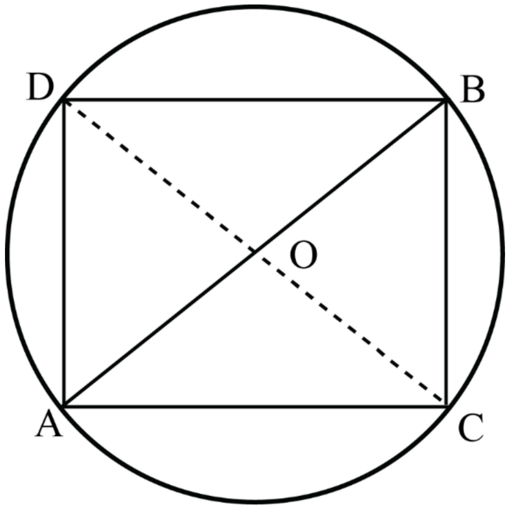
2∠BOD = 2∠BAC

Therefore, ∠BOD = ∠BAC

8. On a common hypotenuse AB, two right triangles ACB and ADB are situated on opposite sides. Prove that ∠BAC = ∠BDC.

Answer: Given, two right triangles ACB and ADB lie on common hypotenuse AB are situated on opposite sides.

We have to prove that ∠BAC = ∠BDC.



Join CD

Let O be the midpoint of AB

We know that the midpoint of the hypotenuse of a right triangle is equidistant from its vertices.

So, OA = OB = OC = OD

Now draw a circle that passes through the points A, B, C and D with O as centre and radius equal to OA.

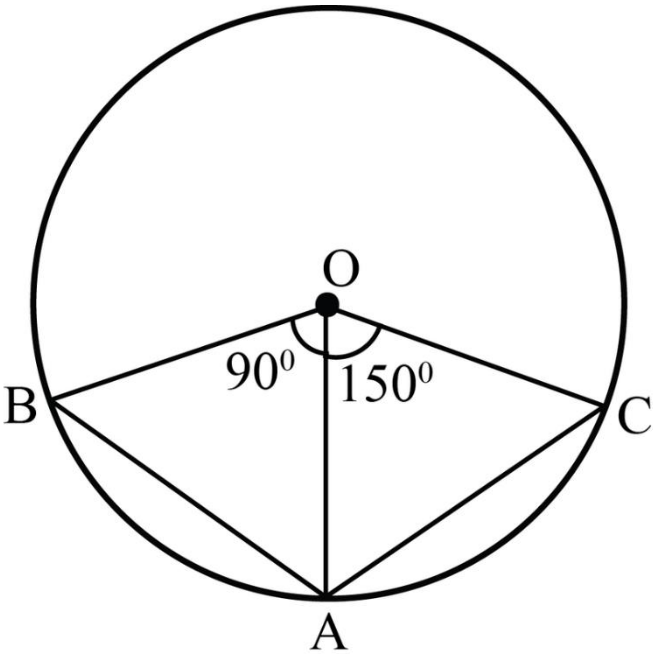
We know that the angles in the same segment of a circle are equal.

From the figure,

∠BAC and ∠BDC are angles of the same segment BC.

Therefore, ∠BAC = ∠BDC

9. Two chords AB and AC of a circle subtend angles equal to 90^0 and 150^0, respectively at the centre. Find ∠BAC, if AB and AC lie on the opposite sides of the centre.



Answer: Given, two chords AB and AC of a circle subtends angles equal to 90º and 150º at the centre.

AB and AC lie on the opposite sides of the centre.

We have to find ∠BAC

Considering triangle BOA,

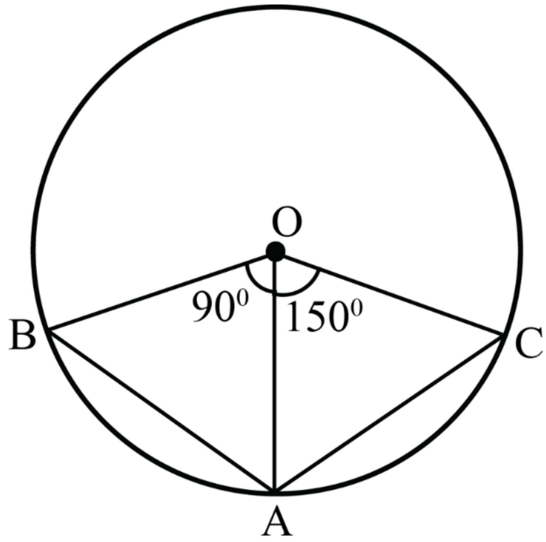
OA = OB = radius of the circle

We know that the angles opposite to the equal sides are equal.

So, ∠OAB = ∠OBA ----------------------- (1)

In triangle OAB,

By angle sum property of a triangle,



∠OBA + ∠OAB + ∠AOB = 180°

From (1),

∠OAB + ∠OAB + ∠AOB = 180°

Given, ∠AOB = 90°

∠OAB + ∠OAB + 90° = 180°

2∠OAB = 180° - 90°

2∠OAB = 90°

∠OAB = 45°

In triangle AOC,

AO = OC = radius of the circle

We know that the angles opposite to the equal sides are equal.

So, ∠OCA = ∠OAC --------------------------- (2)

By angle sum property of a triangle,

∠AOC + ∠OAC + ∠OCA = 180°

Given, ∠AOC = 150°

150° + ∠OAC + ∠OCA = 180°

From (2),

∠OAC + ∠OAC = 180° - 150°

2∠OAC = 30°

∠OAC = 15°

Now, ∠BAC = ∠OAB + ∠OAC

= 45° + 15°

= 60°

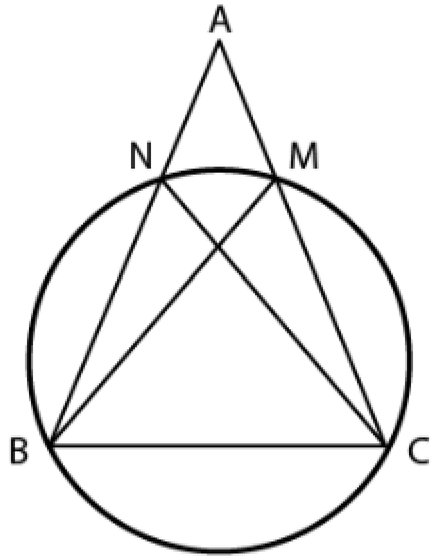
Therefore, ∠BAC = 60°

10. If BM and CN are the perpendiculars drawn on the sides AC and AB of the triangle ABC, prove that the points B, C, M and N are concyclic.

Answer: Given, ABC is a triangle

BM and CN are the perpendiculars drawn on the sides AC and AB

We have to prove that B, C, M and N are concyclic.



Since BM and CN are the perpendiculars drawn on the sides AC and AB of the triangle ABC

∠BMC = ∠BNC = 90°

We know that if a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, then the four points are concyclic.

Here, the line segment BC joining two points B and C subtends equal angles of 90 degrees at M and N.

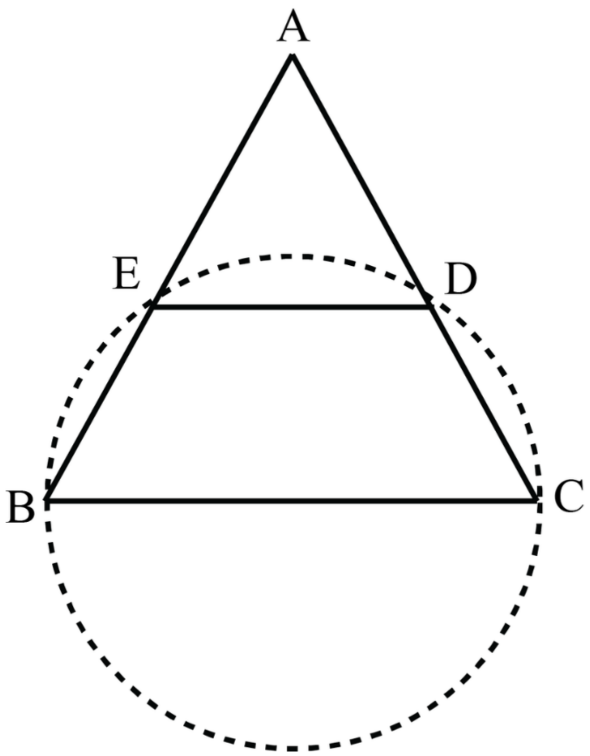
Therefore, the four points B, C , M and N are concyclic.

11. If a line is drawn parallel to the base of an isosceles triangle to intersect its equal sides, prove that the quadrilateral, so formed is cyclic.

Answer: Consider an isosceles triangle ABC with base BC

A line ED is drawn parallel to the base BC to intersect its equal sides AB and AC.

We have to prove that the quadrilateral so formed is cyclic.



Draw a circle that passes through the points B, C, D and E.

Considering triangle ABC,

AB = AC = equal sides of an isosceles triangle

We know that the angles opposite to the equal sides are equal.

∠ACB = ∠ABC -------------------------- (1)

Given, DE || BC

We know that the corresponding angles are equal.

So, ∠ADE = ∠ACB --------------------- (2)

Adding ∠EDC on both sides in (2),

∠ADE + ∠EDC = ∠ACB + ∠EDC

We know that the linear pair of angles is always supplementary.

So, ∠ADE + ∠EDC = 180°

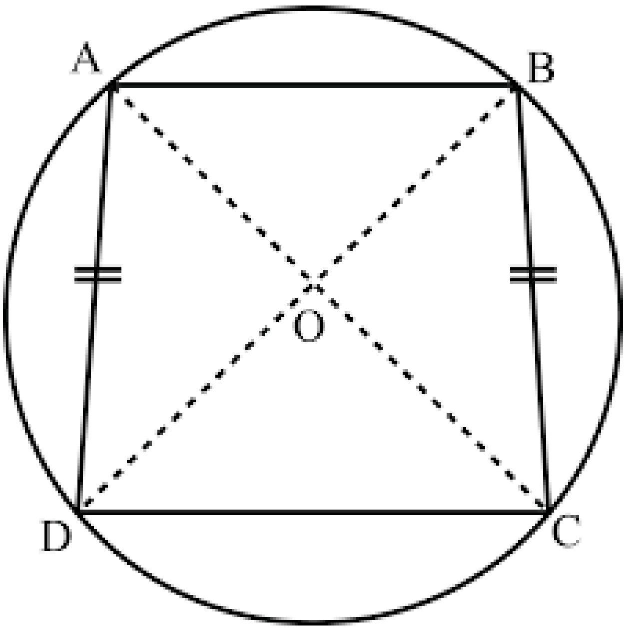
Now, ∠ACB + ∠EDC = 180°

This implies the sum of the opposite angles is equal to 180°

We know that the sum of the opposite angles is always supplementary in a cyclic quadrilateral.

Therefore, BCDE is a cyclic quadrilateral.

12. If a pair of opposite sides of a cyclic quadrilateral are equal, then prove that its diagonals are also equal.



Answer:

Consider a cyclic quadrilateral ABCD

A pair of opposite sides of a cyclic quadrilateral are equal.

Given, AD = BC

Considering triangle AOD and BOC,

We know that the same segment subtends equal angle to the circle

∠OAD = ∠OBC

Given, AD = BC

∠ODA = ∠OCB

ASA criterion states that two triangles are congruent, if any two angles and the side included between them of one triangle are equal to the corresponding angles and the included side of the other triangle.

By ASA criterion, the triangles AOD and BOC are similar.

△AOD ≊ △BOC

Adding triangle DOC on both sides,

△AOD + △DOC ≊ △BOC + △DOC

△ADC ≊ △BCD

The Corresponding Parts of Congruent Triangles are Congruent (CPCTC) theorem states that when two triangles are similar, then their corresponding sides and angles are also congruent or equal in measurements.

By CPCTC,

AC = BD

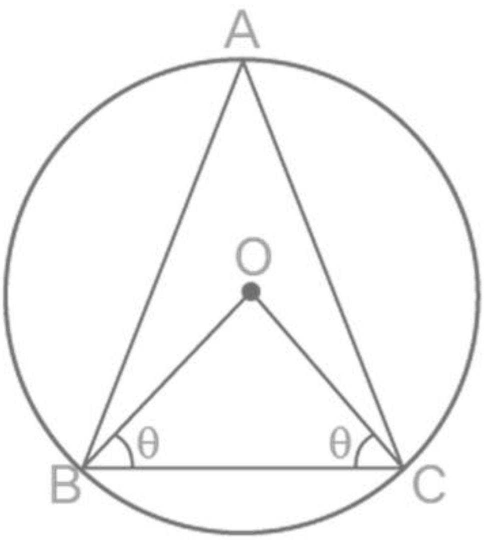
Therefore, it is proven that if a pair of opposite sides of a cyclic quadrilateral are equal, then its diagonals are also equal.

13. The circumcentre of the ΔABC is 0. Prove that ∠OBC + ∠BAC = 90°.

Answer: Given, ABC is a triangle

O is the circumcentre of the triangle

We have to prove that ∠OBC + ∠BAC = 90^0



Join BO and CO.

Let ∠OBC = ∠OCB = θ --------------------------- (1)

Considering triangle OBC,

By angle sum property,

∠BOC + ∠OCB + ∠CBO = 180^0

∠BOC + θ + θ = 180^0

∠BOC = 180 - $2θ^0$ --------------------------------- (2)

We know that in a circle the angle subtended by an arc at the centre is twice the angle subtended by it at the remaining part of the circle.

So, ∠BOC = 2 ∠BAC

∠BAC = 1/2 ∠BOC

From (2),

∠BAC = 180^0 - 2θ/2

∠BAC = 90^0- θ

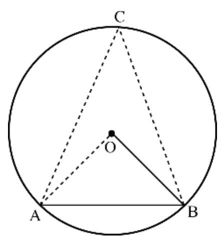
∠BAC + θ = 90^0

From (1),

∠BAC + ∠OBC = 90^0

Therefore, ∠OBC + ∠BAC = 90^0

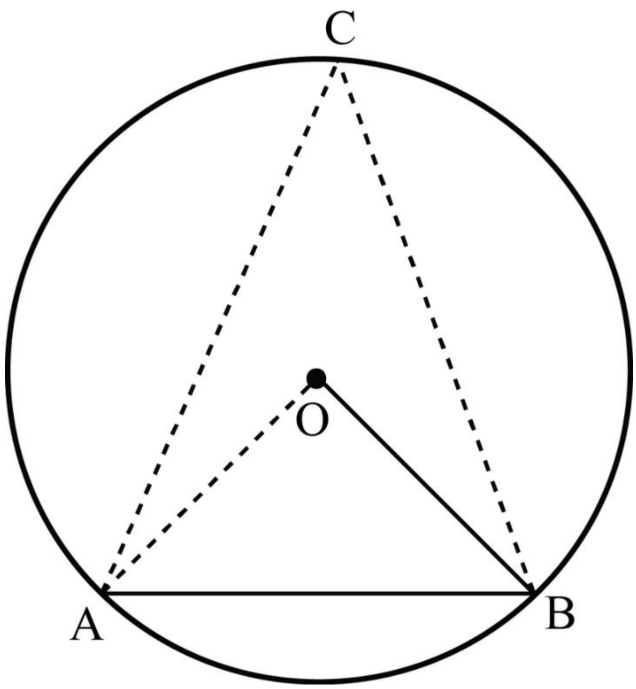
14. A chord of a circle is equal to its radius. Find the angle subtended by this chord at a point in major segment.



Answer: Given, AB is a chord of a circle

AB is equal to the radius of the circle.

So, AB = BO ------------------------ (1)



Join OA, AC and BC.

Radius of circle = OA = OB

So, OA = OB = AB

Considering triangle OAB,

OAB is an equilateral triangle

We know that each angle of an equilateral triangle is equal to 60 degrees.

So, ∠AOB = 60^0

We know that in a circle the angle subtended by an arc at the centre is twice the angle subtended by it at the remaining part of the circle.

So, ∠AOB = 2∠ACB

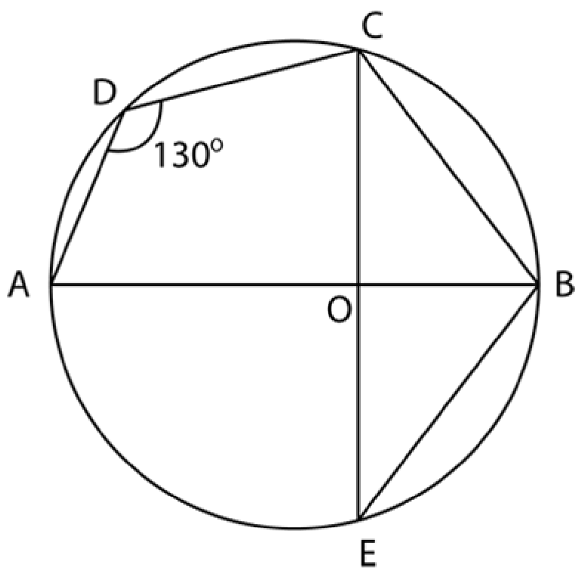
∠ACB = 1/2 ∠AOB

∠ACB = 60^0/2

∠ACB = 30^0

Therefore, the angle subtended by the chord AB at a point in major segment is 60^0.

15. In figure, ∠ADC = 130° and chord BC = chord BE. Find ∠CBE.



Answer: Given, ∠ADC = 130°

Chord BC = chord BE

We have to find ∠CBE

Considering triangle BCO and BEO,

Given, BC = BE

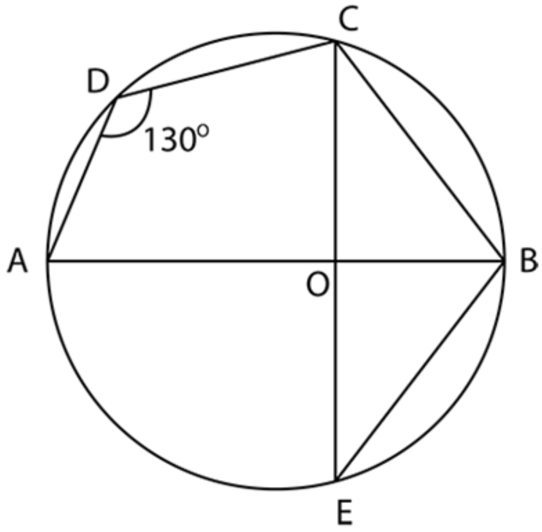
We know that the base angles of equal sides are also equal.

So, ∠BCO = ∠BEO

Common side = BO

By SAS criterion, the triangles BCO and BEO are similar.

By CPCTC,



∠CBO = ∠OBE ------------------------- (1)

From the figure,

ABCD is a cyclic quadrilateral since its points lie on a circle.

We know that the sum of the opposite angles of a cyclic quadrilateral is 180 degrees.

∠ADC + ∠CBA = 180^0

130^0 + ∠CBA = 180^0

∠CBA = 180^0 - 130^0

∠CBA = 50^0

From the figure,

∠CBA = ∠CBO

So, ∠CBO = 50^0

From (1),

∠OBE = 50^0

From the figure,

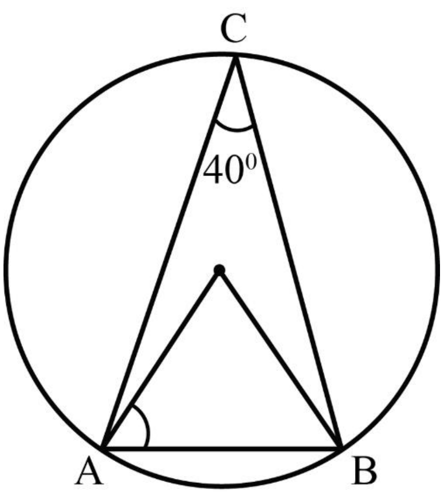
∠CBE = ∠CBO + ∠CBO

= 50^0 + 50^0

= 100^0

Therefore, ∠CBE = 100^0

16. In Fig.10.14, ∠ACB = 40^0. Find ∠OAB.



Answer: Given, ∠ACB = 40^0

We have to find ∠OAB

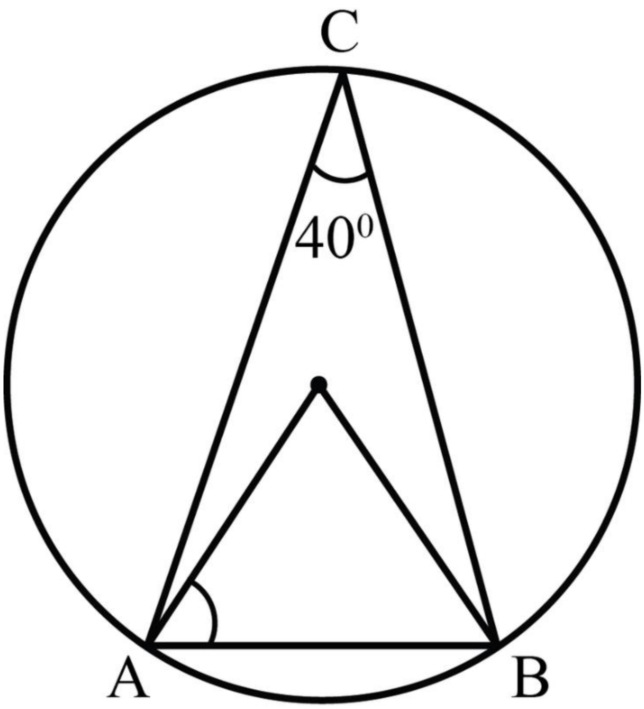
We know that in a circle the angle subtended by an arc at the centre is twice the angle subtended by it at the remaining part of the circle.

∠AOB = 2∠ACB

∠AOB = 2(40º)

∠AOB = 80^0----------------------- (1)

Considering triangle AOB,



OA = OB = radius of the circle

We know that the angles opposite to the equal sides are equal.

∠OBA = ∠OAB ------------------- (2)

By angle sum property of the triangle,

∠AOB + ∠OBA + ∠OAB = 180^0

From (1) and (2),

800 + ∠OAB + ∠OAB = 180^0

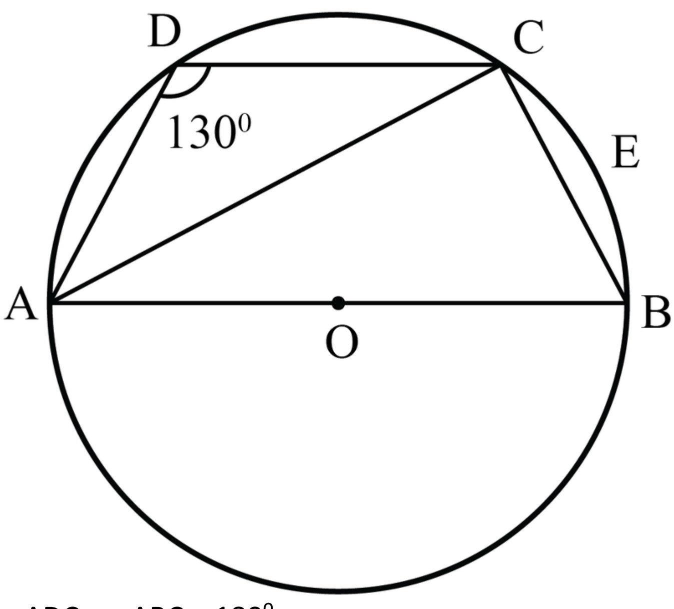
2∠OAB = 180^0 - 80^0

2∠OAB = 100^0

∠OAB = 100^0/2

Therefore, ∠OAB = 50^0

17. A quadrilateral ABCD is inscribed in a circle such that AB is a diameter and ∠ADC = 130°. Find ∠BAC.



Answer: Given, ABCD is a quadrilateral

Quadrilateral ABCD is inscribed in a circle

AB is the diameter of the circle

∠ADC = 130º

We have to find ∠BAC

Given, ABCD is a cyclic quadrilateral as it is inscribed in a circle with center O

We know that the sum of the opposite sides of a cyclic quadrilateral is 180 degrees.

∠ADC + ∠ABC = 180^0

130 + ∠ABC = 180^0

∠ABC = 180^0 - 130^0

∠ABC = 50^0 ------------------------ (1)

We know that the angle subtended by a diameter to the circle is a right angle

So, ∠ACB = 90^0 ------------------ (2)

In triangle ABC,

By angle sum property of a triangle,

∠BAC + ∠ACB + ∠ABC = 1800

From (1) and (2),

∠BAC + 90^0 + 50^0 = 180^0

∠BAC + 140^0 = 180^0

∠BAC = 180^0 - 140^0

Therefore, ∠BAC = 40^0

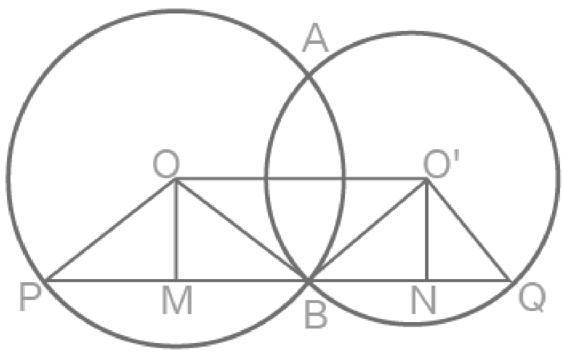
18. Two circles with centres O and O' intersect at two points A and B. A line PQ is drawn parallel to OO' through A(or B) intersecting the circles at P and Q. Prove that PQ = 2 OO'.

Solution:

Given, two circles with centres O and O' intersect at two points A and B

A line PQ is drawn parallel to OO' through A or B intersecting the circle at P and Q.

We have to prove that PQ = 2OO'



Join OO', OP, O'Q, OM and O'N

Considering triangle OPB,

Since OM is the perpendicular bisector of PB

BM = PM ----------------------- (1)

Considering triangle O'BQ,

Since O'N is the perpendicular bisector of BQ

BN = NQ ----------------------- (2)

Adding (1) and (2),

BM + BN = PM + NQ

Adding BM + BN on both sides,

BM + BN + BM + BN = PM + NQ + BM + BN

2(BM + BN) = (BM + PM) + (BN + NQ)

We know, OO' = MN = BM + BN

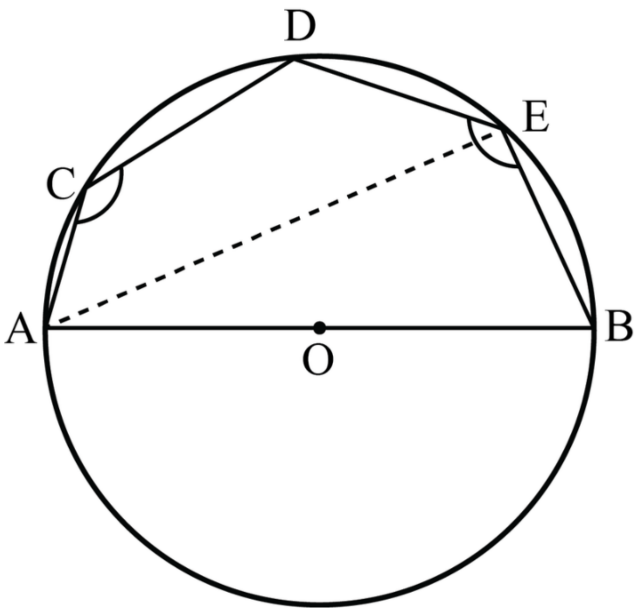
So, 2OO' = BP + BQ

Also, PQ = BP + BQ

2OO' = PQ

Therefore, PQ = 2OO'

19. In Fig.10.15, AOB is a diameter of the circle and C, D, E are any three points on the semi-circle. Find the value of ∠ACD + ∠BED.



Answer: Given, AOB is a diameter of the circle

C, D and E are any three points on the semicircle

We have to find the value of ∠ACD + ∠BED

Join AE

Since the points A, C, D and E lie on the circle

ACDE is a cyclic quadrilateral

We know that the sum of opposite angles of a cyclic quadrilateral is 180 degrees.

∠ACD + ∠AED = 180° ----------------- (1)

AB is the diameter of the circle

We know that the angle subtended by a diameter to the circle is a right angle

So, ∠AEB = 90° ------------------------- (2)

Adding (1) and (2),

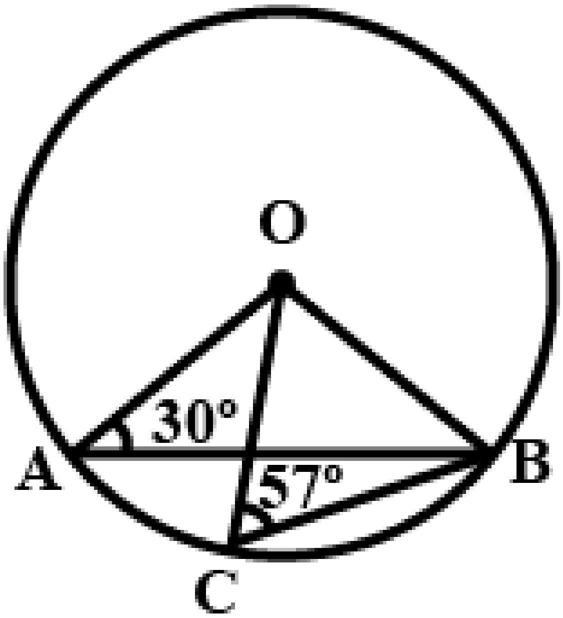
∠ACD + ∠AED + ∠AEB = 180° + 90°

From the figure,

∠BED = ∠AED + ∠AEB

Therefore, ∠ACD + ∠BED = 270°

20. In Fig. 10.16, ∠OAB = 30º and ∠OCB = 57º. Find ∠BOC and ∠AOC.



Answer:

Given, ∠OAB = 30^0

∠OCB = 57^0

We have to find ∠BOC and ∠AOC

Considering triangle AOB,

OA = Ob = radius of the circle

We know that the angles opposite to the equal sides are equal.

∠OBA = ∠BAO

Given, ∠OAB = 30^0

So, ∠OBA = 30^0

In triangle AOB,

By angle sum property of a triangle,

∠AOB + ∠OBA + ∠OAB = 180^0

∠AOB + 30^0 + 30^0 = 180^0

∠AOB + 60^0 = 180^0

∠AOB = 180^0 - 60^0-

∠AOB = 120^0 -------------------------- (1)

Considering triangle OCB,

OC = OB = radius of the circle

We know that the angles opposite to the equal sides are equal.

∠OBC = ∠OCB

Given, ∠OCB = 57^0

So, ∠OBC = 57^0

By angle sum property of a triangle,

∠COB + ∠OBC + ∠OCB = 180^0

∠COB + 57^0 + 57^0 = 180^0

∠COB + 114^0 = 180^0

∠COB = 180^0 - 114^0

∠COB = 66^0------------------------ (2)

From the figure,

∠AOB = ∠AOC + ∠COB

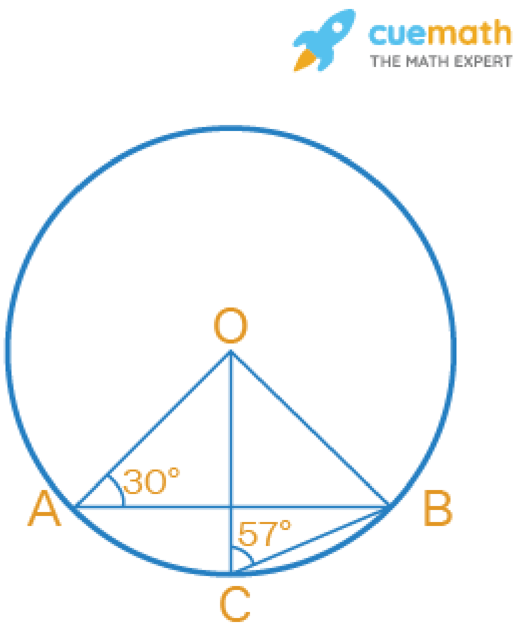
From (1) and (2),

120^0 = ∠AOC + 66^0

∠AOC = 120^0 - 66^0

Therefore, ∠AOC = 54^0

22. In Fig. 10.16, ∠OAB = 30º and ∠OCB = 57º. Find ∠BOC and ∠AOC.



Answer:

Given, ∠OAB = 30^0

∠OCB = 57^0

We have to find ∠BOC and ∠AOC

Considering triangle AOB,

OA = Ob = radius of the circle

We know that the angles opposite to the equal sides are equal.

∠OBA = ∠BAO

Given, ∠OAB = 30^0

So, $\angle OBA = 30^0$

In triangle AOB,

By angle sum property of a triangle,

$\angle AOB + \angle OBA + \angle OAB = 180^0$

$\angle AOB + 30^0 + 30^0 = 180^0$

$\angle AOB + 60^0 = 180^0$

$\angle AOB = 180^0 - 60^0$

$\angle AOB = 120^0$ -------------------------- (1)

Considering triangle OCB,

OC = OB = radius of the circle

We know that the angles opposite to the equal sides are equal.

$\angle OBC = \angle OCB$

Given, $\angle OCB = 57^0$

So, $\angle OBC = 57^0$

By angle sum property of a triangle,

$\angle COB + \angle OBC + \angle OCB = 180^0$

$\angle COB + 57^0 + 57^9 = 180^0$

$\angle COB + 114^0 = 180^0$

$\angle COB = 180^0 - 114^0$

$\angle COB = 66^0$------------------------ (2)

From the figure,

$\angle AOB = \angle AOC + \angle COB$

From (1) and (2),

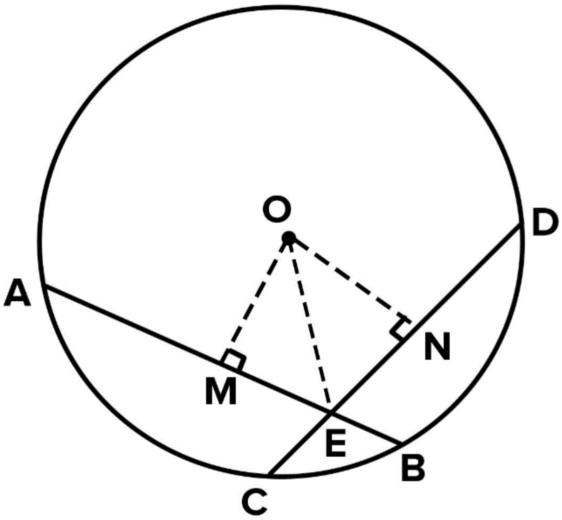
$120^0 = \angle AOC + 66^0$

$\angle AOC = 120^0 - 66^0$

Therefore, $\angle AOC = 54^0$

Exercise 10.4

1. If two equal chords of a circle intersect, prove that the parts of one chord are separately equal to the parts of the other chord.



Answer: Consider two equal chords AB and CD of a circle

The chords AB and CD meet at a point E

We have to prove that the parts of one chord are separately equal to the parts of the other chord.

Draw OM perpendicular to AB and ON perpendicular to CD

Join OE where O is the centre of the circle.

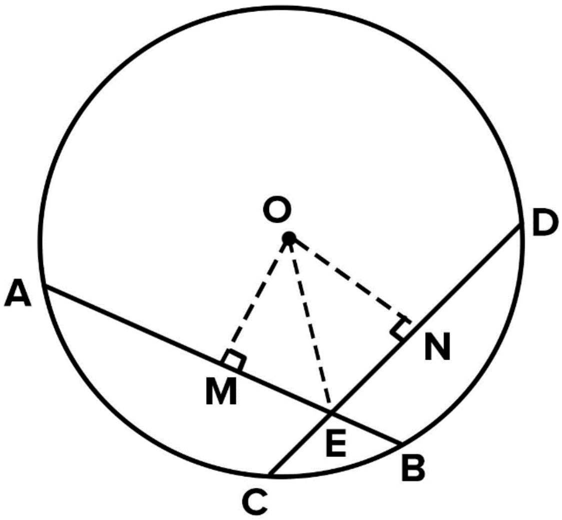
Considering triangles OME and ONE,

We know that the equal chords are equidistant from the centre of a circle.

OM = ON

Common side = OE

∠OME = ∠ONE = 90°



RHS criterion states that if the hypotenuse and side of one right-angled triangle are equal to the hypotenuse and the corresponding side of another right-angled triangle, the two triangles are congruent.

By RHS criterion, the triangles OME and ONE are similar.

The Corresponding Parts of Congruent Triangles are Congruent (CPCTC) theorem states that when two triangles are similar, then their corresponding sides and angles are also congruent or equal in measurements.

By CPCTC,

EM = EN --------------------------- (1)

Now, AB = CD

Dividing both sides by 2,

AB/2 = CD/2

AM = CN ------------------------------- (2)

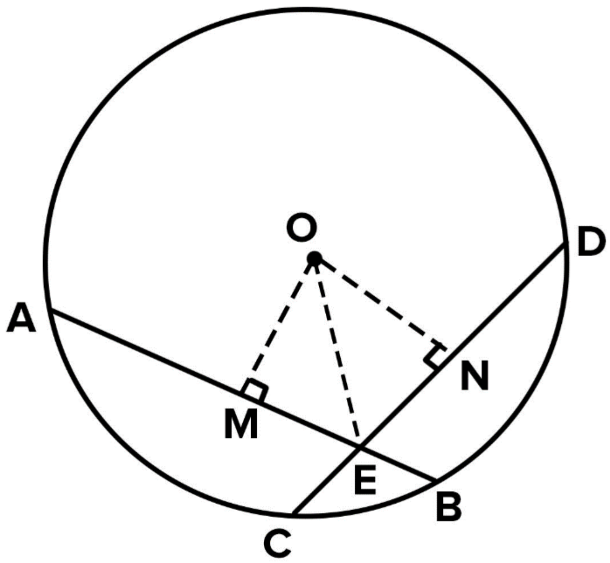
We know that the perpendicular drawn from the centre of the circle to a chord bisects the chord.

So, AM = MB

CN = DN

Adding (1) and (2),

EM + AM = EN + CN



From the figure,

EM + AM = AE

EN + CN = CE

So, AE = CE ---------------------------- (3)

Now, AB = CD

Subtracting AE from both sides,

AB - AE = CD - AE

From the figure,

AB - AE = BE

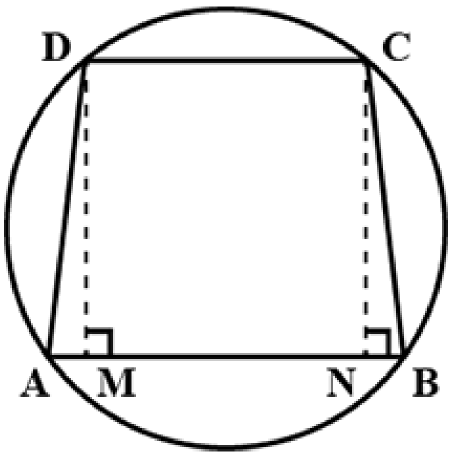
So, BE = CD - AE

From (3),

BE = CD - CE

Therefore, BE = DE

2. If non-parallel sides of a trapezium are equal, prove that it is cyclic.



Answer:

We know that, if the sum of a pair of opposite angles of a quadrilateral is 180°, the quadrilateral is cyclic.

Draw a trapezium ABCD with AB || CD

AD and BC are the non-parallel sides that are equal. AD = BC. Draw AM ⊥ CD and BN ⊥ CD.

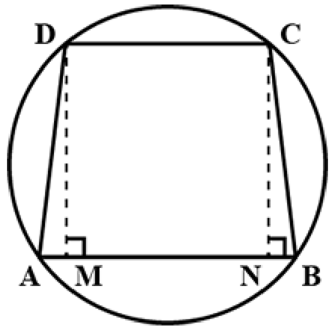
Consider ΔAMD and ΔBNC

AD = BC (Given)

∠AMD = ∠BNC (90°)

AM = BN (Perpendicular distance between two parallel lines is same)

By RHS congruence, ΔAMD ≅ ΔBNC.



Using CPCT, ∠ADC = ∠BCD.....(1)

∠BAD and ∠ADC are on the same side of transversal AD.

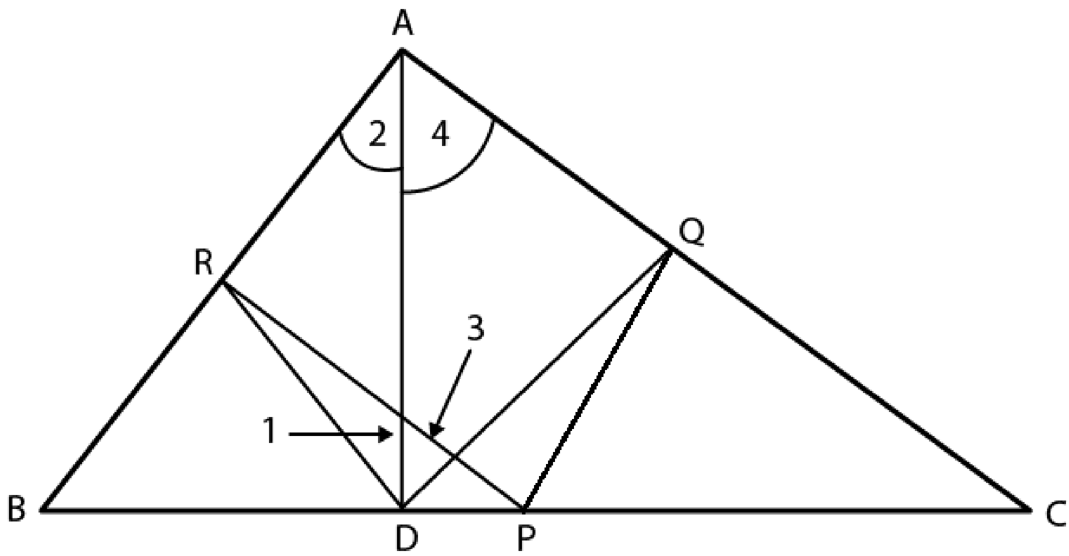
∠BAD + ∠ADC = 180°

∠BAD + ∠BCD = 180° [From equation(1)]

This equation proves that the sum of opposite angles is supplementary. Hence, ABCD is a cyclic quadrilateral.

3. If P, Q and R are the mid-points of the sides BC, CA and AB of a triangle and AD is the perpendicular from A on BC, prove that P, Q, R and D are concyclic.

Answer: To prove: R, D, P and Q are concyclic.



Construction: Join RD, QD, PR and PQ.

RP joins the mid-point of AB, i.e., R, and the mid-point of BC, i.e., P.

Using midpoint theorem,

RP||AC

Similarly,

PQ||AB.

So, we get,

ARPQ is a parallelogram.

So, ∠RAQ = ∠RPQ [Opposite angles of a ||gm]...(1)

ABD is a right angled triangle and DR is a median,

RA = DR and ∠1 = ∠2 …(2)

Similarly ∠3 = ∠4 …(3)

Adding equations (2) and (3),

We get,

∠1+ ∠3 = ∠2 +∠4

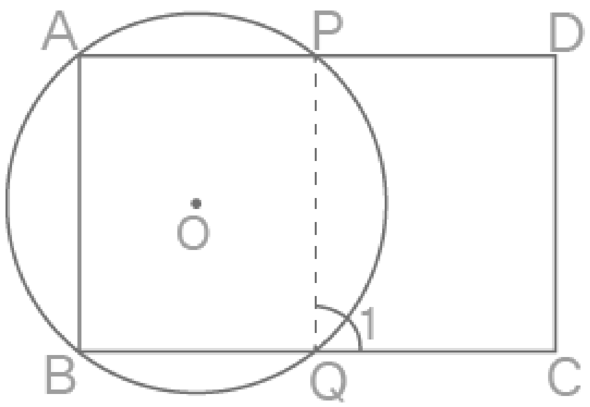
⇒ ∠RDQ = ∠RAQ

∠RPQ [Proved above]

Since ∠D and ∠P are subtended by RQ on the same side of it, we get the points R, D, P and Q concyclic.

Hence, R, D, P and Q are concyclic.

4. ABCD is a parallelogram. A circle through A, B is so drawn that it intersects AD at P and BC at Q. Prove that P, Q, C and D are concyclic.



Answer: Given, ABCD is a parallelogram

A circle through A, B is drawn so that it intersects AD at P and BC at Q.

We have to prove that P, Q, C and D are concyclic.

Join PQ

By exterior angle property of a cyclic quadrilateral,

∠1 = ∠A

We know that the opposite angles of a parallelogram are equal

∠A = ∠C

So, ∠1 = ∠C --------------------------- (1)

Now, AD || BC and PQ is a transversal,

We know that the sum of co interior angles on the same side is 180 degrees.

So, ∠C+ ∠D = 180°

From (1),

∠1+ ∠D = 180°

We know that the sum of opposite angles of a cyclic quadrilateral is equal to 180 degrees.

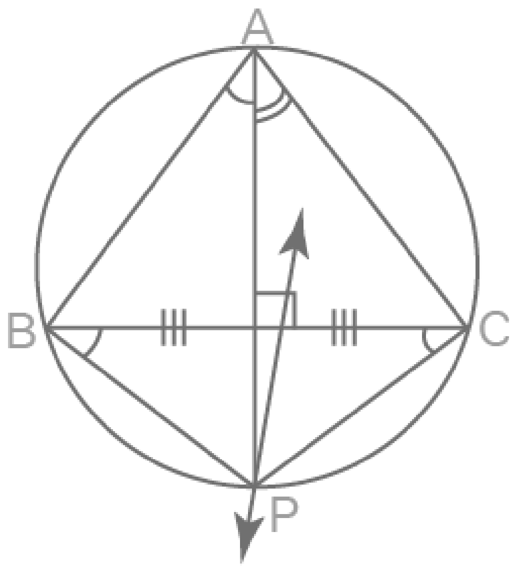
So, the quadrilateral QCDP is cyclic.

Therefore, the points P, Q, C and D are con-cyclic.

5. Prove that angle bisector of any angle of a triangle and perpendicular bisector of the opposite side, if intersect, they will intersect on the circumcircle of the triangle.

Answer:

Consider a triangle ABC



We have to prove that the angle bisector of ∠A and perpendicular bisector of BC intersect on the circumcircle of ΔABC.

Let the angle bisector of ∠A intersect circumcircle of ΔABC at P.

Join BP and CP.

We know that the angles in the same segment of a circle are equal

So, ∠BAP = ∠BCP

Since, AP is bisector of ∠A.

∠BAP = ∠BCP

∠A = ∠BAP + ∠BCP

So, ∠BAP = ∠BCP = 1/2 ∠A -------------------------- (1)

Similarly, ∠PAC = ∠PBC = 1/2 ∠A ------------------- (2)

From (1) and (2),

∠BCP = ∠PBC

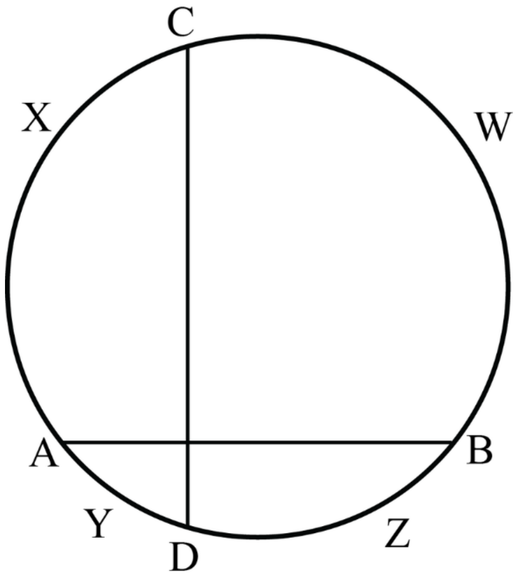
We know that, if the angles subtended by two Chords of a circle at the centre are equal, then the chords are equal.

So, BP = CP

P lies on the perpendicular bisector of BC.

Therefore, angle bisector of ∠A and perpendicular bisector of BC intersect on the circumcircle of ΔABC.

6. If two chords AB and CD of a circle AYDZBWCX intersect at right angles (see Fig.10.18), prove that arc CXA + arc DZB = arc AYD + arc BWC = semi-circle.



Answer:

Given, two chords AB and CD intersect at right angles in a circle AYDZBWCX

We have to prove arc CXA + arc DZB = arc AYD + arc BWC = Semi-circle.

Draw a diameter EF parallel to the CD having centre M.

Since, CD||EF

arc EC = arc DF --------------------- (1)

arc ECXA = arc EWB

arc AF = arc BF ---------------------- (2)

We know that arc ECXAYDF = semicircle

Now, arc EA + arc AF = semicircle

From (2),

arc EC + arc CXA + arc BF = semicircle

From (1),

arc DF + arc CXA + arc BF = semicircle

arc DF + arc BF + arc CXA = semicircle

From the figure,

arc DF + arc BF = arc DZB

So, arc DZB + arc CXA = semicircle

We know that the circle divides itself into two semicircles, the remaining portion of the circle is also equal to the semicircle.

Therefore, arc AYD + arc BWC = semicircle.

7. If ABC is an equilateral triangle inscribed in a circle and P be any point on the minor arc BC which does not coincide with B or C, prove that PA is angle bisector of ∠BPC.

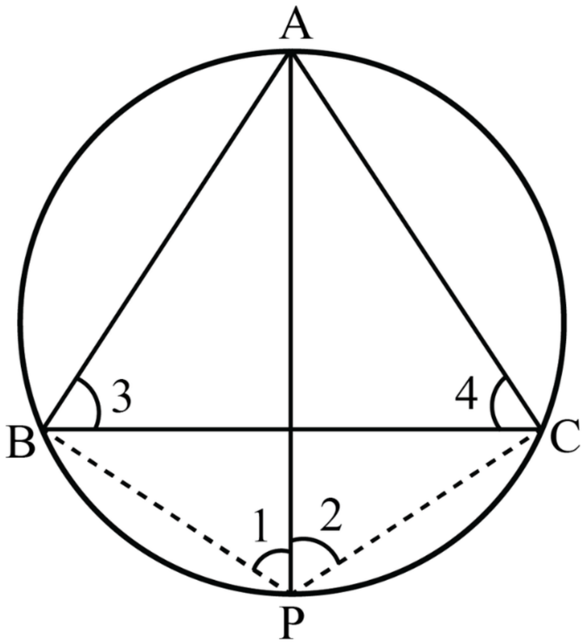
Answer:

Given, ABC is an equilateral triangle inscribed in a circle

P is any point on the minor arc BC which does not coincide with B or C

We have to prove that PA is the angle bisector of ∠BPC.

Join PB and PC



We know that in an equilateral triangle each angle is equal to 60 degrees.

∠3 = ∠4 = 60°

We know that angles in the same segment of a circle are equal.

Considering segment AB

∠1 = ∠4 = 60° ----------------------------- (1)

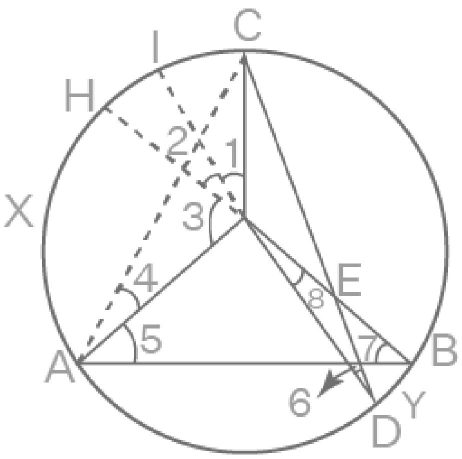
Considering segment AC

∠2 = ∠3 = 60° --------------------------- (2)

So, ∠1 = ∠2 = 60°

Therefore, PA is the bisector of ∠BPC.

8. In Fig. 10.19, AB and CD are two chords of a circle intersecting each other at point E. Prove that ∠AEC = 1/2 (Angle subtended by arc CXA at centre + angle subtended by arc DYB at the centre).



Answer:

Given, AB and CD are two chords of a circle intersecting each other at point E.

We have to prove that ∠AEC = 1/2 (Angle subtended by arc CXA at centre + angle subtended by arc DYB at the centre).

Extend the line OD and OB at the points I and H on the circle.

Join AC

We know that the angle subtended by an arc at the centre of a circle is twice the angle subtended by it at the remaining part of the circle.

∠1 = 2∠6 ------------------- (1)

∠3 = 2∠7 ------------------ (2)

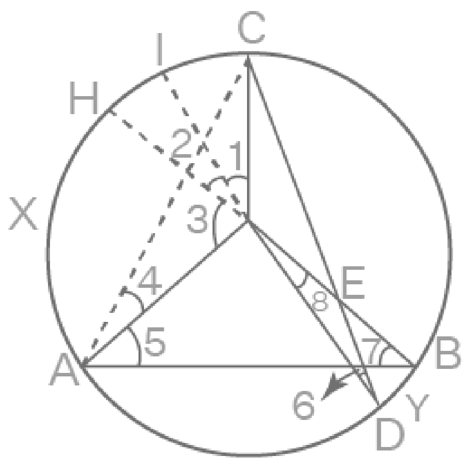
In triangle AOC,

OC = OA = radius of the circle

We know that the angles opposite to the equal sides are equal.

∠OCA = ∠4

By angle sum property of a triangle,



∠AOC + ∠OCA + ∠4 = 180°

∠AOC + ∠4 + ∠4 = 180°

∠AOC + 2∠4 = 180°

∠AOC = 180° - 2∠4 --------------------- (3)

In triangle OCD,

OC = OD

We know that the angles opposite to the equal sides are equal.

∠ECO = ∠6 ---------------------------------- (4)

Similarly, in triangle ∠5 = ∠7 ----------- (5)

In triangle AEC,

By angle sum property of a triangle,

∠AEC + ∠EAC + ∠ECA = 180°

∠AEC = 180° - (∠EAC + ∠ECA)

Now, ∠AEC = 180° - [(∠ECO + ∠OCA) + (∠CAO + ∠OAE)]

From (4),

∠AEC = 180° - [(∠6 + ∠4) + (∠4 + ∠5)]

∠AEC = 180° - (2∠4 + ∠6 + ∠5)

∠AEC = 180° - 2∠4 - ∠6 - ∠5

From (3),

∠AEC = ∠AOC - ∠6 - ∠5

From (5),

∠AEC = ∠AOC - ∠6 - ∠7

From (1) and (2),

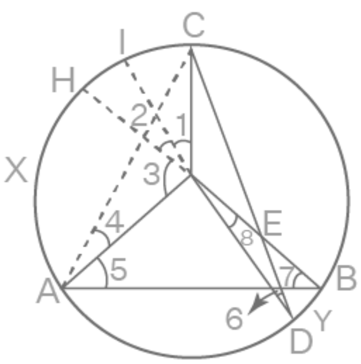
∠AEC = ∠AOC - 1/2 ∠1 - 1/2 ∠3

By adding and subtracting ∠2/2,

∠AEC = ∠AOC - ∠1/2 - ∠2/2 - ∠3/2 + ∠2/2

= ∠AOC - 1/2 (∠1 + ∠2 + ∠3) + ∠2/2

We know that the vertically opposite angles are equal



∠2 = ∠8

So, ∠AEC = ∠AOC - 1/2 (∠1 + ∠2 + ∠3) + ∠8/2

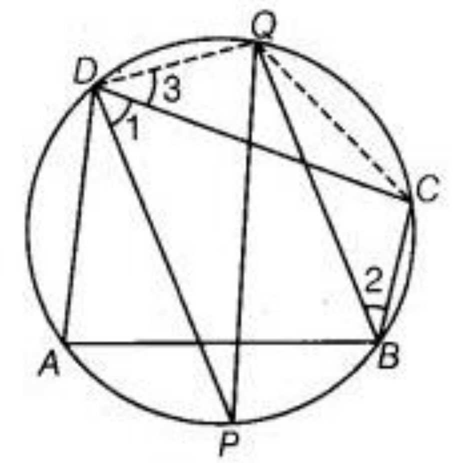
= ∠AOC - ∠AOC/2 + ∠DOB/2

∠AEC = ∠AOC/2 + ∠DOB/2

∠AEC = 1/2 (∠AOC + ∠DOB)

Therefore, ∠AEC = 1/2 (Angle subtended by arc CXA at centre + angle subtended by arc DYB at the centre).

9. If bisectors of opposite angles of a cyclic quadrilateral ABCD intersect the circle, circumscribing it at the points P and Q, prove that PQ is a diameter of the circle.



Answer: Given, ABCD is a cyclic quadrilateral

The bisectors of opposite angles of a cyclic quadrilateral ABCD intersect the circle, circumscribing it at the points P and Q.

We have to prove that PQ is a diameter of the circle.

Join QD and QC.

We know that the sum of opposite angles of a cyclic quadrilateral is equal to 180 degrees.

∠CDA + ∠CBA = 180°

Dividing by 2 on both sides,

1/2 ∠CDA + 1/2 ∠CBA = 1/2 (180°)

Given, the bisectors of opposite angles of a cyclic quadrilateral ABCD intersect the circle

∠1 = 1/2 ∠CDA

∠2 = 1/2 ∠CBA

So, ∠1 + ∠2 = 90° --------------------------- (1)

We know that the angles in the same segment of a circle are equal.

Considering segment QC,

∠2 = ∠3 --------------------------------------- (2)

From (1) and (2),

∠1 + ∠3 = 90°

∠PDQ = 90°

We know that the diameter of the circle subtends a right angle at the circumference

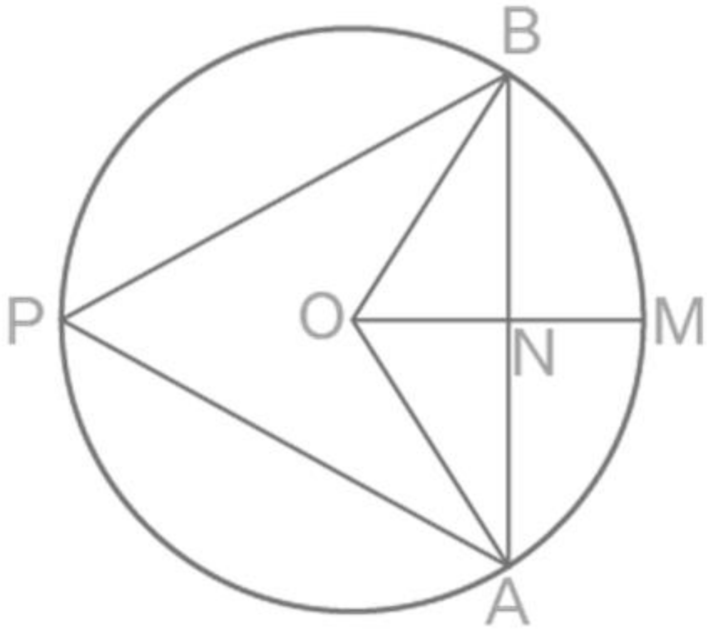
Therefore, PQ is the diameter of a circle.

10. A circle has a radius √2 cm. It is divided into two segments by a chord of length 2 cm. Prove that the angle subtended by the chord at a point in the major segment is 45^0.

Answer: Given, a circle has a radius of √2 cm

The circle is divided into two segments by a chord of length 2 x m

We have to prove that the angle subtended by the chord at a point in the major segment is 45°



Draw a circle having centre O

Let AB = 2 cm be a chord of a circle

A chord AB is divided by the line OM into two equal segments.

Now, AN = NB = 1 cm

OB = √2 cm

In triangle ONB,

By Pythagorean theorem,

$OB^2 = ON^2 + NB^2$

$(\sqrt{2})^2 = ON^2 + (1)^2$

$ON^2 = 2 - 1$

ON = 1 cm

Since ON is the perpendicular bisector of the chord AB

∠ONB = 90°

So, ∠NOB = ∠NBO = 45°

Similarly, ∠AON = 45°

Now, ∠AOB = ∠AON + ∠NOB

= 45° + 45°

= 90°

We know that the chord subtends an angle to the circle equal to half the angle subtended by it to the centre.

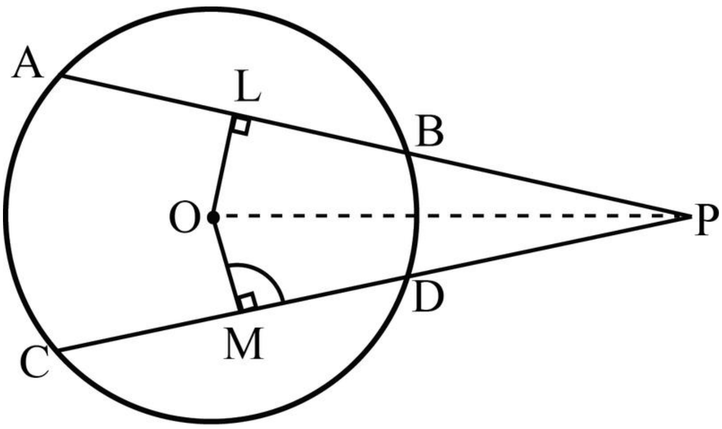
∠APB = 1/2 ∠AOB

= 1/2 (90°)

= 45°

Therefore, ∠APB = 45°

11. Two equal chords AB and CD of a circle when produced intersect at a point P. Prove that PB = PD.



Answer: Given, AB and CD are two equal chords of a circle.

The chords AB and CD intersect at a point P.

We have to prove that PB = PD

Join OP

Draw OL perpendicular to AB and OM perpendicular to CD

We know that the two equal chords are equidistant from the centre of a circle.

Now, AB = CD

OL = OM

Considering triangles OLP and OMP,

OL = OM

∠OLP = ∠OMP

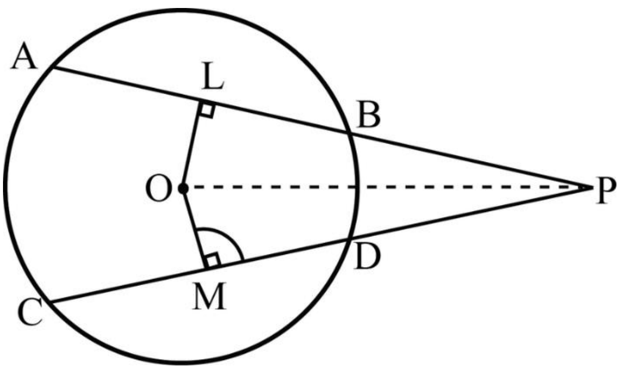
Common side = OP

By RHS criterion, the triangles OLP and OMP are similar.

By CPCTC,

LP = MP -------------------------- (1)

We know that the perpendicular drawn from centre to the circle bisects the chord.



AL = LB

CM = MD

Now, AB = CD

Dividing by 2 on both sides,

1/2 AB = 1/2 CD

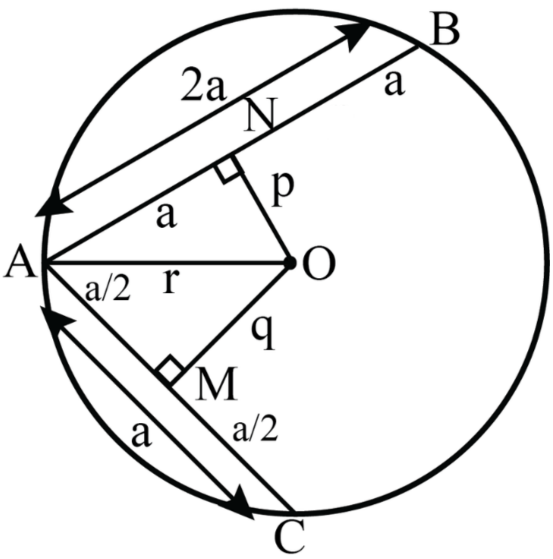
So, BL = DM ---------------------- (2)

Subtracting (1) and (2),

LP - BL = MP - DM

Therefore, PB = PD

12. AB and AC are two chords of a circle of radius r such that AB = 2AC. If p and q are the distances of AB and AC from the centre, prove that $4q^2 = p^2 + 3r^2$.



Answer:

Given, AB and AC are two chords of a circle of radius r

AB = 2AC

The distance of AB and AC from the centre are p and q.

We have to prove that $4q^2 = p^2 + 3r^2$

Let AC = a

AB = 2a

A perpendicular is drawn to the chords AC and AB from centre O at M and N

AM = MC = a/2

AN = NB = a

In triangle OAM,

By Pythagorean theorem,

$AO^2 = AM^2 + MO^2$

$AO^2 = (a/2)^2 + q^2$ ------------------------ (1)

In triangle OAN,

By Pythagorean theorem,

$AO^2 = AN^2 + NO^2$

$AO^2 = a^2 + p^2$ ----------------------- (2)

In triangle OAN,

By Pythagorean theorem,

$r^2 = a^2 + p^2$ --------------------------- (3)

From (1) and (2),

$(a/2)^2 + q^2 = a^2 + p^2$

$a^2/4 + q^2 = a^2 + p^2$

$a^2 + 4q^2 = 4a^2 + 4p^2$

$4q^2 = 4a^2 - a^2 + 4p^2$

$4q^2 = 3a^2 + 4p^2$

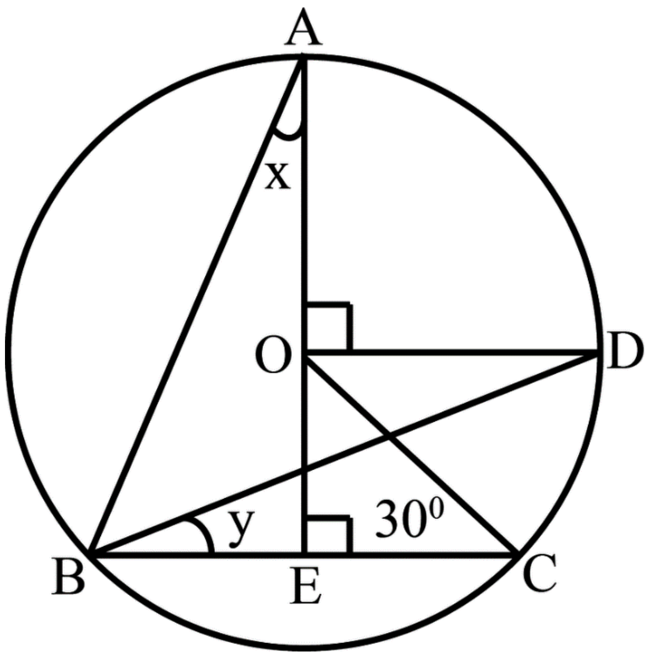
$4q^2 = p^2 + 3a^2 + 3p^2$

$4q^2 = p^2 + 3(a^2 + p^2)$

From (3),

Therefore, $4q^2 = p^2 + 3r^2$

13. In Fig. 10.20,O is the centre of the circle, ∠BCO = 30°. Find x and y.



Answer:

Given, O is the centre of the circle

∠BCO = 30°

We have to find x and y.

Join OB and AC

In triangle BOC,

OC = OB = radius of the circle

We know that the angles opposite to equal sides are equal

∠OBC = ∠OCB = 30°

By angle sum property of a triangle,

∠BOC + ∠OBC + ∠OCB = 180°

∠BOC = 180° - (∠OBC + ∠OCB)

= 180° - (30° + 30°)

= 180° - 60°

= 120°

So, ∠BOC = 120°

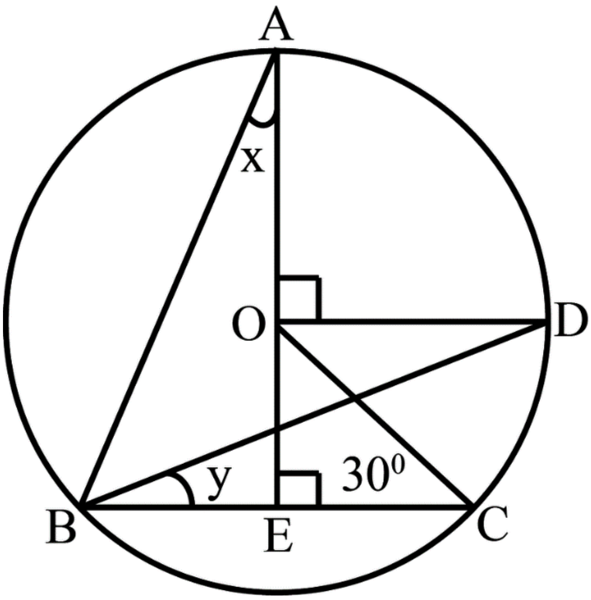
We know that an angle subtended by an arc at the centre is twice the angle subtended by it at the remaining part of the circle.

∠BOC = 2∠BAC

∠BAC = 1/2 ∠BOC

∠BAC = 120°/2

∠BAC = 60°



AE is the angle bisector of ∠A

So, ∠BAE = ∠CAE = 30°

∠BAE = x

So, x = 30°

In triangle ABE,

By angle sum property of a triangle,

∠BAE + ∠EBA + ∠AEB = 180°

30° + ∠EBA + 90° = 180°

∠EBA = 180° - 120°

∠EBA = 60°

We know that the chord subtends an angle to the circle equal to half the angle subtended by it to the centre.

∠ABD = 1/2 ∠AOD

Now, ∠ABD + y = 60°

1/2 ∠AOD + y = 60°

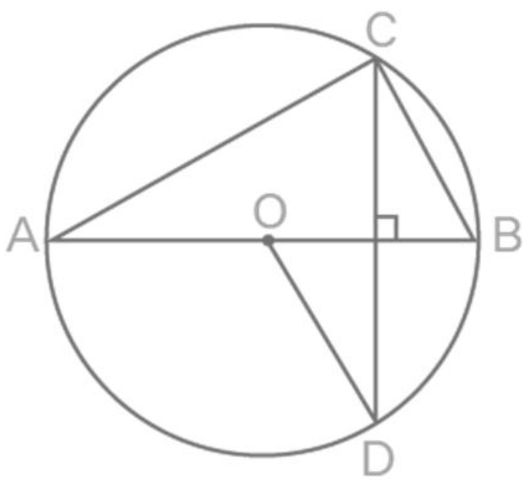
90°/2 + y = 60°

45° + y = 60°

y = 60° - 45°

y = 15°

14. In Fig. 10.21, O is the centre of the circle, BD = OD and CD ⊥ AB. Find ∠CAB.



Answer:

Given, O is the centre of the circle

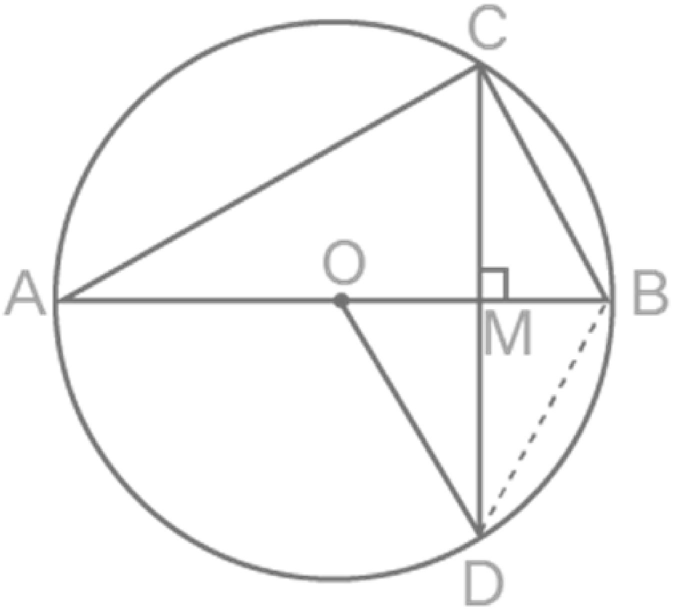
BD = OD

CD ⊥ AB

We have to find ∠CAB

In triangle OBD,

Given, BD = OD



OD = OB = radius of the circle

OB = OD = BD

Therefore, ODB is an equilateral triangle

We know that in an equilateral triangle each angle is equal to 60 degrees.

∠BOD = ∠OBD = ∠ODB = 60°

In triangles MBC and MBD,

Common side = MB

∠CMB = ∠BMD = 90°

We know that in a circle any perpendicular drawn on a chord also bisects the chord.

CM = MD

SAS criterion states that if two sides of one triangle are proportional to two sides of another triangle and their included angles are congruent, then the triangles are similar.

By SAS criterion, the triangles MBC and MBD are similar.

The Corresponding Parts of Congruent Triangles are Congruent (CPCTC) theorem states that when two triangles are similar, then their corresponding sides and angles are also congruent or equal in measurements.

By CPCTC,

∠MBC = ∠MBD

∠MBC = ∠OBD = 60°

Since AB is a diameter of a circle

∠ACB = 90°

In triangle ACB,

By angle sum property of a triangle,

∠CAB = ∠CBA + ∠ACB = 180°

∠CAB + 60° + 90° = 80°

∠CAB + 60° = 180° - 90°

∠CAB = 90° - 60°

Therefore, ∠CAB = 30°